Minimal Length From QM and Classical GR

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Why do we believe in a minimal length?

A minimal length from QM and GR

Claim: GR and QM imply that no operational procedure exists which can measure a distance less than the Planck length.

Assumptions:

- Hoop Conjecture (GR): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.
- Quantum Mechanics: uncertainty relation.

Minimal Ball of uncertainty:

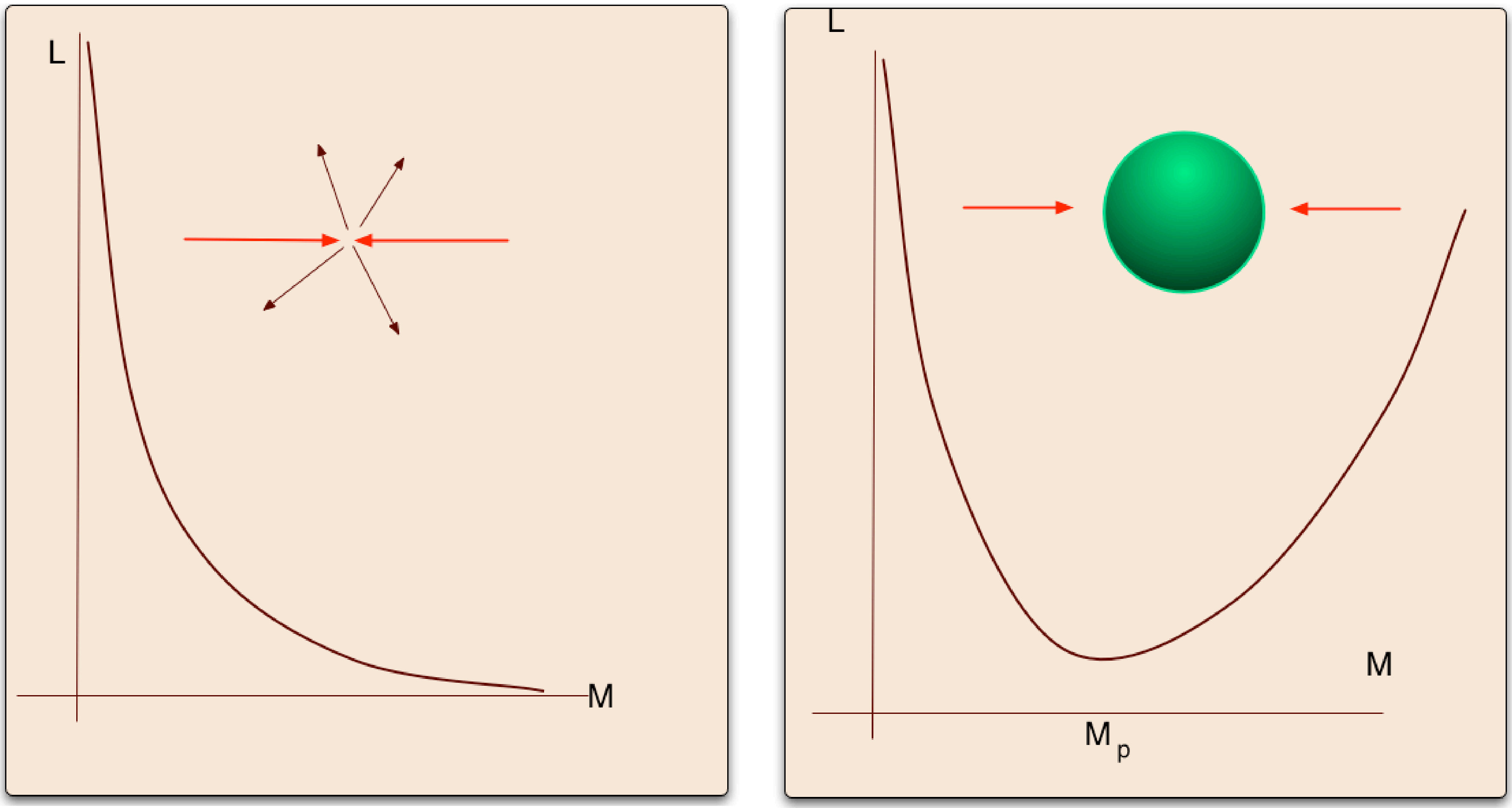
Consider a particle of Energy E which is not already a black hole. Its size r must satisfy:

$$r \gtrsim \max\left[1/E\,,\,E\right]$$

where 1/E is the Compton wavelength and E comes from the Hoop Conjecture. We find:

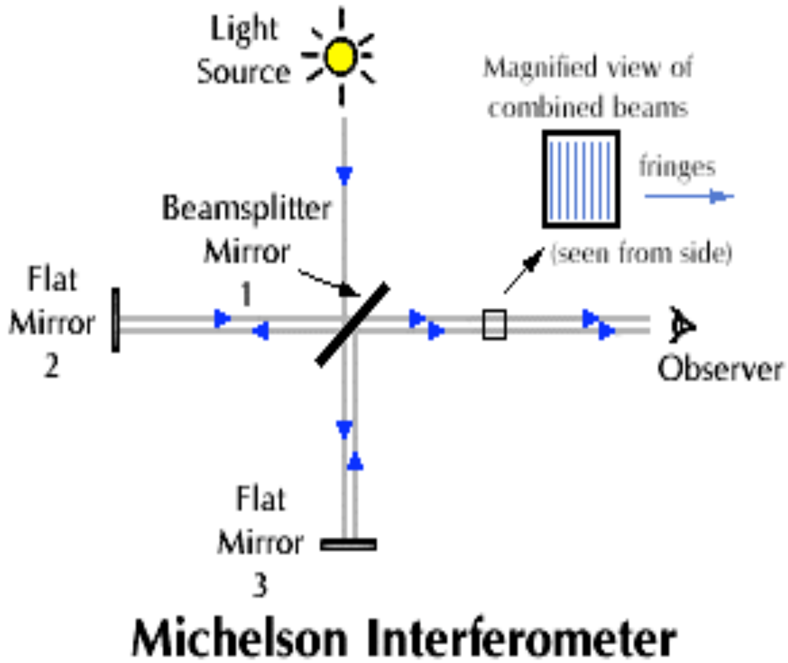
$$r \sim l_P$$

This is basically the argument you have heard from Professor Susskind on Thursday:
probing short distance requires high energy



But, this is not enough to exclude a length shorter than the Planck length

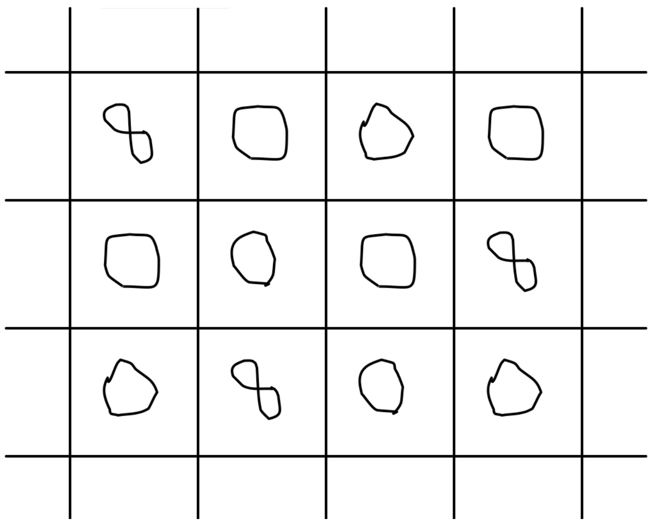
Could an interferometer do better?



Wave length of light is much bigger than the path length sensitivity!

Our concrete model:

We assume that the position operator has discrete eigenvalues separated by a distance l_P or smaller.



- Let us start from the standard inequality:

$$(\Delta A)^2(\Delta B)^2 \geq -\frac{1}{4}(\langle[A,B]\rangle)^2$$

- Suppose that the position of a test mass is measured at time t=0 and again at a later time. The position operator at a later time t is:

$$x(t) = x(0) + p(0)\frac{t}{M}$$

- The commutator between the position operators at t=0 and t is

$$[x(0), x(t)] = i\frac{t}{M}$$

- Using the standard inequality we have:

$$|\Delta x(0)||\Delta x(t)| \geq \frac{t}{2M}$$

- At least one of the uncertainties Δx(0) or Δx(t) must be larger than:

$$\sqrt{t/2M}$$

- A measurement of the discreteness of x(0) requires two position measurements, so it is limited by the greater of Δx(0) or Δx(t):

$$\Delta x \equiv max\left[\Delta x(0), \Delta x(t)\right] \geq \sqrt{\frac{t}{2M}}$$

- This is the bound we obtain from Quantum Mechanics.

- To avoid gravitational collapse, the size R of our measuring device must also grow such that R > M.
- However, by causality R cannot exceed t.
- GR and causality imply:

$$t > R > M$$

- Combined with the QM bound, they require $\Delta x > 1$ in Planck units or

$$\Delta x > l_P$$

- This derivation was not specific to an interferometer - the result is device independent: no device subject to quantum mechanics, gravity and causality can exclude the quantization of position on distances less than the Planck length.

Conclusions

- If you accept Quantum Mechanics and Classical General Relativity: there must be a minimal length in Nature in the sense that you could not probe a length shorter than this minimal length.
- Talking about lengths shorter than the Planck length makes no sense within that framework.
- One challenge is to implement the notion of a minimal length in gauge theories and gravity: one option is a noncommutative spacetime (but really quantized volume)
- Thank you for your attention.