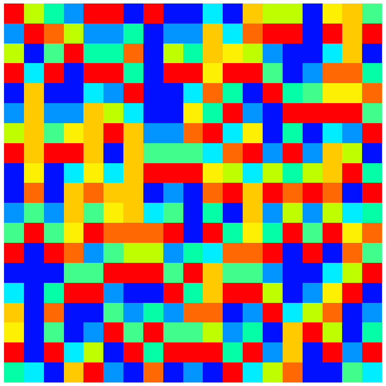
Complexity of chaotic strings and standard model parameters



Christian Beck
Queen Mary, University of London

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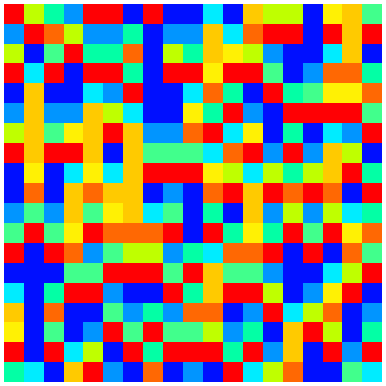

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References:

- C.B., Chaotic strings and standard model parameters, Physica D 171, 72 (2002)
- C.B., Spatio-temporal chaos and vacuum fluctuations of quantized fields, World Scientific (2002)
  (50-page summary at hep-th/0207081)
- C.B., Phys. Rev. D 69, 123515 (2004)

1 Intro 1: Coupled map lattices

large 1-dim lattices, lattice sites i. Dynamics given by

$$\Phi_{n+1}^{i} = (1-a)T(\Phi_n^i) + \frac{a}{2}(T(\Phi_n^{i-1}) + T(\Phi_n^{i+1}))$$

i: discrete spatial coordinate (periodic boundary conditions)
n: discrete time
a: coupling constant
T: local map, e.g. $T(\Phi) = 2\Phi^2 - 1$ (negative Ulam map) (strongly chaotic!)

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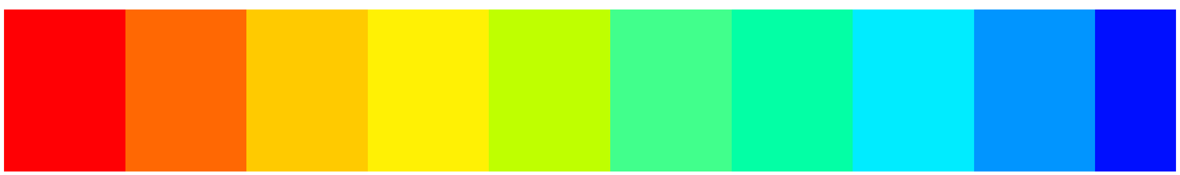

-1.0 -0.8 -0.6 -0.4 -0.2 0.00 +0.2 +0.4 +0.6 +0.8 +1.0

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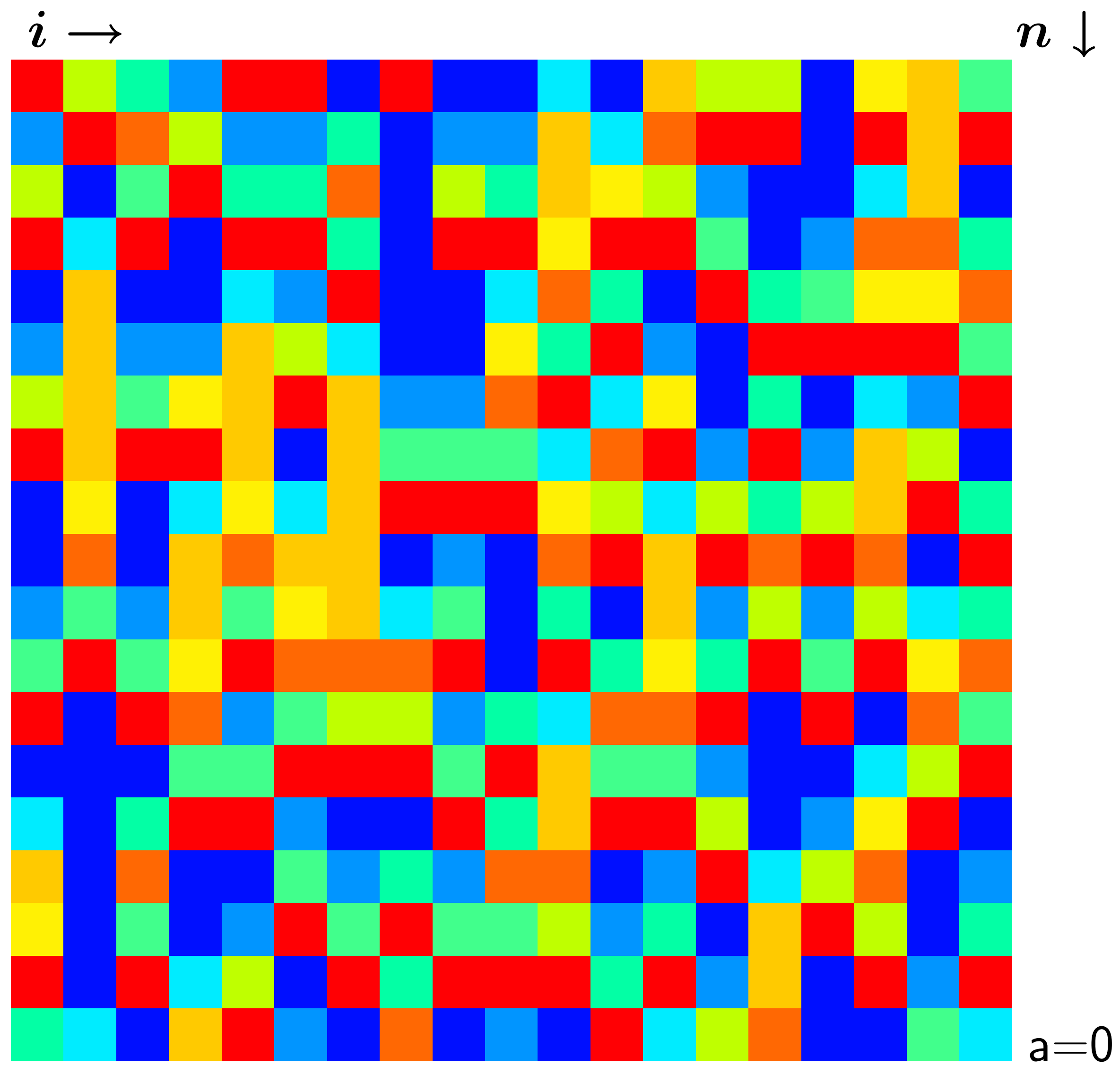
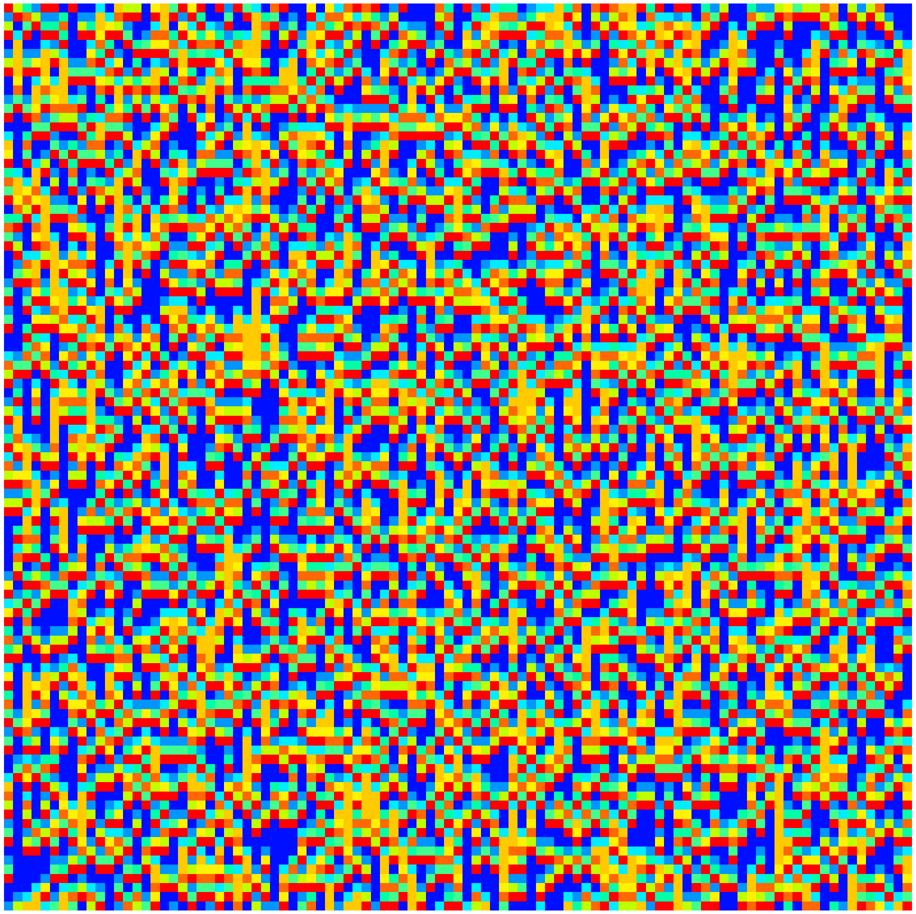
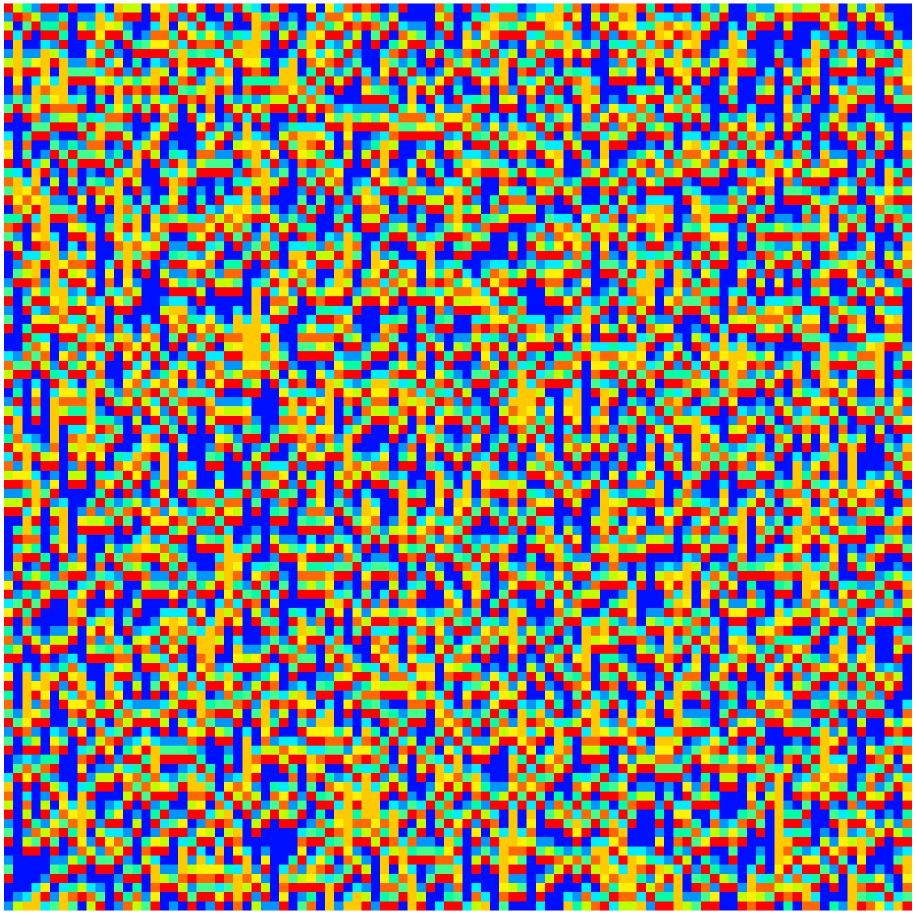
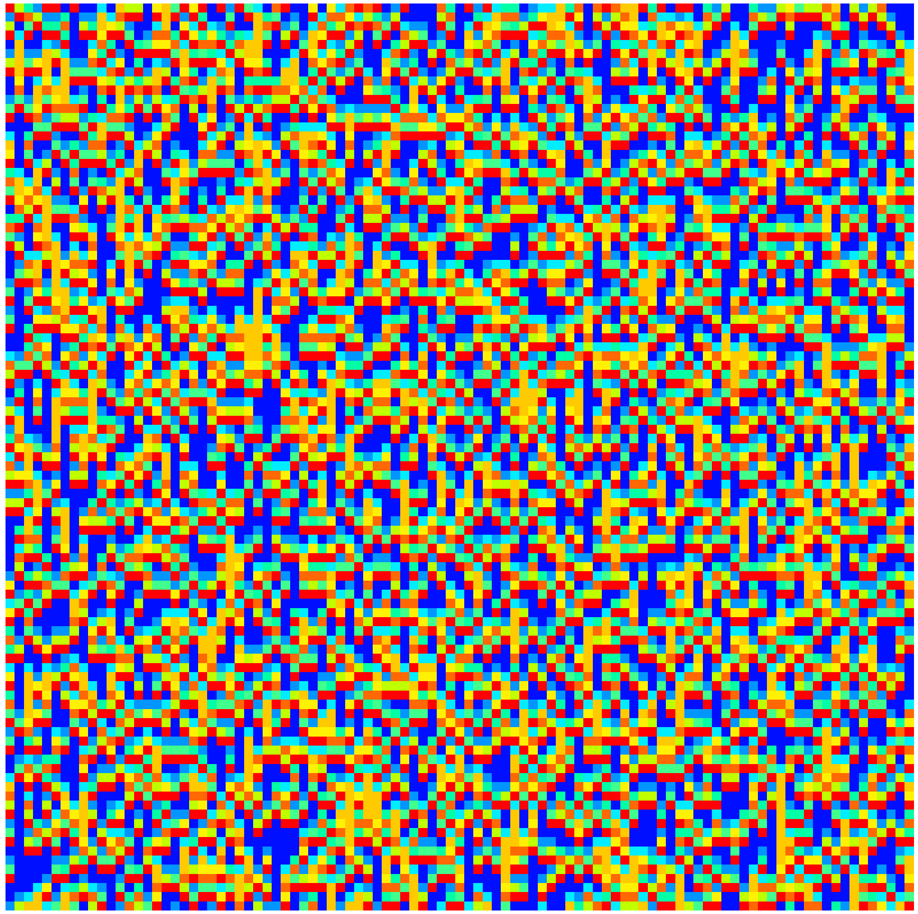


(larger lattice)  a=0 (larger lattice)  a=0

Ulam map conjugated to tent map, iterates satisfy a Central Limit Theorem for a=0:

$\frac{1}{\sqrt{M}} \sum_{n=1}^{M} \Phi_n^i \longrightarrow$ Gaussian $(M \longrightarrow \infty)$

(larger lattice)  a=0

Ulam map conjugated to tent map, iterates satisfy a Central Limit Theorem for a=0:

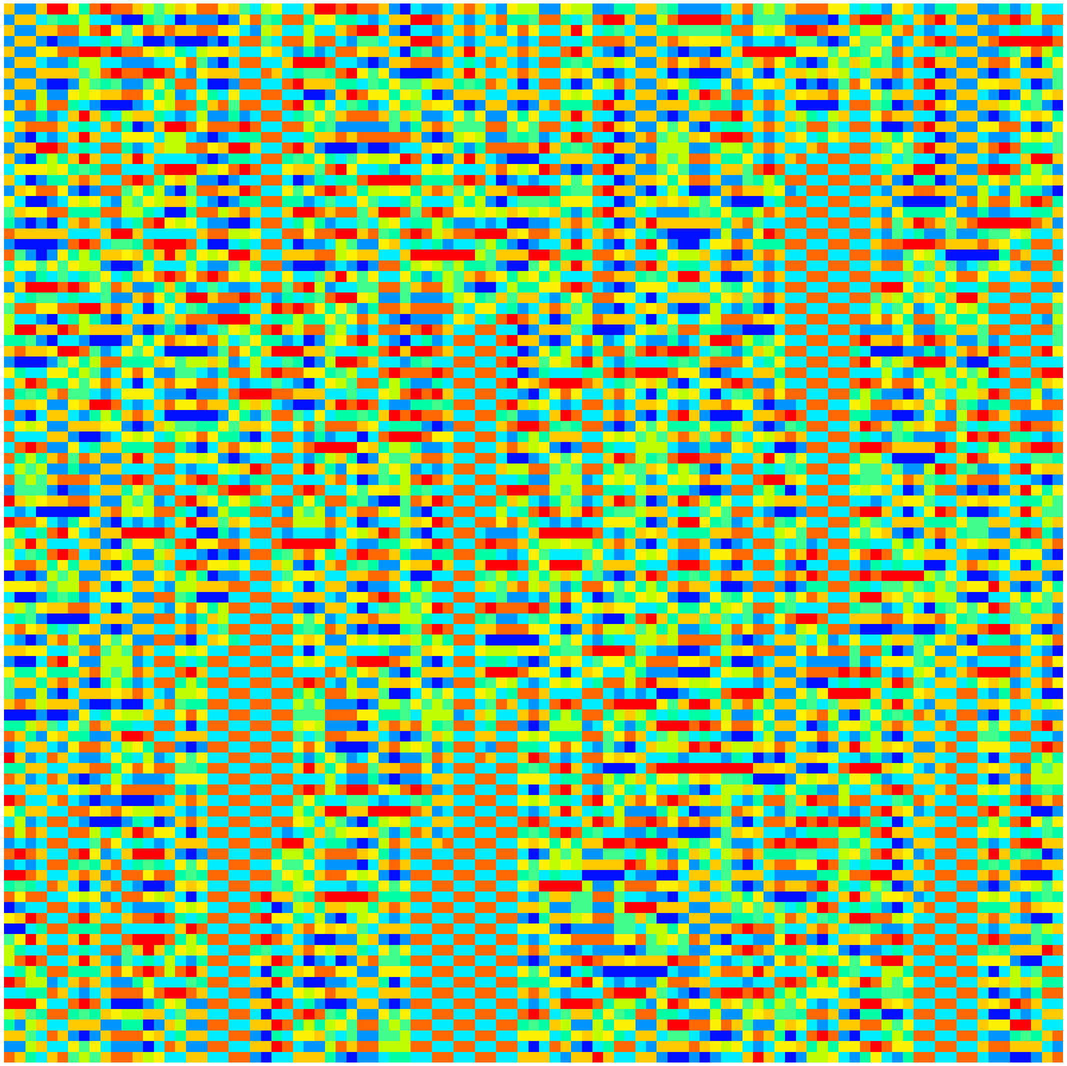
$\frac{1}{\sqrt{M}} \sum_{n=1}^{M} \Phi_n^i \longrightarrow$ Gaussian $(M \longrightarrow \infty)$

Can also prove a functional central limit theorem (convergence to Wiener processd under rescaling)

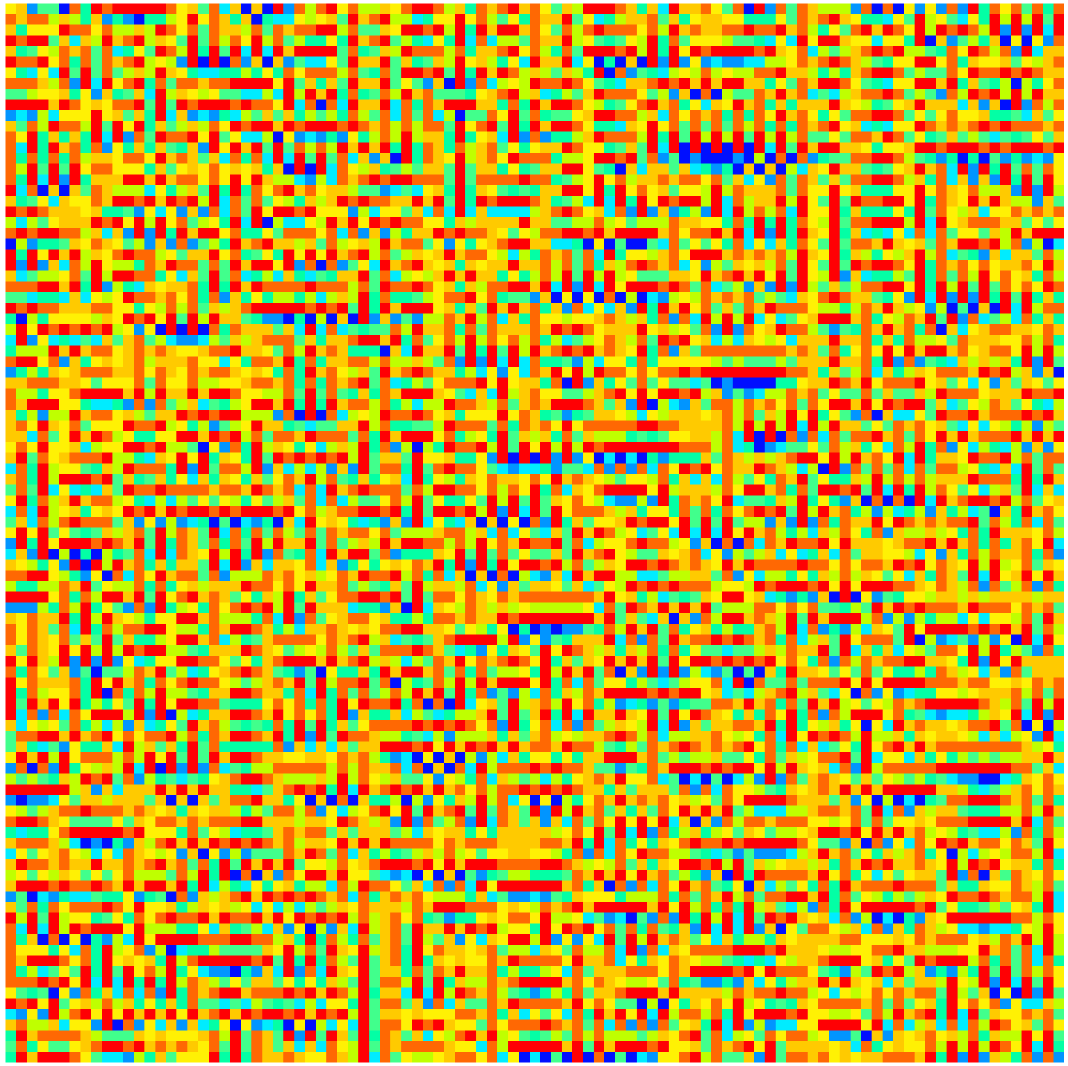
Meaning: On large scales the deterministic chaotic fluctuations look like Gaussian white noise.

$$\Phi^i_{n+1} = (1-a)T(\Phi^i_n) + \frac{a}{2}(T(\phi^{i-1}_n) + T(\Phi^{i+1}_n))$$

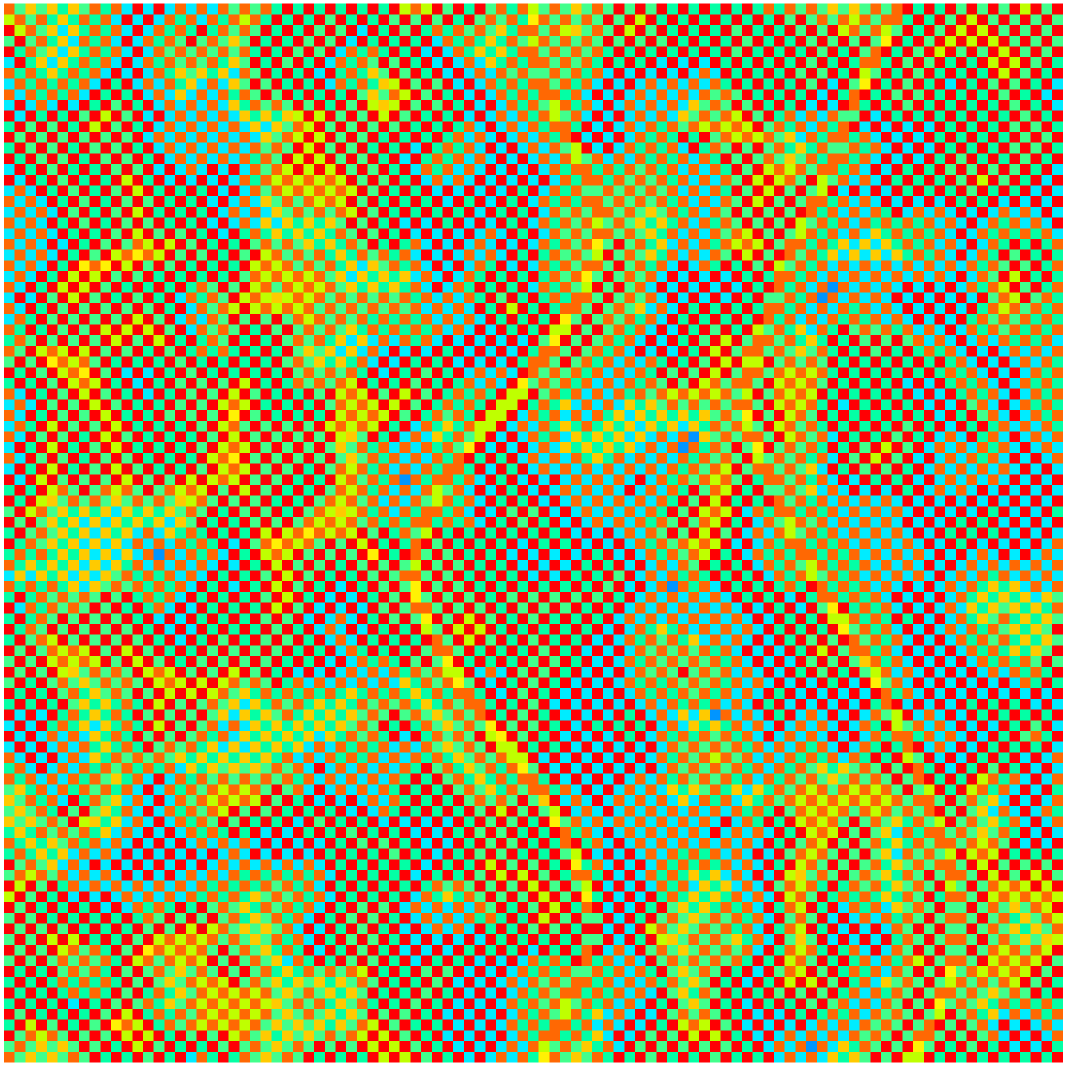
$a = 0.375$



$\boldsymbol{a = 1}$



a=0.5, 2-dim lattice



(snapshot at fixed time n)

We are particularly interested in cases where the local map exhibits strongest possible chaotic behaviour, e.g. Tchebyscheff maps $\boldsymbol{T_N}$ of $\boldsymbol{N}$-th order:

$$\boldsymbol{T_2(\Phi) = 2\Phi^2 - 1} \quad (1)$$

$$\boldsymbol{T_3(\Phi) = 4\Phi^3 - 3\Phi} \quad (2)$$

$$\boldsymbol{... = ...} \quad (3)$$

$$\boldsymbol{T_N(\Phi) = \cos(N \arccos \Phi)} \quad (4)$$

conjugated to a Bernoulli shift of $\boldsymbol{N}$ symbols.

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What is a Bernoulli shift? Chaotic dynamics $\Phi_{n+1} = T_N(\Phi_n)$ equivalent (in suitable coordinates) to a shift of N symbols:

e.g. for $T_2(\Phi) = 2\Phi^2 - 1$ you get 0011010100011111001....

or for $T_3(\Phi) = 4\Phi^3 - 3\Phi$ you get 2011210012101221120.........

Each iteration is like throwing away the first digit and moving the remaining sequence one step to the left. ($\Phi_0 \in [-1, 1]$).

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For more details on Bernoulli shifts and chaotic dynamics see e.g.

C.B., F.Schloegl, Thermodynamics of Chaotic Systems, Cambridge University Press (1993)

Single Tchebyscheff map: Invariant density (prob.density of iterates) given by $\rho_0(x) = \frac{1}{\pi\sqrt{1-\phi^2}}$.
CML with $a = 0$: The invariant density for all M lattice sites is (of course) given by

$$\rho_0(\Phi^1, \Phi^2, \ldots, \Phi^M) = \prod_{i=1}^{M} \frac{1}{\pi\sqrt{1-(\Phi^i)^2}} \tag{5}$$

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Note that this is like a generalized canonical ensemble (product of q-Gaussians) in nonextensive statistical mechanics with $q = 3$, energy $\epsilon = \frac{1}{2}\Phi^2$, $\beta = 1$ (C. Tsallis, J. Stat. Phys. 1988)

(recall $e_q(x) := (1 + (q-1)x)^{-\frac{1}{q-1}}$, hence $\rho_0(\phi) = \frac{1}{\pi} e_q^{-\beta\epsilon}$)

Emergence this formally q-deformed statistical mechanics can be rigorously understood from fixed point of Perron-Frobenius operator.

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For finite $a > 0$ the density changes and is not a product of single-site densities any more. Numerics necessary.

We are interested in averages of some test functions (observables) $\boldsymbol{h(\Phi^i)}$:

$$\langle \boldsymbol{h(\Phi)} \rangle_a = \lim_{M\to\infty, J\to\infty} \frac{1}{MJ} \sum_{n=1}^{M} \sum_{i=1}^{J} \boldsymbol{h(\Phi_n^i)}. \tag{6}$$

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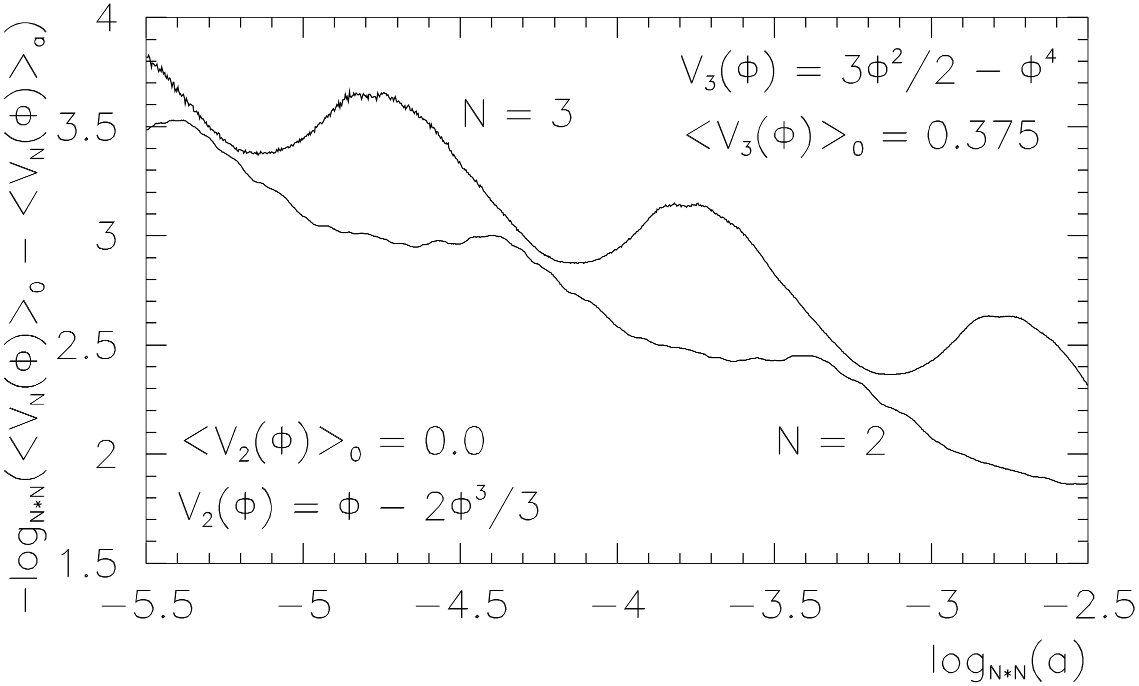
For $a \longrightarrow 0$ one numerically observes the scaling behaviour

$$\langle h(\Phi)\rangle_a - \langle h(\Phi)\rangle_0 = \sqrt{a} \cdot F^{(N)}(\log a) \tag{7}$$

where $F^{(N)}$ is a periodic function of $\log a$ with period $\log N^2$ (proof: S. Groote, C.B., nlin.CD/0603397)



2 Stochastic quantization

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Parisi-Wu (1981): Obtain 2nd quantized equation of motion by considering a Langevin equation in fictitious time s:

$$\frac{\partial}{\partial s}\varphi(x, s) = -\frac{\delta S}{\delta \varphi}(x, s) + L(x, s) \tag{9}$$

$x = (x^1, x^2, x^3, x^4) = x^\mu$ point in Euclidean space-time

$x^4 = t$ physical time

$L(x, s)$ spatio-temporal Gaussian white noise

$$\langle L(x,s)\rangle = 0 \tag{10}$$
$$\langle L(x,s)L(x',s')\rangle = 2\delta(x-x')\delta(s-s') \tag{11}$$

Parisi and Wu: Quantum mechanical expectations = expectations of Langevin process for $s \longrightarrow \infty$.

Example: φ^4-theory

Action:

$$S[\varphi] = \int d^4x \left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4\right) \tag{12}$$

Classical field equation:

$$(-\partial^2 + m^2)\varphi(x) + \lambda\varphi^3(x) = 0 \tag{13}$$

2nd quantized version:

$$\frac{\partial}{\partial s}\varphi(x,s) = (\partial^2 - m^2)\varphi(x,s) - \lambda\varphi^3(x,s) + L(x,s) \tag{14}$$

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Could this give a 'selection principle' for the universe to find out of the $\mathbf{10^{120}}$ possible vacua of M-theory the right one?

3 Chaotic quantization/ chaotic strings

4 Vacuum energy of chaotic strings

5 Another derivation of the 'chaotic string'

(C. B., Phys. Rev. D 69, 123515 (2004))
Consider quantized scalar field φ in Robertson-Walker metric:

$$\frac{\partial}{\partial s}\varphi = \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) + L(s,t), \tag{15}$$

where H is the Hubble parameter, V is the potential under consideration and $L(s,t)$ is Gaussian white noise t physical time, s fictitious time. Discretize

$$s = n\tau \tag{16}$$

$$t = i\delta \tag{17}$$

τ: fictitious time lattice constant, δ: physical time lattice constant. We obtain

$$\frac{\varphi^i_{n+1} - \varphi^i_n}{\tau} = \frac{1}{\delta^2}(\varphi^{i+1}_n - 2\varphi^i_n + \varphi^{i-1}_n) + 3\frac{H}{\delta}(\varphi^i_n - \varphi^{i-1}_n) + V'(\varphi^i_n) + noise \tag{18}$$

This can be written as the following recurrence relation for the field φ_n^i

$$\varphi_{n+1}^i = (1-\alpha)\left\{\varphi_n^i + \frac{\tau}{1-\alpha}V'(\varphi_n^i)\right\} + 3\frac{H\tau}{\delta}(\varphi_n^i - \varphi_n^{i-1}) + \frac{\alpha}{2}(\varphi_n^{i+1} + \varphi_n^{i-1}) + \tau \cdot noi \tag{19}$$

where a dimensionless coupling constant α is introduced as

$$\alpha := \frac{2\tau}{\delta^2}. \tag{20}$$

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Introduce dimensionless field variable Φ_n^i by writing $\varphi_n^i = \Phi_n^i p_{max}$, where p_{max} is some (so far) arbitrary energy scale. $\Longrightarrow$

$$\Phi_{n+1}^i = (1-\alpha)T(\Phi_n^i) + \frac{3}{2}H\delta\alpha(\Phi_n^i - \Phi_n^{i-1}) + \frac{\alpha}{2}(\Phi_n^{i+1} + \Phi_n^{i-1}) + \tau \cdot noise, \tag{21}$$

where the local map T is given by

$$T(\Phi) = \Phi + \frac{\tau}{p_{max}(1-\alpha)}V'(p_{max}\Phi). \tag{22}$$

Note that a symmetric diffusively coupled map lattice (Kaneko 1984)

$$\Phi^i_{n+1} = (1-\alpha)T(\Phi^i_n) + \frac{\alpha}{2}(\Phi^{i+1}_n + \Phi^{i-1}_n) + \tau \cdot noise \tag{23}$$

is obtained if $H\delta << 1$, equivalent to

$$\delta << H^{-1} \tag{24}$$

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The main result of our consideration is that iteration of a coupled map lattice of the form (23) with a given map T has physical meaning: It means that one is considering the second-quantized dynamics of a self-interacting real scalar field φ with a force V' given by

$$V'(\varphi) = \frac{1-\alpha}{\tau}\left\{-\varphi + p_{max}T\left(\frac{\varphi}{p_{max}}\right)\right\}. \tag{25}$$

Integration yields

$$V(\varphi) = \frac{1-\alpha}{\tau}\left\{-\frac{1}{2}\varphi^2 + p_{max}\int d\varphi\, T\left(\frac{\varphi}{p_{max}}\right)\right\} + const. \tag{26}$$

In terms of the dimensionless field Φ this can be written as

$$V(\varphi) = \frac{1-\alpha}{\tau}p_{max}^2\left\{-\frac{1}{2}\Phi^2 + \int d\Phi T(\Phi)\right\} + const. \tag{27}$$

Distinguished example of a φ^4-theory generating strongest possible chaotic behaviour:

$$\Phi_{n+1} = T_{-3}(\Phi_n) = -4\Phi_n^3 + 3\Phi_n \tag{28}$$

on the interval $\Phi \in [-1, 1]$. T_{-3} is the negative third-order Tchebyscheff map, a standard example of a map exhibiting strongly chaotic behaviour. It is conjugated to a Bernoulli shift. The corresponding potential is given by

$$V_{-3}(\varphi) = \frac{1-\alpha}{\tau} \left\{ \varphi^2 - \frac{1}{p_{max}^2} \varphi^4 \right\} + const, \tag{29}$$

or, in terms of the dimensionless field Φ,

$$V_{-3}(\varphi) = \frac{1-\alpha}{\tau} p_{max}^2 (\Phi^2 - \Phi^4) + const. \tag{30}$$

We obtain by second quantization a field φ that rapidly fluctuates in fictitious time on some finite interval, provided that initially $\varphi_0 \in [-p_{max}, p_{max}]$.

Of physical relevance are the expectations of suitable observables with respect to the ergodic chaotic dynamics. For example, the expectation $\langle V_{-3}(\varphi)\rangle$ of the potential is a possible candidate for vacuum energy in our universe. One obtains

$$\langle V_{-3}(\varphi)\rangle = \frac{1-\alpha}{\tau} p_{max}^2 (\langle \Phi^2 \rangle - \langle \Phi^4 \rangle) + const. \tag{31}$$

For uncoupled Tchebyscheff maps ($\alpha = 0$), expectations of any observable A can be evaluated as the ergodic average

$$\langle A \rangle = \int_{-1}^{+1} A(\Phi) d\mu(\Phi), \tag{32}$$

with the natural invariant measure being given by

$$d\mu(\Phi) = \frac{d\Phi}{\pi\sqrt{1-\Phi^2}} \tag{33}$$

From eq. (33) one obtains $\langle \Phi^2 \rangle = \frac{1}{2}$ and $\langle \Phi^4 \rangle = \frac{3}{8}$, thus

$$\langle V_{-3}(\varphi)\rangle = \frac{1}{8} \frac{p_{max}^2}{\tau} + const. \tag{34}$$

Alternatively, we may consider the positive Tchebyscheff map $T_3(\Phi) = 4\Phi^3 - 3\Phi$. This basically exhibits the same dynamics as T_{-3}, up to a sign. Repeating the same calculation we obtain

$$V_3(\varphi) = \frac{1-\alpha}{\tau}\left\{-2\varphi^2 + \frac{1}{p_{max}^2}\varphi^4\right\} + const \tag{35}$$

and

$$V_3(\varphi) = \frac{1-\alpha}{\tau}p_{max}^2(-2\Phi^2 + \Phi^4). \tag{36}$$

For the expectation of the vacuum energy one gets

$$\langle V_3(\varphi)\rangle = \frac{1-\alpha}{\tau}p_{max}^2(-2\langle\Phi^2\rangle + \langle\Phi^4\rangle) + const, \tag{37}$$

which for $\alpha = 0$ reduces to

$$\langle V_3(\varphi)\rangle = -\frac{5}{8}\frac{p_{max}^2}{\tau} + const. \tag{38}$$

Symmetry considerations between T_{-3} and T_3 suggest to take the additive constant $const$ as

$$const = +\frac{1-\alpha}{\tau}p_{max}^2\frac{1}{2}\langle\Phi^2\rangle. \tag{39}$$

One obtains the fully symmetric equation

$$\langle V_{\pm 3}(\varphi)\rangle = \pm\frac{1-\alpha}{\tau} p_{max}^2 \left\{-\frac{3}{2}\langle\Phi^2\rangle + \langle\Phi^4\rangle\right\}, \tag{40}$$

which for $\alpha \longrightarrow 0$ reduces to

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The simplest model for dark energy in the universe, as generated by a chaotic φ^4-theory, would be to identify $\frac{3}{8}p_{max}^2/\tau = \rho_\Lambda$, the constant vacuum energy density corresponding to a classical cosmological constant Λ, which stays constant during the expansion of the universe.

The ratio p_{max}^2/τ could well be determined by an anthropic principle.

For a more sophisticated model (including late-time symmetry breaking due to structure formation, and tracking behaviour in the early universe) see C.B., Phys. Rev. D 69, 123515 (2004)

In this approach the chaotic string dynamics represents a rapidly fluctuating spatially homogeneous field, which again has distinguished local minima and zeros of the vacuum energy.

The sense of dark energy in this model would be to fix and stabilize the observed standard model parameters.

Again one would assume that gauge coupling constants of the standard model are driven to the stable zeros of the interaction energy of the chaotic field, or to local minima of the self energy, as before.

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Could such a scalar field theory be embedded into string theory, or M theory, or some other candidate theory of quantum gravity, and possibly provide a 'selection principle' to find the right vacuum in the landscape?

6 Conclusions

- An enourmous number of numerical coincidences was found between minima of the vacuum energy of the chaotic fields and standard model couplings that are realised in nature.
- A random coincidence can be excluded. Joint probability $\sim \mathbf{10^{-60}}$. Hence there is the need to embed these chaotic dynamics in future theories of particle physics in one way or another.
- The chaotic string is completely different from the SM field equations, but is seems to encode the most important properties of the standard model. It is like a 'DNA string of the universe'.
- Some theoretical predictions coming out of this are particularly interesting and should be checked in future experiments:

  $$\boldsymbol{m_H = 154.4(4) GeV, m_t = 174.4(3) GeV, \alpha_s(m_Z) = 0.11780(1)}$$
- There is no evidence from chaotic strings for low-energy supersymmetry.