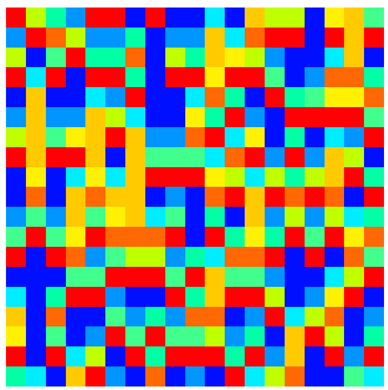
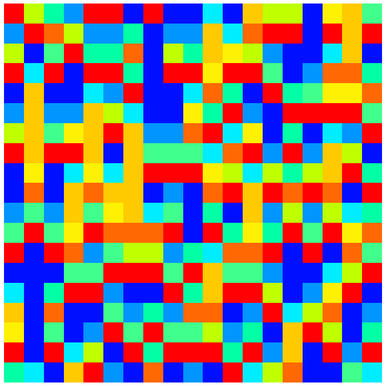
Complexity of chaotic strings and standard model parameters



Christian Beck
Queen Mary,
University of
London

Complexity of chaotic strings and standard model parameters



Christian Beck
Queen Mary, University of London

Contents

Contents

References:

- C.B., Chaotic strings and standard model parameters, Physica D 171, 72 (2002)
- C.B., Spatio-temporal chaos and vacuum fluctuations of quantized fields, World Scientific (2002)
  (50-page summary at hep-th/0207081)
- C.B., Phys. Rev. D 69, 123515 (2004)

1 Intro 1: Coupled map lattices

large 1-dim lattices, lattice sites i. Dynamics given by

$$\Phi_{n+1}^{i} = (1-a)T(\Phi_n^i) + \frac{a}{2}(T(\Phi_n^{i-1}) + T(\Phi_n^{i+1}))$$

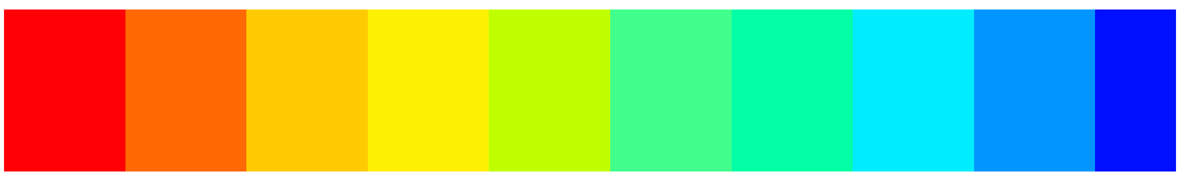
i: discrete spatial coordinate (periodic boundary conditions)
n: discrete time
a: coupling constant
T: local map, e.g. $T(\Phi) = 2\Phi^2 - 1$ (negative Ulam map) (strongly chaotic!)

1 Intro 1: Coupled map lattices

large 1-dim lattices, lattice sites i. Dynamics given by

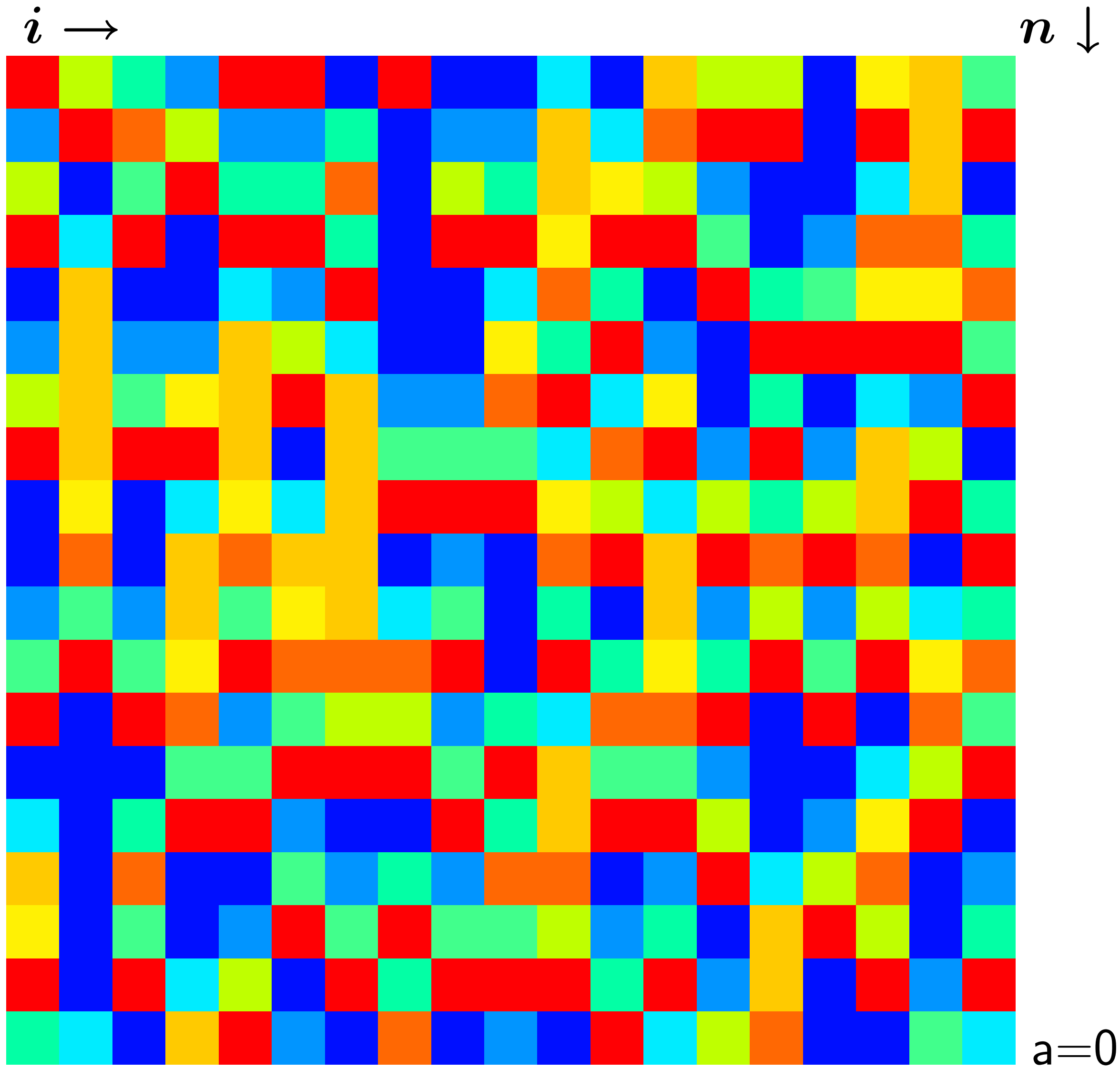
$$\Phi_{n+1}^{i} = (1-a)T(\Phi_n^i) + \frac{a}{2}(T(\Phi_n^{i-1}) + T(\Phi_n^{i+1}))$$

i: discrete spatial coordinate (periodic boundary conditions)
n: discrete time
a: coupling constant
T: local map, e.g. $T(\Phi) = 2\Phi^2 - 1$ (negative Ulam map) (strongly chaotic!)
Colour coding:

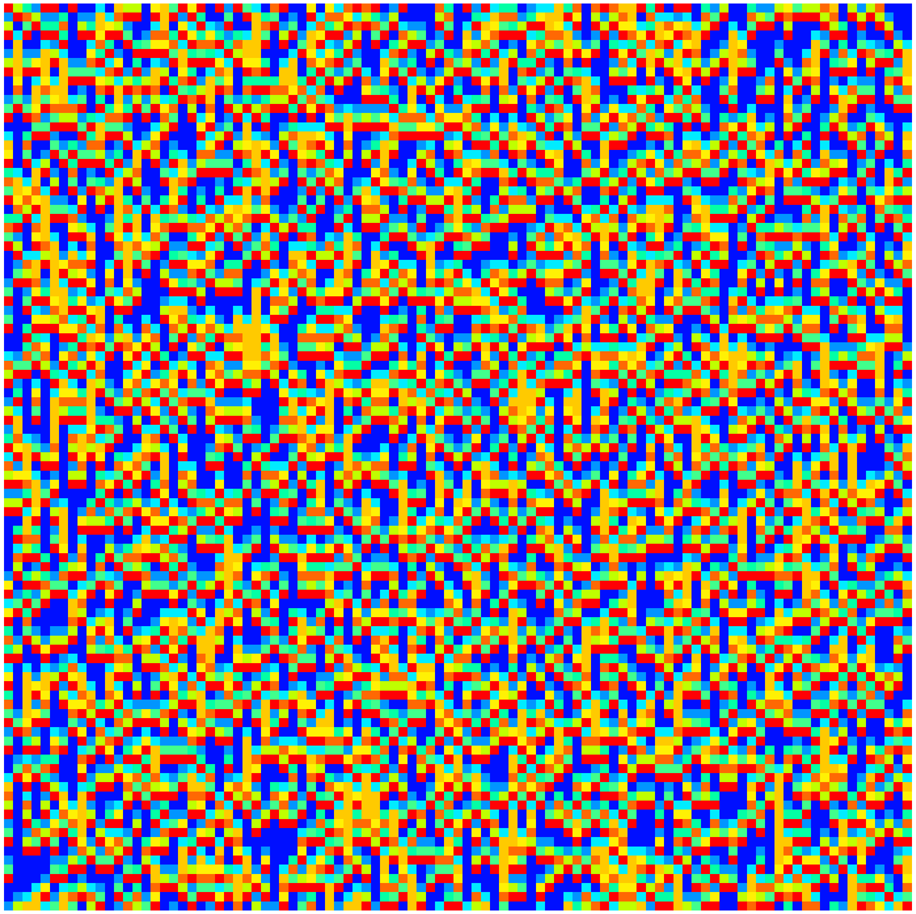


-1.0 -0.8 -0.6 -0.4 -0.2 0.00 +0.2 +0.4 +0.6 +0.8 +1.0

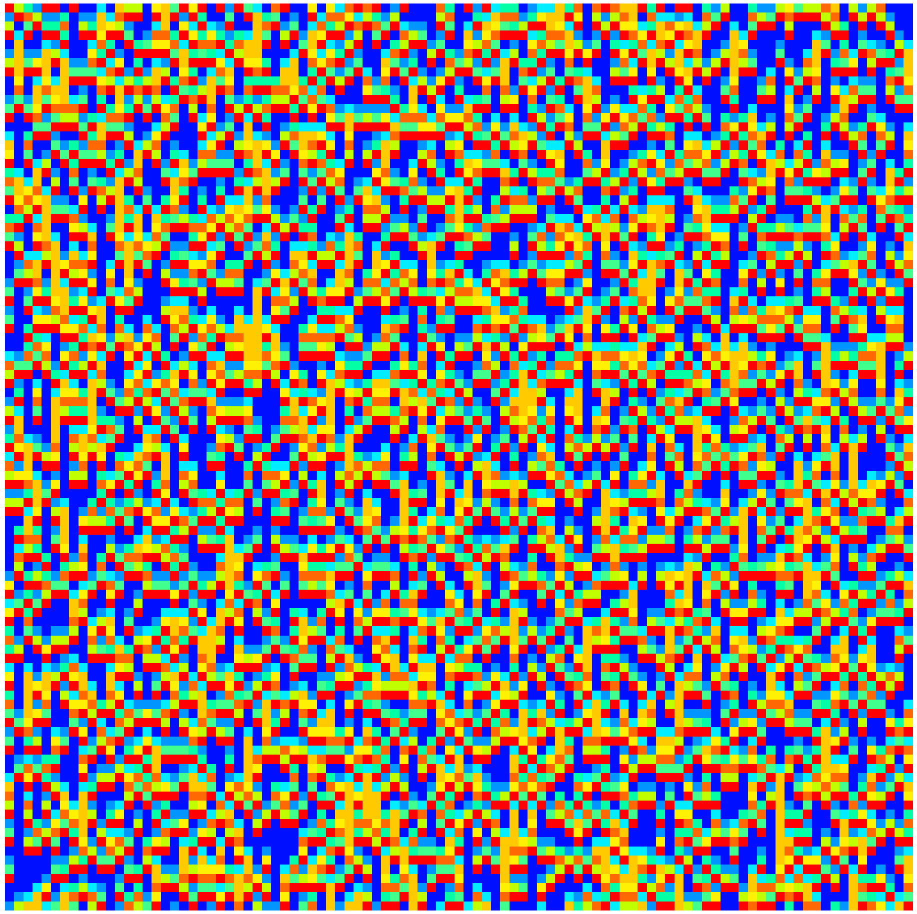
$$\Phi^i_{n+1} = (1-a)T(\Phi^i_n) + \frac{a}{2}(T(\phi^{i-1}_n) + T(\Phi^{i+1}_n))$$



(larger lattice)

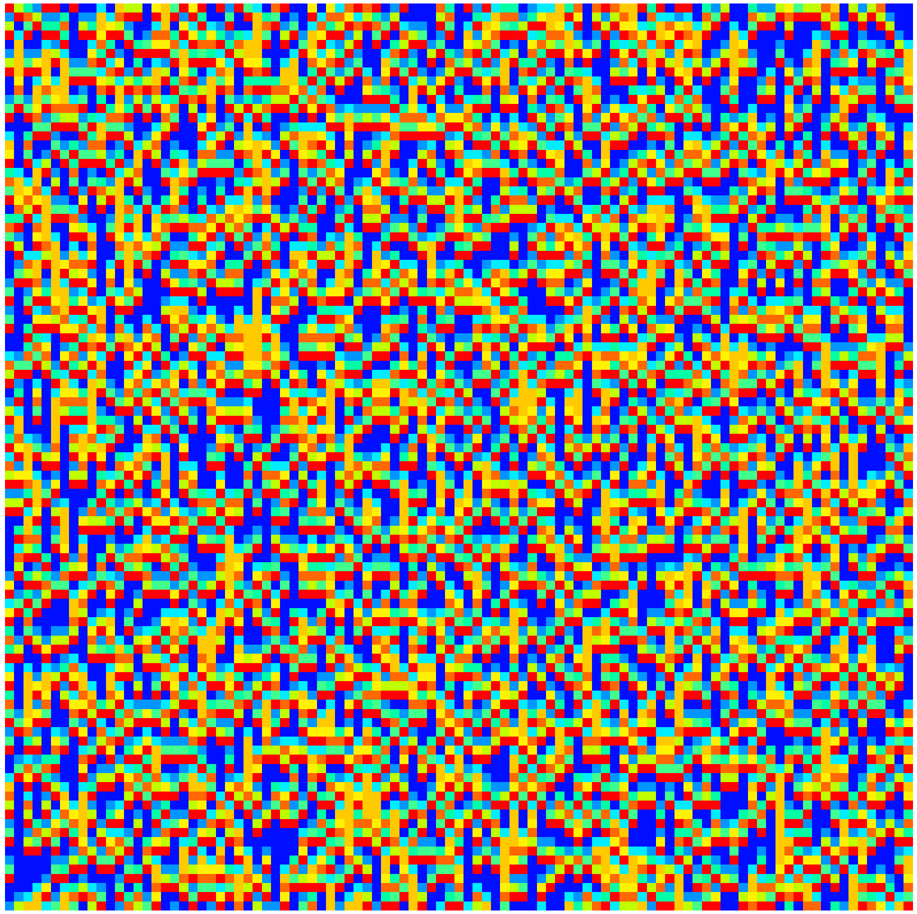


a=0

(larger lattice)  a=0

Ulam map conjugated to tent map, iterates satisfy a Central Limit Theorem for a=0:

$\frac{1}{\sqrt{M}} \sum_{n=1}^{M} \Phi_n^i \longrightarrow$ Gaussian $(M \longrightarrow \infty)$

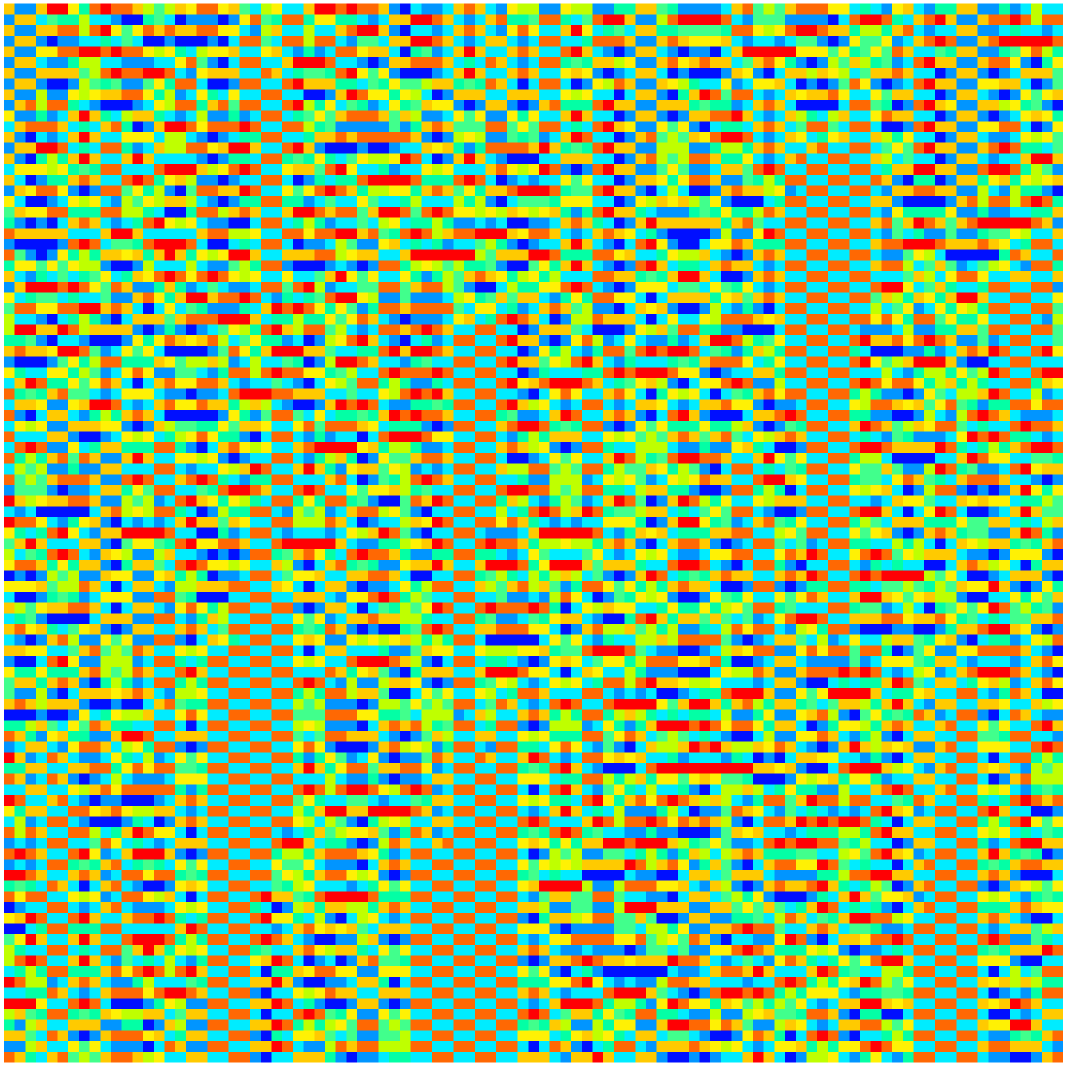
(larger lattice)  a=0

Ulam map conjugated to tent map, iterates satisfy a Central Limit Theorem for a=0:
$\frac{1}{\sqrt{M}}\sum_{n=1}^{M}\Phi_n^i \longrightarrow$ Gaussian $(M \longrightarrow \infty)$

Can also prove a functional central limit theorem (convergence to Wiener processd under rescaling)

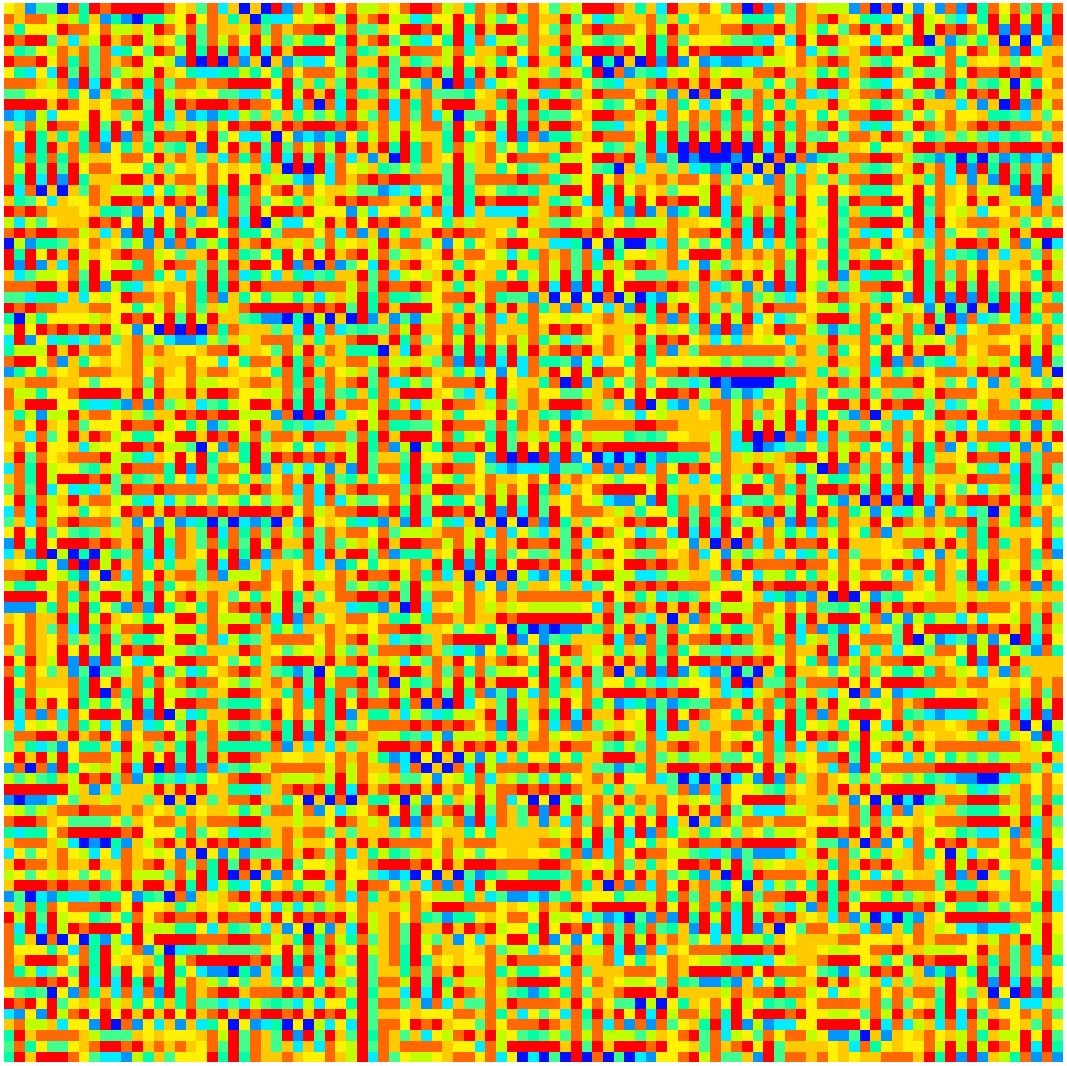
Meaning: On large scales the deterministic chaotic fluctuations look like Gaussian white noise.

$$\Phi_{n+1}^{i} = (1-a)T(\Phi_n^i) + \frac{a}{2}(T(\phi_n^{i-1}) + T(\Phi_n^{i+1}))$$

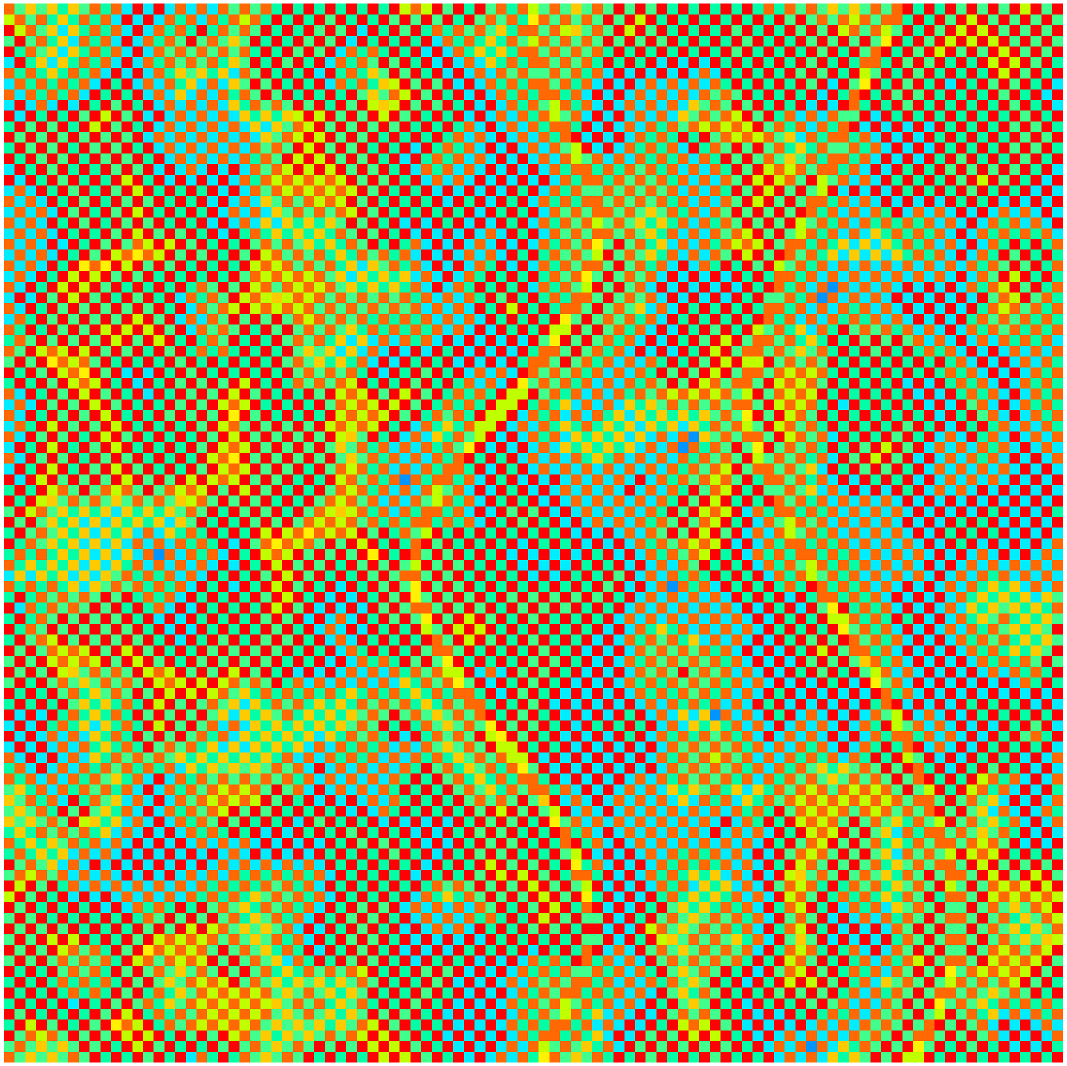


$a = 0.375$

$a = 1$



a=0.5, 2-dim lattice



(snapshot at fixed time n)

We are particularly interested in cases where the local map exhibits strongest possible chaotic behaviour, e.g. Tchebyscheff maps T_N of N-th order:

$$\begin{aligned} T_2(\Phi) &= 2\Phi^2 - 1 && (1) \\ T_3(\Phi) &= 4\Phi^3 - 3\Phi && (2) \\ \ldots &= \ldots && (3) \\ T_N(\Phi) &= \cos(N \arccos \Phi) && (4) \end{aligned}$$

conjugated to a Bernoulli shift of N symbols.

We are particularly interested in cases where the local map exhibits strongest possible chaotic behaviour, e.g. Tchebyscheff maps T_N of N-th order:

$$\begin{aligned} T_2(\Phi) &= 2\Phi^2 - 1 && (1) \\ T_3(\Phi) &= 4\Phi^3 - 3\Phi && (2) \\ ... &= ... && (3) \\ T_N(\Phi) &= \cos(N \arccos \Phi) && (4) \end{aligned}$$

conjugated to a Bernoulli shift of N symbols.

What is a Bernoulli shift? Chaotic dynamics $\Phi_{n+1} = T_N(\Phi_n)$ equivalent (in suitable coordinates) to a shift of N symbols:

e.g. for $T_2(\Phi) = 2\Phi^2 - 1$ you get 0011010100011111001....

or for $T_3(\Phi) = 4\Phi^3 - 3\Phi$ you get 2011210012101221120.........

Each iteration is like throwing away the first digit and moving the remaining sequence one step to the left. ($\Phi_0 \in [-1, 1]$).

We are particularly interested in cases where the local map exhibits strongest possible chaotic behaviour, e.g. Tchebyscheff maps T_N of N-th order:

$$T_2(\Phi) = 2\Phi^2 - 1 \quad (1)$$
$$T_3(\Phi) = 4\Phi^3 - 3\Phi \quad (2)$$
$$... = ... \quad (3)$$
$$T_N(\Phi) = \cos(N \arccos \Phi) \quad (4)$$

conjugated to a Bernoulli shift of N symbols.

What is a Bernoulli shift? Chaotic dynamics $\Phi_{n+1} = T_N(\Phi_n)$ equivalent (in suitable coordinates) to a shift of N symbols:

e.g. for $T_2(\Phi) = 2\Phi^2 - 1$ you get 0011010100011111001....

or for $T_3(\Phi) = 4\Phi^3 - 3\Phi$ you get 2011210012101221120.........

Each iteration is like throwing away the first digit and moving the remaining sequence one step to the left. ($\Phi_0 \in [-1, 1]$).

This is like DNA! (N=4)

We are particularly interested in cases where the local map exhibits strongest possible chaotic behaviour, e.g. Tchebyscheff maps T_N of N-th order:

$$T_2(\Phi) = 2\Phi^2 - 1 \quad (1)$$
$$T_3(\Phi) = 4\Phi^3 - 3\Phi \quad (2)$$
$$... = ... \quad (3)$$
$$T_N(\Phi) = \cos(N \arccos \Phi) \quad (4)$$

conjugated to a Bernoulli shift of N symbols.

What is a Bernoulli shift? Chaotic dynamics $\Phi_{n+1} = T_N(\Phi_n)$ equivalent (in suitable coordinates) to a shift of N symbols:

e.g. for $T_2(\Phi) = 2\Phi^2 - 1$ you get 00110101000111111001....

or for $T_3(\Phi) = 4\Phi^3 - 3\Phi$ you get 2011210012101221120.........

Each iteration is like throwing away the first digit and moving the remaining sequence one step to the left. ($\Phi_0 \in [-1, 1]$).

This is like DNA! (N=4)

For more details on Bernoulli shifts and chaotic dynamics see e.g.

C.B., F.Schloegl, Thermodynamics of Chaotic Systems, Cambridge University Press (1993)

Single Tchebyscheff map: Invariant density (prob.density of iterates) given by $\rho_0(x) = \frac{1}{\pi\sqrt{1-\phi^2}}$.
CML with $a = 0$: The invariant density for all M lattice sites is (of course) given by

$$\rho_0(\Phi^1, \Phi^2, \ldots, \Phi^M) = \prod_{i=1}^{M} \frac{1}{\pi\sqrt{1-(\Phi^i)^2}} \tag{5}$$

Single Tchebyscheff map: Invariant density (prob.density of iterates) given by $\rho_0(x) = \frac{1}{\pi\sqrt{1-\phi^2}}$.

CML with $a = 0$: The invariant density for all M lattice sites is (of course) given by

$$\rho_0(\Phi^1, \Phi^2, \ldots, \Phi^M) = \prod_{i=1}^{M} \frac{1}{\pi\sqrt{1-(\Phi^i)^2}} \tag{5}$$

Note that this is like a generalized canonical ensemble (product of q-Gaussians) in nonextensive statistical mechanics with $q = 3$, energy $\epsilon = \frac{1}{2}\Phi^2$, $\beta = 1$ (C. Tsallis, J. Stat. Phys. 1988)

(recall $e_q(x) := (1 + (q-1)x)^{-\frac{1}{q-1}}$, hence $\rho_0(\phi) = \frac{1}{\pi} e_q^{-\beta\epsilon}$)

Emergence this formally q-deformed statistical mechanics can be rigorously understood from fixed point of Perron-Frobenius operator.

Single Tchebyscheff map: Invariant density (prob.density of iterates) given by $\rho_0(x) = \frac{1}{\pi\sqrt{1-\phi^2}}$.

CML with $a = 0$: The invariant density for all M lattice sites is (of course) given by

$$\rho_0(\Phi^1, \Phi^2, \ldots, \Phi^M) = \prod_{i=1}^{M} \frac{1}{\pi\sqrt{1-(\Phi^i)^2}} \tag{5}$$

Note that this is like a generalized canonical ensemble (product of q-Gaussians) in nonextensive statistical mechanics with $q = 3$, energy $\epsilon = \frac{1}{2}\Phi^2$, $\beta = 1$ (C. Tsallis, J. Stat. Phys. 1988)

(recall $e_q(x) := (1 + (q-1)x)^{-\frac{1}{q-1}}$, hence $\rho_0(\phi) = \frac{1}{\pi} e_q^{-\beta\epsilon}$)

Emergence this formally q-deformed statistical mechanics can be rigorously understood from fixed point of Perron-Frobenius operator.

For finite $a > 0$ the density changes and is not a product of single-site densities any more. Numerics necessary.

We are interested in averages of some test functions (observables) $\boldsymbol{h(\Phi^i)}$:

$$\langle \boldsymbol{h(\Phi)} \rangle_{\boldsymbol{a}} = \lim_{\boldsymbol{M\to\infty, J\to\infty}} \frac{\mathbf{1}}{\boldsymbol{MJ}} \sum_{\boldsymbol{n=1}}^{\boldsymbol{M}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{J}} \boldsymbol{h(\Phi_n^i)}. \tag{6}$$

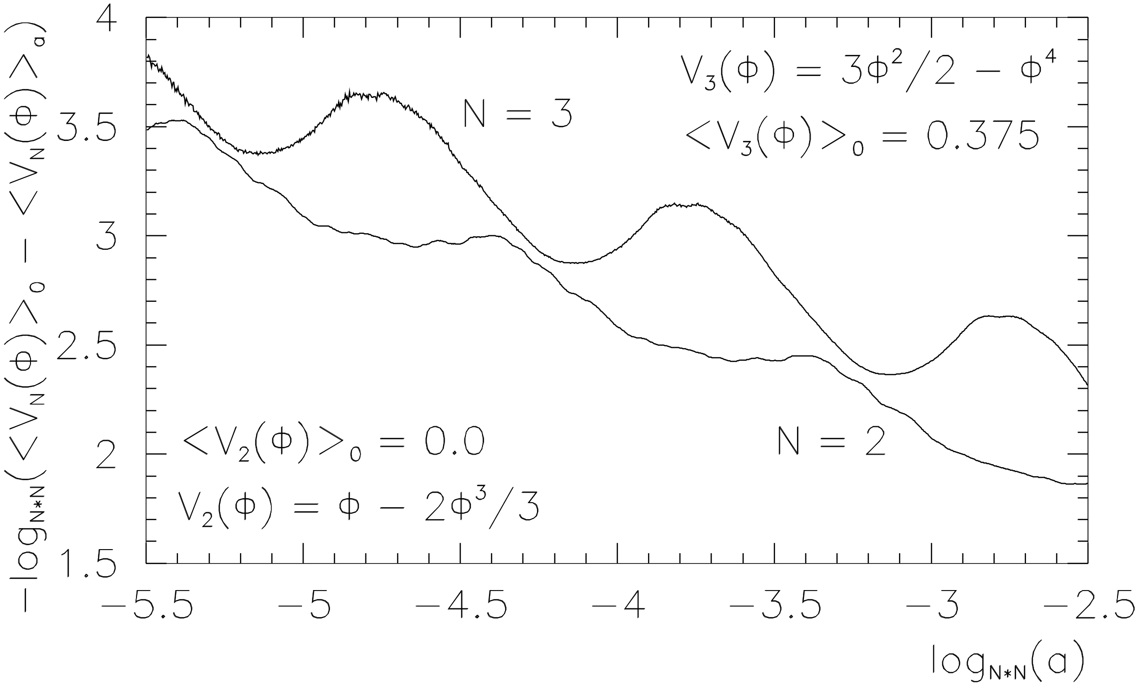
We are interested in averages of some test functions (observables) $h(\Phi^i)$:

$$\langle h(\Phi)\rangle_a = \lim_{M\to\infty, J\to\infty} \frac{1}{MJ} \sum_{n=1}^{M} \sum_{i=1}^{J} h(\Phi_n^i). \tag{6}$$

For $a \longrightarrow 0$ one numerically observes the scaling behaviour

$$\langle h(\Phi)\rangle_a - \langle h(\Phi)\rangle_0 = \sqrt{a} \cdot F^{(N)}(\log a) \tag{7}$$

where $F^{(N)}$ is a periodic function of $\log a$ with period $\log N^2$ (proof: S. Groote, C.B., nlin.CD/0603397)



2 Stochastic quantization

How can chaotic coupled map lattices be relevant in quantum field theories?

2 Stochastic quantization

How can chaotic coupled map lattices be relevant in quantum field theories?

Stochastic Quantization.

Consider classical field decribed by an action $S[\varphi]$. Classical field equation:

$$\frac{\delta S}{\delta \varphi} = 0 \tag{8}$$

meaning: Action has an extremum.

2 Stochastic quantization

How can chaotic coupled map lattices be relevant in quantum field theories?

Stochastic Quantization.

Consider classical field decribed by an action $S[\varphi]$. Classical field equation:

$$\frac{\delta S}{\delta \varphi} = 0 \tag{8}$$

meaning: Action has an extremum.

Parisi-Wu (1981): Obtain 2nd quantized equation of motion by considering a Langevin equation in fictitious time s:

$$\frac{\partial}{\partial s}\varphi(x,s) = -\frac{\delta S}{\delta \varphi}(x,s) + L(x,s) \tag{9}$$

$x = (x^1, x^2, x^3, x^4) = x^\mu$ point in Euclidean space-time

$x^4 = t$ physical time

$L(x,s)$ spatio-temporal Gaussian white noise

$$\langle L(x,s)\rangle = 0 \tag{10}$$
$$\langle L(x,s)L(x',s')\rangle = 2\delta(x-x')\delta(s-s') \tag{11}$$

Parisi and Wu: Quantum mechanical expectations = expectations of Langevin process for $s \longrightarrow \infty$.

Example: φ^4-theory

Action:

$$S[\varphi] = \int d^4x \left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4\right) \tag{12}$$

Classical field equation:

$$(-\partial^2 + m^2)\varphi(x) + \lambda\varphi^3(x) = 0 \tag{13}$$

2nd quantized version:

$$\frac{\partial}{\partial s}\varphi(x,s) = (\partial^2 - m^2)\varphi(x,s) - \lambda\varphi^3(x,s) + L(x,s) \tag{14}$$

The basic idea in the following is:

Replace the Gaussian white noise of the Parisi-Wu approach by something suitable deterministic chaotic on smallest scales.

'Chaotic quantization'

The basic idea in the following is:

Replace the Gaussian white noise of the Parisi-Wu approach by something suitable deterministic chaotic on smallest scales.

'Chaotic quantization'

On large scales, the chaotic noise will just look like ordinary Gaussian white noise ($\Longrightarrow$ ordinary path integrals, ordinary QFT, ordinary string theory)

The basic idea in the following is:

Replace the Gaussian white noise of the Parisi-Wu approach by something suitable deterministic chaotic on smallest scales.

'Chaotic quantization'

On large scales, the chaotic noise will just look like ordinary Gaussian white noise ($\Longrightarrow$ ordinary path integrals, ordinary QFT, ordinary string theory)

But on smallest scales the dynamics is different and a complex amendment to ordinary quantization schemes.

The basic idea in the following is:

Replace the Gaussian white noise of the Parisi-Wu approach by something suitable deterministic chaotic on smallest scales.

'Chaotic quantization'

On large scales, the chaotic noise will just look like ordinary Gaussian white noise ($\Longrightarrow$ ordinary path integrals, ordinary QFT, ordinary string theory)

But on smallest scales the dynamics is different and a complex amendment to ordinary quantization schemes.

The chaotic dynamics (suitably chosen!) will distinguish standard model parameters, in the sense that they mininimize the vacuum energy of the chaotic noise dynamics.

The basic idea in the following is:

Replace the Gaussian white noise of the Parisi-Wu approach by something suitable deterministic chaotic on smallest scales.

'Chaotic quantization'

On large scales, the chaotic noise will just look like ordinary Gaussian white noise ($\Longrightarrow$ ordinary path integrals, ordinary QFT, ordinary string theory)

But on smallest scales the dynamics is different and a complex amendment to ordinary quantization schemes.

The chaotic dynamics (suitably chosen!) will distinguish standard model parameters, in the sense that they mininimize the vacuum energy of the chaotic noise dynamics.

In other words: If the standard model parameters (electroweak couplings, masses, strong couplings,...) had other values than those observed in the accelerator experiments, larger vacuum energy would arise in the chaotic noise sector.

The basic idea in the following is:

Replace the Gaussian white noise of the Parisi-Wu approach by something suitable deterministic chaotic on smallest scales.

'Chaotic quantization'

On large scales, the chaotic noise will just look like ordinary Gaussian white noise ($\Longrightarrow$ ordinary path integrals, ordinary QFT, ordinary string theory)

But on smallest scales the dynamics is different and a complex amendment to ordinary quantization schemes.

The chaotic dynamics (suitably chosen!) will distinguish standard model parameters, in the sense that they mininimize the vacuum energy of the chaotic noise dynamics.

In other words: If the standard model parameters (electroweak couplings, masses, strong couplings,...) had other values than those observed in the accelerator experiments, larger vacuum energy would arise in the chaotic noise sector.

Could this give a 'selection principle' for the universe to find out of the $\mathbf{10^{120}}$ possible vacua of M-theory the right one?

3 Chaotic quantization/ chaotic strings

4 Vacuum energy of chaotic strings

5 Standard model parameters

6 Connection to dark energy