The hidden SUSY face of QCD

I

G. Veneziano @

How and where to go beyond the Standard Model

(Erice, Aug. 29 - Sept. 7, 2004)
Purpose of the two lectures*

* based on work done at CERN with A. Armoni and M. Shifman

(hep-th/0302163, 0307097, 0309013)

and reviewed in ASV, hep-th/0403071
Part I

1. Large-N expansions in QCD: a reminder
2. QCD_F vs. QCD_OR
3. Planar equivalence in perturbation theory and beyond
Part II

1. A quick review of SYM
2. SUSY relics in $N_f=1$ QCD
3. Analytic estimate of $\langle \psi \psi \rangle$ in QCD
4. Extensions
I.1 Large-N expansions *) in QCD: a reminder

*) For those wishing to go deeper, S. Coleman (Erice 197?) gave an excellent full course on large N
Planar, quenched limit ('tHooft, 1974)
= $1/N$ expansion @ fixed $\lambda = g^2N$ and $N_f$
Leading diagrams can be obtained easily by
using 'tHooft's single/double-line notation
for quarks/gluons in SU(N) ~ U(N)

Corrections: $O(N_f/N_c)$ from q-loops,
$O(1/N_c^2)$ from non-planar diagrams
A simpler example and a traffic analogue

\[ g^6 N^4 \sim N \]
Properties at leading order

1. Resonances have zero width
2. U(1) problem not solved, but...
3. Multiparticle production not allowed
Resonances have zero width

Can be seen in two ways:

1. Finite width comes from decay of $qq\bar{q}$ meson into (at least) two particles. That means creating (at least) one $qq\bar{q}$ pair, a subleading process at large $N$

2. The coupling of three mesons can be easily shown to be $O(N^{-1/2})$, hence $\Gamma = O(N^{-1})$

3. More generally, in terms of a canonical $\phi$, $S_{\text{eff}} \sim N F(\phi N^{-1/2}) = \phi^2 + N^{-1/2} \phi^3 + ..$
U(1) problem not solved, but..

- The bad news: at leading order nothing distinguishes the non-flavour-singlet pseudoscalar meson ($\pi$) from the flavour-singlet pseudoscalar meson ($\eta'$). They are both massless in the chiral limit $m_q \to 0$. This is the U(1)-problem.

- The good news: at next-to-leading order the $\eta'$ gets a mass that can be predicted (WV) in terms of $F_{\pi}$ and of the so-called topological susceptibility of the pure YM theory. Latter can be computed on the lattice (e.g. using GW fermions) and leads to a good value for $m_{\eta'}$ and for the pseudoscalar mixing angle.
Multiparticle production not allowed

The reason is obviously the same as the one for $\Gamma = 0$
To conclude on 't Hooft expansion

Theoretically, if not phenomenologically, appealing: should give the tree-level of some string theory

Unfortunately, even in 't Hooft's limit QCD proved hard to solve, except in $D=2$

('t Hooft 1974 + ...)

Planar unquenched limit (GV '74--'76)

\[ = \frac{1}{N} \text{expansion} \at \text{fixed} \lambda = g^2N \text{and} \frac{N_f}{N} \]

Leading diagrams obtained easily by using one colour+one flavour line for quarks. Since flavour line NOT coupled to gluons, leading diagrams now include “empty” q-loops

Corrections: \( O\left(\frac{1}{N^2}\right) \) from non-planar diagrams, hence name of topological expansion
Properties of leading order in TE

1. Resonance widths are $O(1)$

2. $U(1)$ problem solved, but..

3. Multiparticle production allowed
Widths become $O(1)$

- Quite obvious since quark loops are not suppressed. Indeed:

  $$\Gamma = O(N_f/N) = 0(1)$$
U(1) problem solved to leading order, but...

We have no way to relate the $\eta'$ mass to something else that we can compute.

Success of WV formula*) becomes accidental since it should suffer $O(1)$ corrections.

*) Better justified through a small-$N_f$ expansion?
Multiparticle production allowed

Probably the most interesting use of the TE

- The planar diagrams generate Regge-pole-like behaviour (including the vacuum trajectory, the Pomeron, whose intercept naturally comes near 1)
- $1/N$-suppressed non-planar corrections generate Regge cuts in agreement with Gribov’s RFT
- The resulting RFT is naturally of the super-critical type ($\alpha(0)-1 > g_p^2$), which is experimentally favoured
- Developed into the so-called dual parton model of soft hadronic processes (C&TTV, K&tM,CIT)
Conclusion: topological expansion perhaps phenomenologically more appealing than 'tHooft's but even harder to solve...
I.2 $QCD_F$ vs. $QCD_{OR}$
Basic (though trivial) Observation

We can generalize QCD to an SU(N) gauge group in different ways by playing with matter representation.

So far we considered QCD_F, i.e. we kept quarks in fundamental + antifundamental (N +N*)

The one we shall consider now is called, for stringy reasons, QCD_OR (OR for Orientifold: see e.g. P.Di Vecchia et al. hep-th/0407038)
In simpler terms: put quarks in the 2-index-antisymmetric (AS)-tensor rep. of SU(N) + its complex conjugate

\[ \psi_\alpha \rightarrow \psi_{\alpha\beta} = - \psi_{\beta\alpha} \quad (\alpha = 1, 2, N) \]

Total number of left-handed quarks = \( N_f N(N-1) \)

As in 'tHooft’s expansion \( \lambda = g^2 N \) and \( N_f \) are both kept fixed

NB. For \( N=3 \) this is still ordinary QCD
Leading diagrams are planar, and include “filled” q-loops since the fermions too have two colour lines.
Properties at leading order

Widths are zero, no particle production

U(1) problem solved, since anomaly is a leading effect

Phenomenologically interesting?
Theoretically more manageable? Yes, I claim.
Let us Compare $\text{QCD}_F$ to $\text{QCD}_{OR}$

- They certainly agree for $N = 3$
- How do they differ at smaller $N$?
- How do they differ at large $N$?

Let us start by doing some “theoretical phenomenology” i.e. by comparing some standard perturbative quantities in the two theories
<table>
<thead>
<tr>
<th>th coeff</th>
<th>YM</th>
<th>$QCD_F$</th>
<th>$QCD_{OR}$</th>
<th>Large-N, $N_f=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>$11N/3$</td>
<td>$(11N-2N_f)/3$</td>
<td>$(11N-2(N-2)N_f)/3$</td>
<td>$3N$</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$17N^2/3$</td>
<td>$17N^2/3 - N_f(13N/6 - 1/2N)$</td>
<td>$17N^2/3 - N_f(N-2)x\left(5N + 3(N-2)(N+1)/N\right)/3$</td>
<td>$3N^2$</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>$X$</td>
<td>$3(N^2-1)/2N$</td>
<td>$3(N-2)(N+1)/N$</td>
<td>$3N$</td>
</tr>
</tbody>
</table>

$QCD_{OR}$ as an interpolating theory:
@ $N=2$ it coincides with pure YM (fermions decouple)
@ $N=3$ it coincides with QCD
... and at large $N$?
I.3 Planar equivalence in perturbation theory and beyond

Let us introduce yet another sequence of QCD-like theories, $\text{QCD}_{\text{Adj}}$, i.e. QCD with $N_f$ Majorana fermions in the adjoint representation of $SU(N)$.

NB: for $N_f = 1$ and $m=0$, $\text{QCD}_{\text{Adj}}$ coincides with $\text{SYM}$, the supersymmetric generalization of pure Yang-Mills theory.
In the large-N limit the bosonic sector of $QCD_{OR}$ is equivalent to that of $QCD_{Adj}$

Important corollary
For $N_f = 1$ and $m = 0$, $QCD_{OR}$ is planar-equivalent to SYM theory
Some properties of the latter should show up in $N_f = 1$ QCD ... if $N=3$ is large enough

NB: Since SYM has $(N^2-1)$ left-handed fermions and $QCD_{OR}$ has $(N^2-N)$, expected accuracy is only $1/N$
Perturbative Argument

Draw a planar diagram on sphere

Differ by an even number of - signs...
Non-perturbative Argument (sketch)

- Integrate out fermions (after having included masses, bilinear sources)
- Use gauge invariance of $\det(\mathcal{D}+m+J)$ to express it in terms of Wilson-loops
- Use large-N factorization to write adjoint and OR Wilson loop as product of fundamental and/or antifundamental Wilson loops
- Use equality of fundamental and antifundamental Wilson loops
Before moving to SUSY..

It would be interesting to check numerically what happens to $QCD_{OR}$ and to $QCD_{Adj}$ as we increase $N$ even for

- $m \neq 0, N_f \neq 1$,
- quenched limit

The two theories should approach each other

Another numerical (analytic?) check could be comparing fermionic determinants in both theories as $N$ is increased
Summary, part I

- We reviewed two large-N expansions of QCD in which we keep quarks in the fundamental rep. of SU(N).
- We considered a new generalization of QCD to arbitrary N where quarks are in the AS rep. of SU(N) and called it QCD$_{OR}$.
- We argued that, at large N, QCD$_{OR}$ \(\Rightarrow\) QCD$_{Adj}$ (in their resp. bosonic sectors).
- We remarked that, in a special case, QCD$_{Adj}$ \(\Rightarrow\) SYM, a supersymmetric theory on which we have better control.
Conclusions

- Postponed till after Lecture II