QUARK MASSES IN QCD

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QCD

$$\alpha_s(Q^2) = \frac{-2/\beta_1}{\ln(Q^2/\mu^2)} \qquad m_q(Q^2) = \frac{\widehat{m}_q}{[\ln(Q^2/\mu^2)]^{\frac{-\gamma_1}{\beta_1}}}$$

$m_q(\mu)|_{\text{MS-bar}}$

- **QCD** Green function $\Leftrightarrow$ DATA (Hadronic spectrum)

QUARK MASSES

- CPT: Light quark mass ratios
- Lattice QCD
- QCD Sum Rules

THIS TALK

- **LIGHT QUARK MASSES**

- **m_u (μ) m_d (μ) m_s(μ)**

  μ = 2 GeV MS-bar

  HEAVY QUARKS

  $m_{c,b}$ (μ) μ =3 (10) GeV

QCD SUM RULES

(Shifman, Vainshtein, Zakharov)

1979 – to date

(a few kP)

- **ANALYTICAL METHOD TO *STUDY* QCD AT FERMI SCALES**
- **OPERATOR PRODUCT EXPANSION OF CURRENT CORRELATORS AT SHORT DISTANCES**
- **CAUCHY THEOREM IN THE COMPLEX ENERGY PLANE**
- **(QUARK-HADRON DUALITY)**
- **EXTENDS DOMAIN OF CHIRAL PT**
- **COMPLEMENTARY TOOL TO LATTICE QCD**

Q C D SUM RULES

Shifman-Vainshtein-Zakharov

(1979)

$$\Pi(q^2) = i\int d^4x\, e^{iqx} < 0|T(J(x)\, J^+(0))|0 >$$

$$J(x) : \overline{\psi}(x)\gamma_\mu\psi(x);\ \overline{\psi}(x)\gamma_\mu\gamma_5\psi(x);\ G^a_{\mu\nu}(x)G^a_{\mu\nu}(x); etc.$$

$$\Pi(q^2)_{QCD} \Leftrightarrow \Pi(q^2)_{HAD}$$

QCD

$$\Pi(q^2) = i \int d^4x \; e^{iqx} < 0 | T(J(x) J^+(0)) | 0 >$$

$$J(x) \Rightarrow \partial^\mu A_\mu(x)\Big|_j^i = (m_i + m_j)\, \overline{\psi}^{\,i}(x)\, i\gamma_5 \psi_j(x)$$

HADRONIC

$$\Pi(q^2) = i \int d^4x\, e^{iqx} < 0 | T(J(x)\, J^+(0)) | 0 >$$

$$J(x) \Rightarrow \partial^\mu A_\mu(x)|_j^i \propto f_{\pi/K}\, M_{\pi/K}^2$$

CONFINEMENT

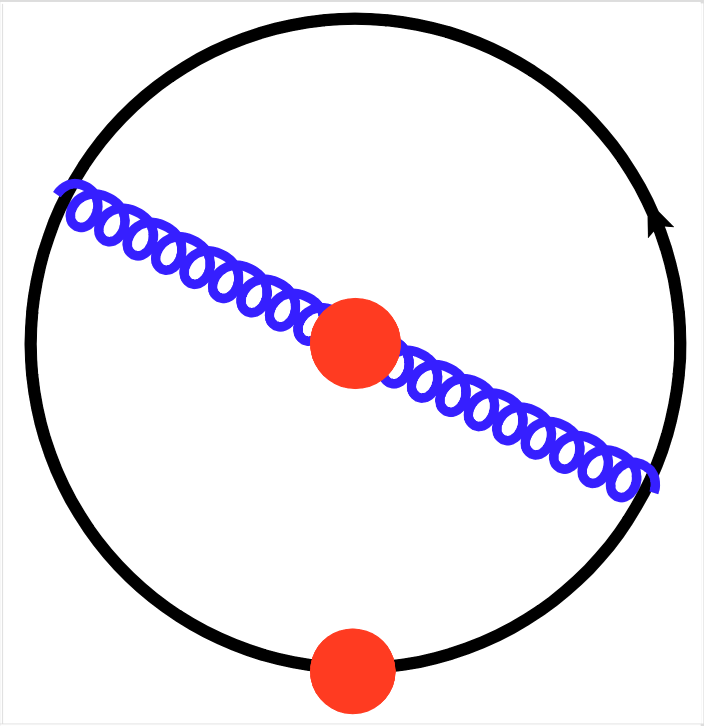
- STRONG MODIFICATION TO QUARK & GLUON PROPAGATORS NEAR THE MASS SHELL

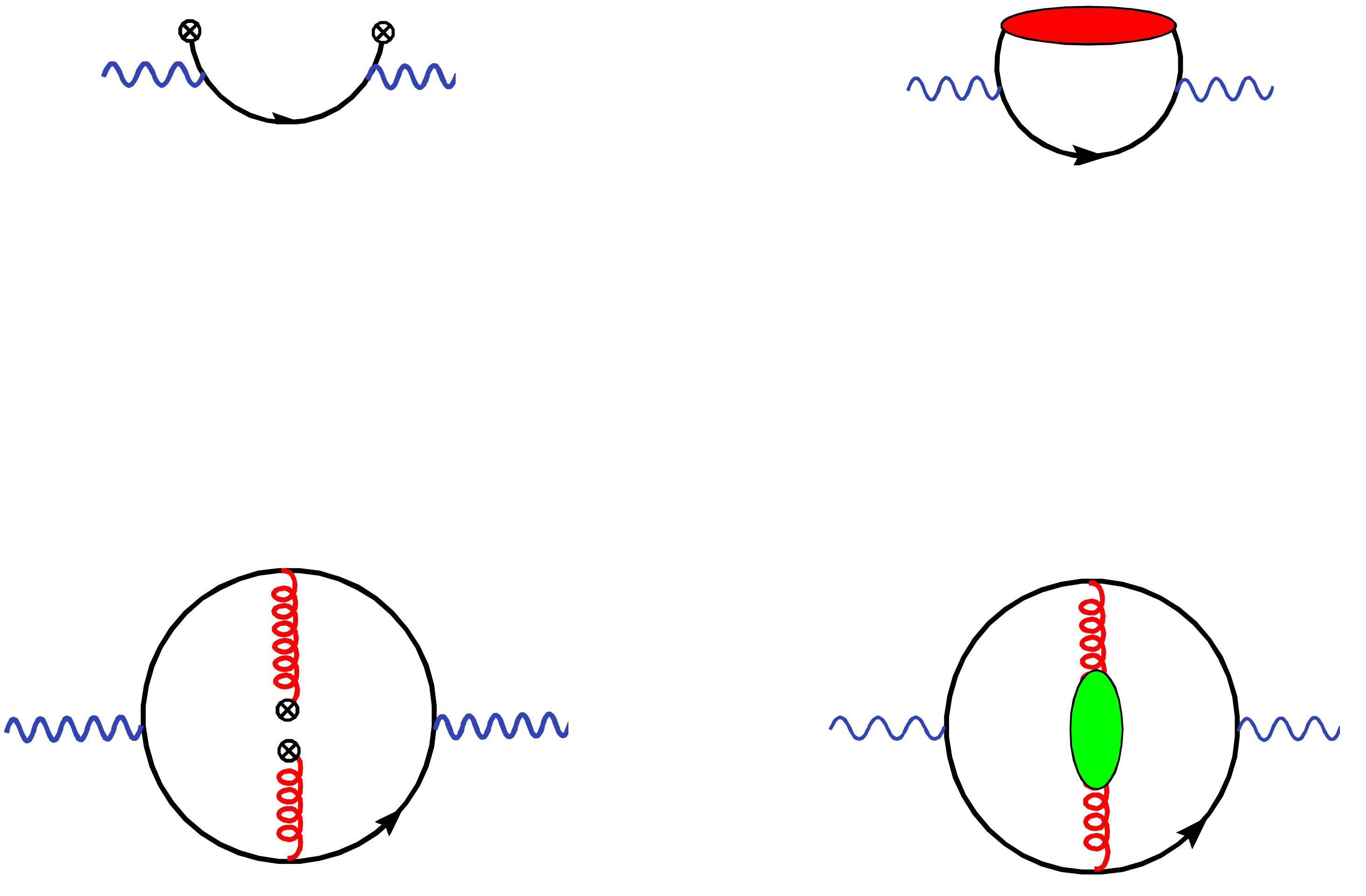
- INCORPORATE CONFINEMENT THROUGH A PARAMETRIZATION OF PROPAGATOR CORRECTIONS

IN TERMS OF QUARK & GLUON VACUUM CONDENSATES

$$S_F = \frac{i}{\not{p} - m} \quad \Rightarrow \quad \frac{i}{\not{p} - m + \Sigma(p^2)}$$

$$D_F = \frac{i}{k^2} \quad \Rightarrow \quad \frac{i}{k^2 + \Lambda(p^2)}$$





QUARK CONDENSATE

$$< 0|\, \bar{q}\, q\, |0 >$$

GLUON CONDENSATE

$$\left\langle 0 \,|\, \alpha_s \, G^a_{\mu\nu} \, G^a_{\mu\nu} \,|\, 0 \right\rangle$$

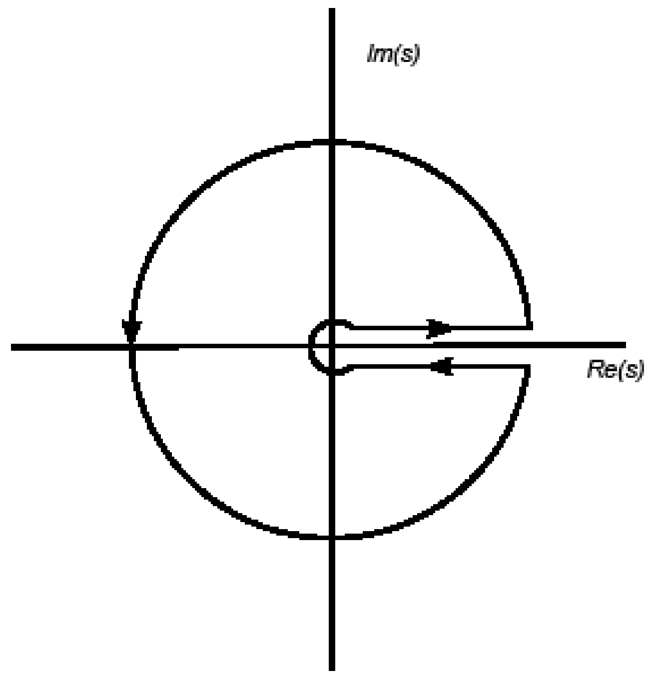
Q C D SUM RULES (SVZ)

$$\Pi(q^2) = \int d^4x\, e^{iqx} < 0 | T(J(x) J^+(0) | 0 >$$

$$\Pi(q^2)|_{QCD} = I + \sum_{N=0} C_{2N+2}(q^2, \mu^2) < 0 | \hat{O}_{2N+2}(\mu^2) | 0 >$$

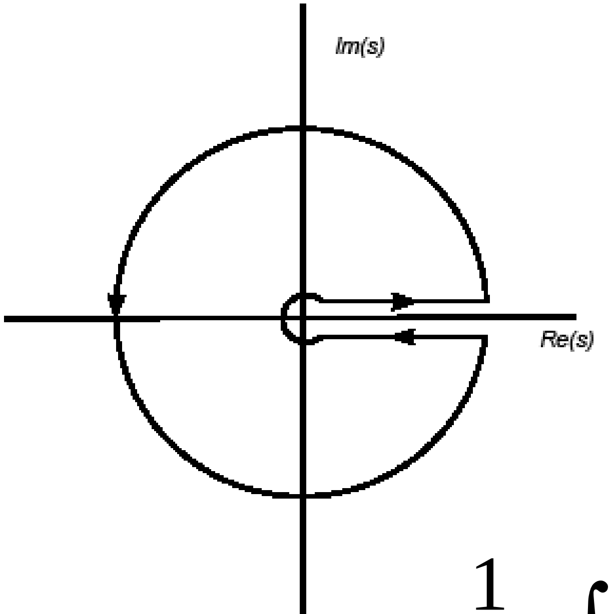
$$I \Rightarrow O(\alpha_s^4) \qquad\qquad C_{2N+2} \Rightarrow \frac{1}{(-q)^{2N+2}}$$

$$m_q < 0 | \bar{q}\, q | 0 >, \quad < 0 | \alpha_s\, G_{\mu\nu} G^{\mu\nu} | 0 >, \quad etc.$$



Im(s)
Re(s)

QUARK-HADRON DUALITY



$$\oint_C \Pi(s)\, ds = 0$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi(s) = \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\,\Pi_{QCD}(s) = \int_{s_{th}}^{s_0} ds\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)\,|_{HAD}$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\,\Pi_{QCD}(s) = 2 f_P^2 M_P^4 + \int_{s_{th}}^{s_0} ds\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi\,(s)|_{RES}$$

$$P = \pi / K$$

$$(m_i + m_j)^2 [1 + \sum_n c_n \, \alpha_s^n + NPQCD]$$

$$= pole + resonances$$

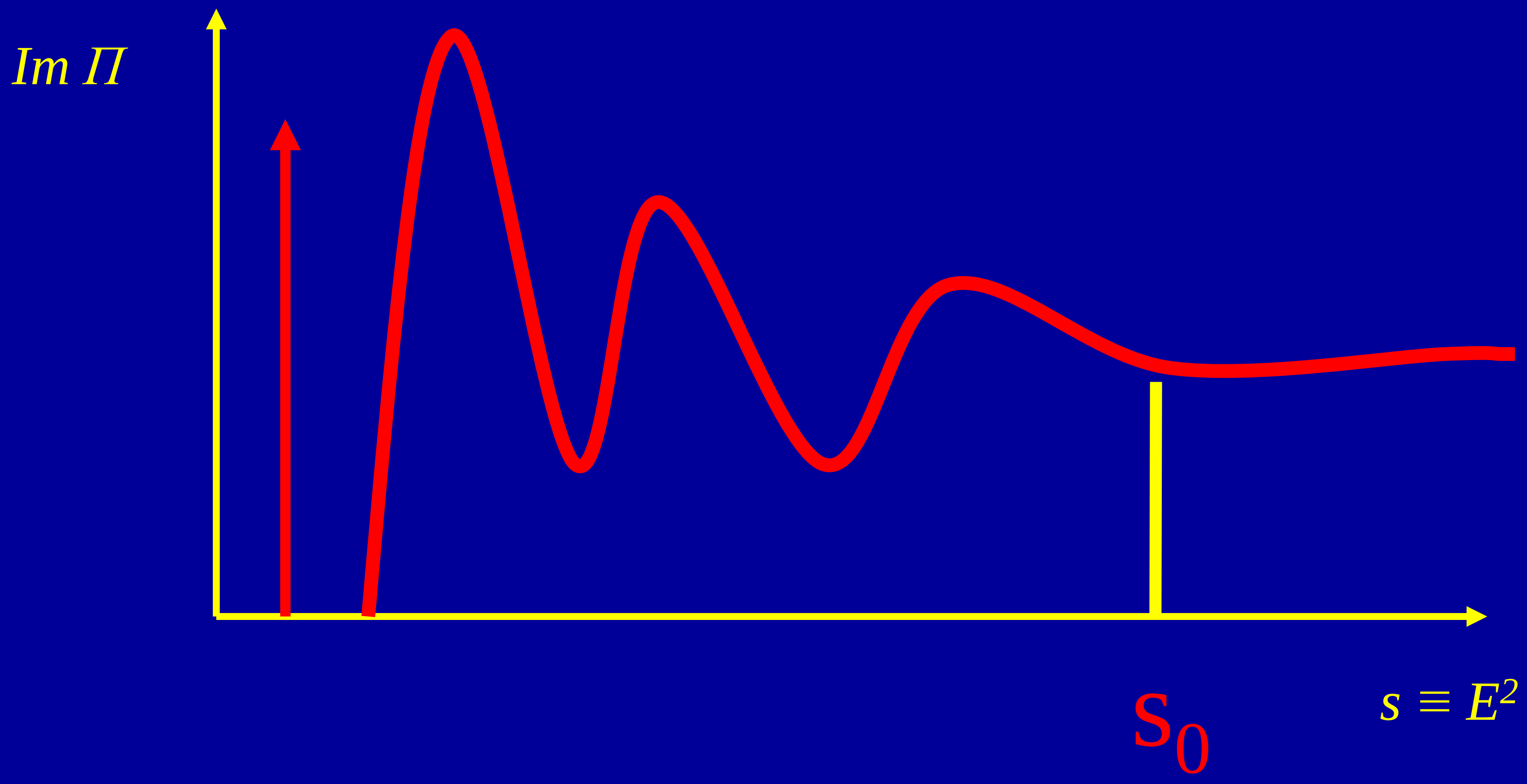
PROBLEM WITH *Im* $\Pi(S)|_{resonance}$

- $e^+ e^- \rightarrow hadrons \Longrightarrow Im\ \Pi(s)|_V$
- $\tau \rightarrow hadrons \Longrightarrow Im\ \Pi(s)|_V\ \&\ Im\ \Pi(s)|_A$

- PSEUDOSCALAR CHANNEL (beyond pole):
- Not measured & not (realistically) measurable

- **SYSTEMATIC UNCERTAINTY**

Realistic Spectral Function



PION (KAON) RADIAL EXCITATIONS

- π (1300): $M = 1300 \pm 100$ MeV
- $\Gamma = 200 - 600$ MeV

- π (1800): $M = 1812 \pm 14$ MeV
- $\Gamma = 207 \pm 13$ MeV

- K (1460) & K (1830) $\Gamma \approx 250$ MeV

SYSTEMATIC UNCERTAINTY

- MASS & WIDTH OF RESONANCES:

- NOT ENOUGH TO RECONSTRUCT HADRONIC SPECTRAL FUNCTION !!!

- HADRONIC BACKGROUND & INELASTICITY & CONSTRUCTIVE/DESTRUCTIVE INTERFERENCE

  - COMPLETELY UNKNOWN

HADRONIC RESONANCE MODEL

(CAD 1984)

- **π_1 & π_2 : 3 π - resonances** (Breit-Wigner)
- **Threshold behaviour controlled by CPT**

SYSTEMATIC UNCERTAINTY

1980's – 2007-2008

CAD, Nasrallah, Schilcher (2007)
CAD, Nasrallah, Röntsch, Schilcher (2008)

INTEGRATION KERNEL

$$\Delta_5 (s)$$

Analytic function

$$\oint ds \; Im \; \Pi(s) \; \Delta_5(s) = 0$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi_{QCD}(s)\,\Delta_5(s) = 2\, f_P^2\, M_P^4\, \Delta_5(M_P^2)$$

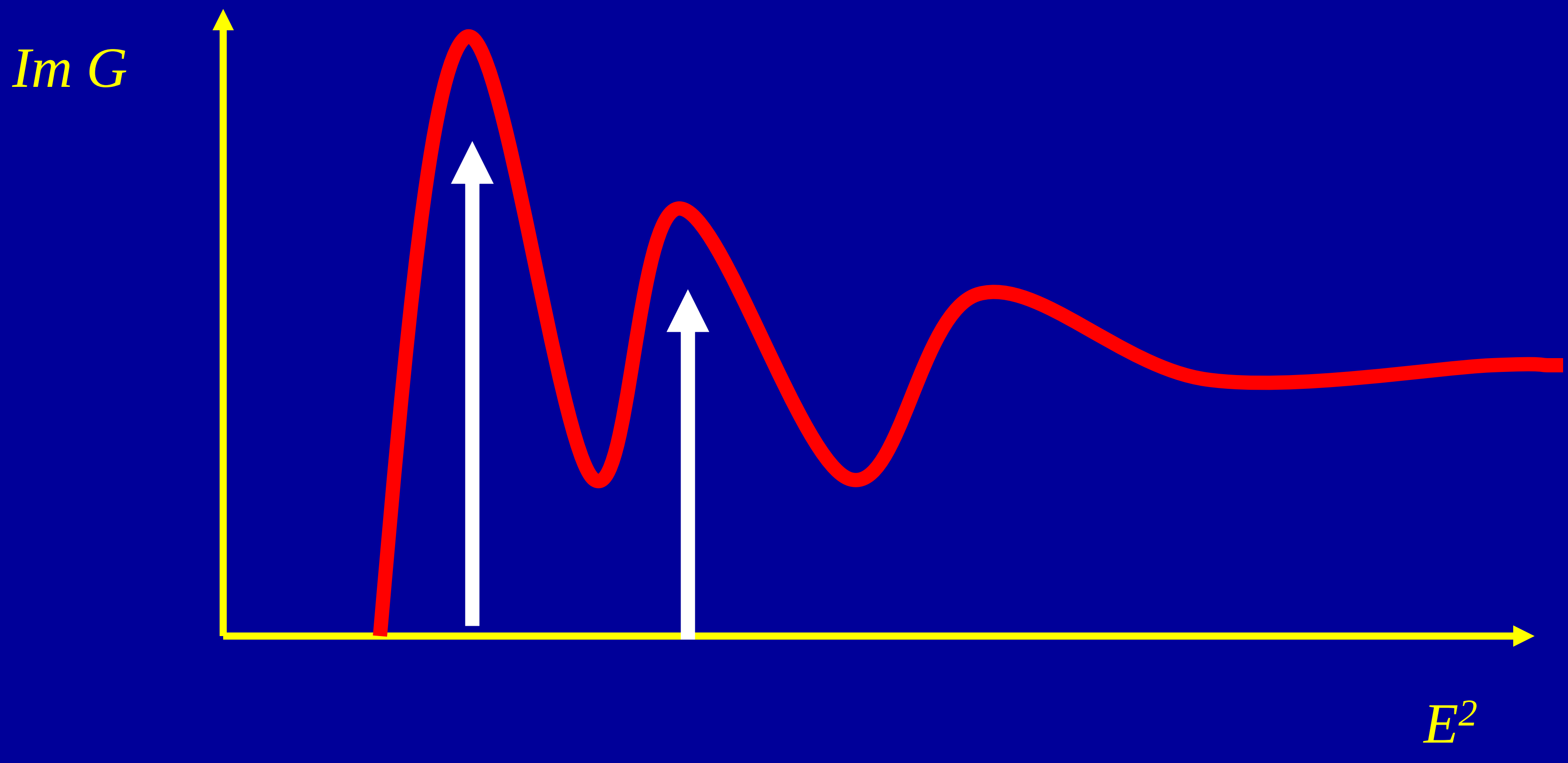
$$+\int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\,\Pi\;(s)\big|_{RES}\; \Delta_5(s)$$

Δ_5 (s)

- $\Delta_5 (s) = 1 - a_0 s - a_1 s^2$
- $\Delta_5 (M_1^2) = \Delta_5 (M_2^2) = 0$

Realistic Spectral Function

IMPACT OF KERNEL $\Delta_5(s)$



$$m_q^2 \propto \frac{\delta_5\,(pole) + \delta_5\,(resonances)}{\delta_5\,(QCD)}$$

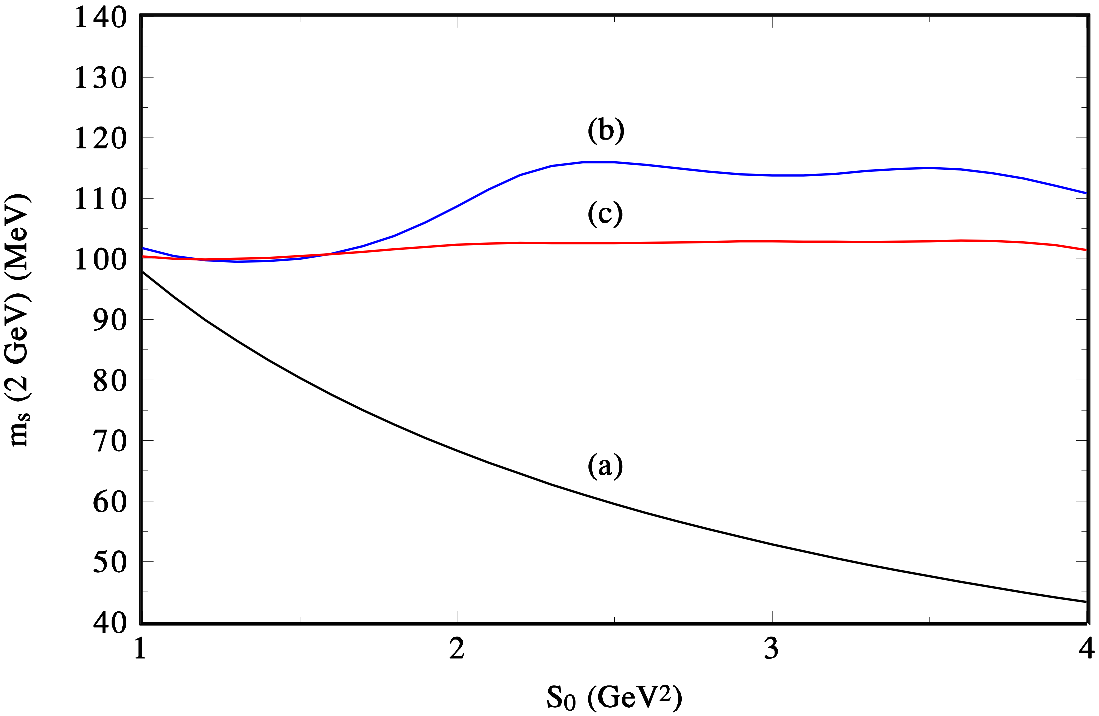
$$\delta_5^{QCD} \propto \oint_{C(|s_0|)} ds\; \psi_5^{QCD}\; \Delta_5\,(s)$$

$$\delta_5^{HAD} \propto \int_0^{s_0} ds\; \mathrm{Im}\,\psi_5^{HAD}\,(s)\; \Delta_5\,(s)$$

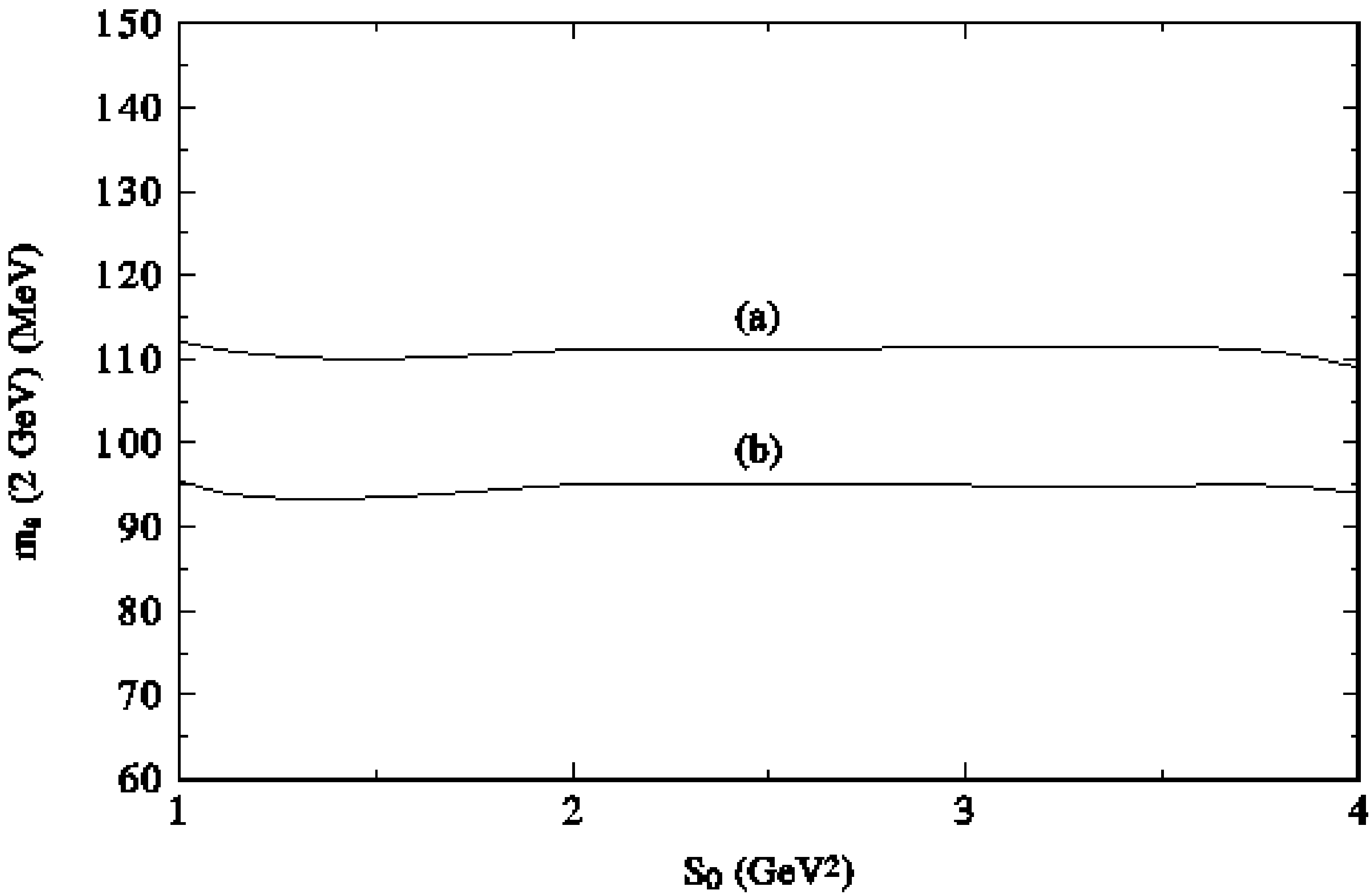
S_0 DEPENDENCE

PHYSICAL QUANTITIES ARE INDEPENDENT OF S_0

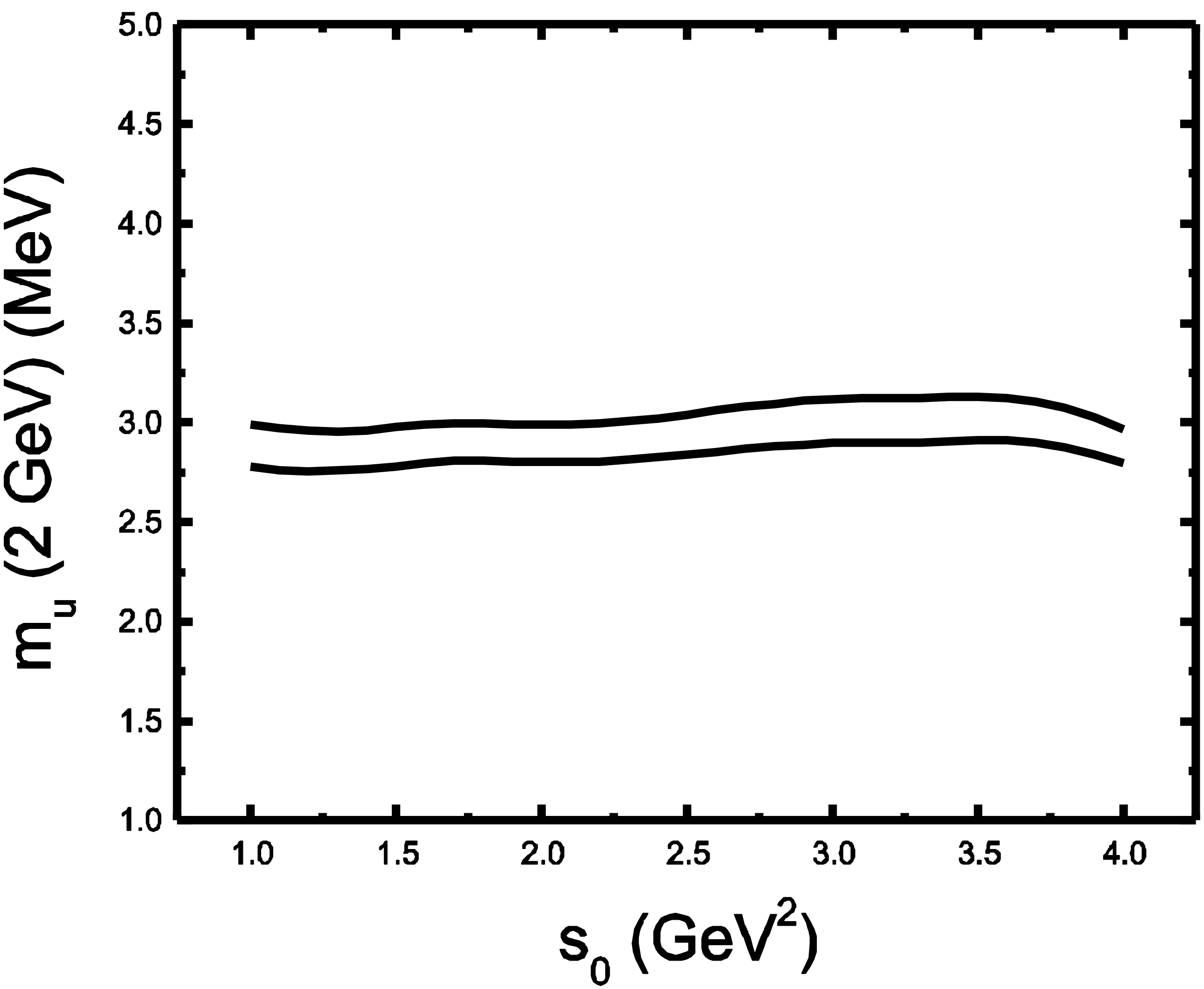
IN PRACTICE : $S_0 \approx 1 - 3\ \mathrm{GeV}^2$

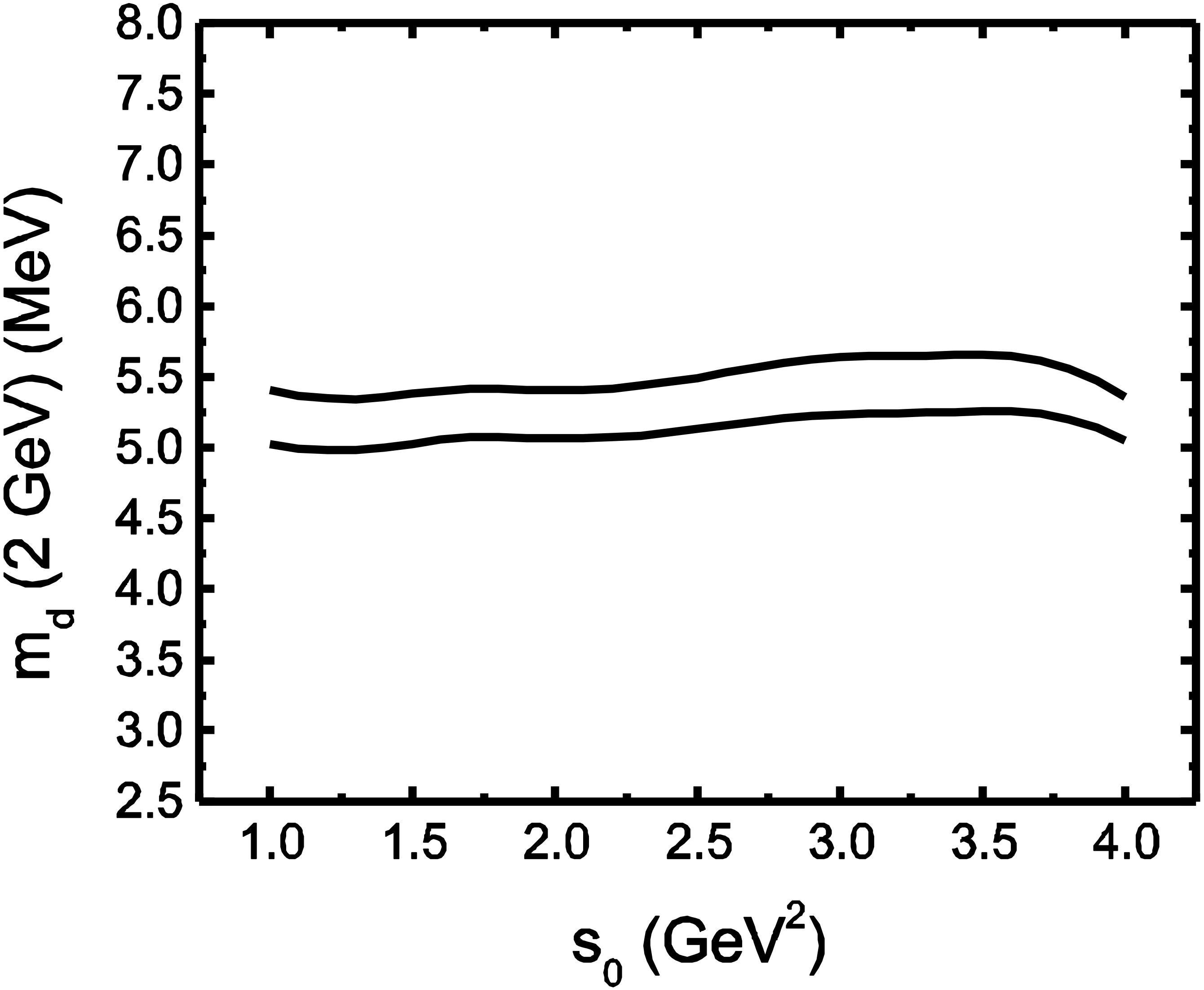


140
130
120
110
100
90
80
70
60
50
40
1
2
3
4
(b)
(c)
(a)
m_s (2 GeV) (MeV)
S_0 (GeV^2)



150
140
130
120
110
100
90
80
70
60
m_q (2 GeV) (MeV)
(a)
(b)
1
2
3
4
S_0 (GeV^2)


5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
m_u (2 GeV) (MeV)
s_0 (GeV^2)



8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
m_d (2 GeV) (MeV)
1.0
1.5
2.0
2.5
3.0
3.5
4.0
s_0 (GeV^2)

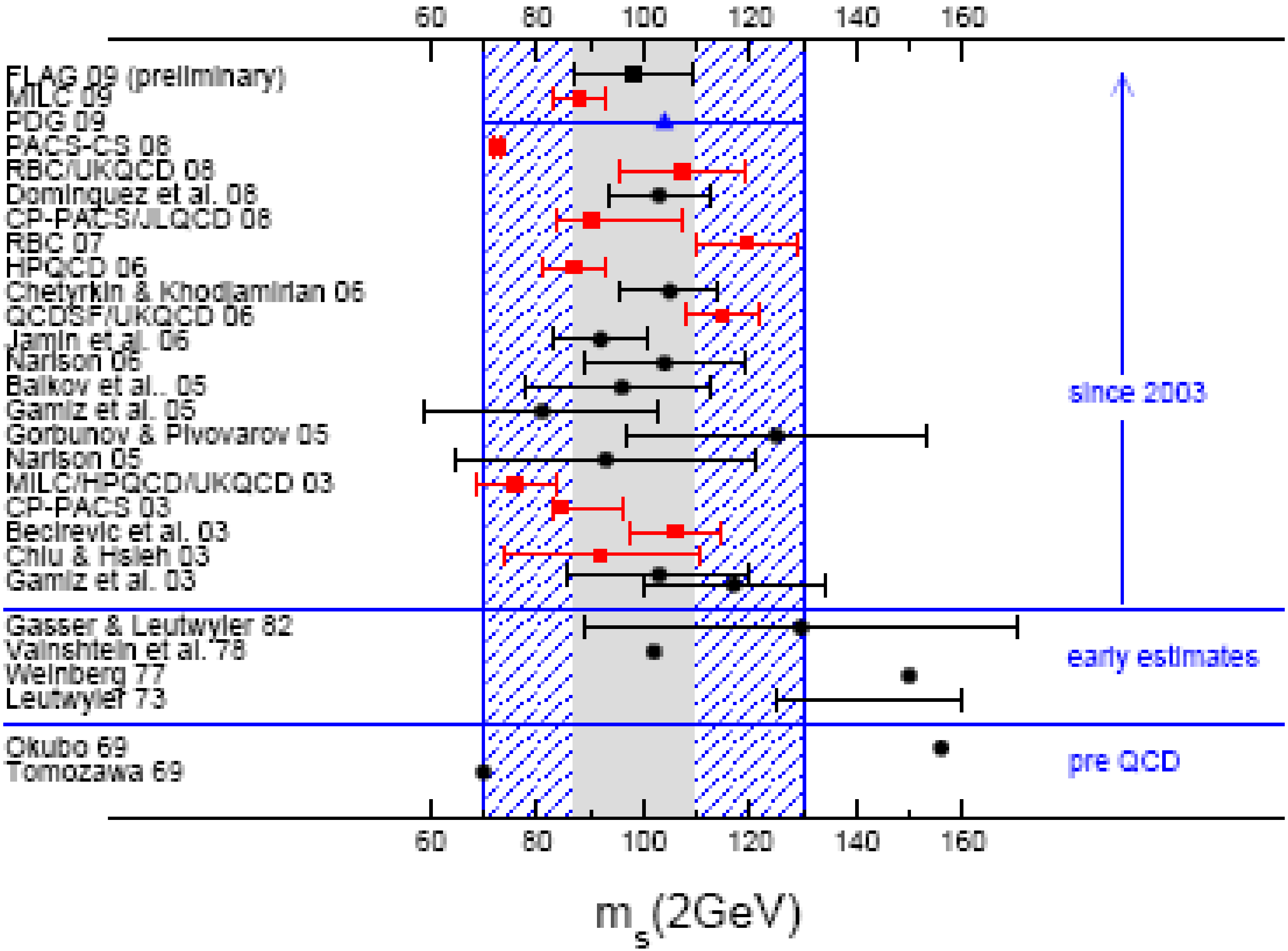
IMPACT OF HADRONIC RESONANCE SPECTRAL FUNCTION

Δ_5 (s)

- Factor 5 smaller than PQCD
- $\pm$ 30 % variation $\Longrightarrow$ < 1% change in m_q

RESULTS

- m_s (2 GeV) = 102 ± 8 MeV
- m_d (2 GeV) = 5.6 ± 0.4 MeV
- m_u (2 GeV) = 2.6 ± 0.3 MeV
- $(m_u + m_d)/2$ = 4.1 ± 0.3 MeV



from: H. Leutwyler

SUMMARY

- A method to decrease substantially the systematic uncertainties from the hadronic resonance sector
- Future improvement from more precise Λ_{QCD} & higher loop order in PQCD
- Comparison with earlier determinations ??

HEAVY QUARK MASSES

QCD

$$\Pi(q^2) = i \int d^4x \; e^{iqx} < 0 | T(J(x) \, J^+(0)) | 0 >$$

$$J(x) \Rightarrow V_\mu(x)|_j^i = \bar{\psi}^i(x) \, i \gamma_\mu \psi_j(x)$$

Karlsruhe Group:

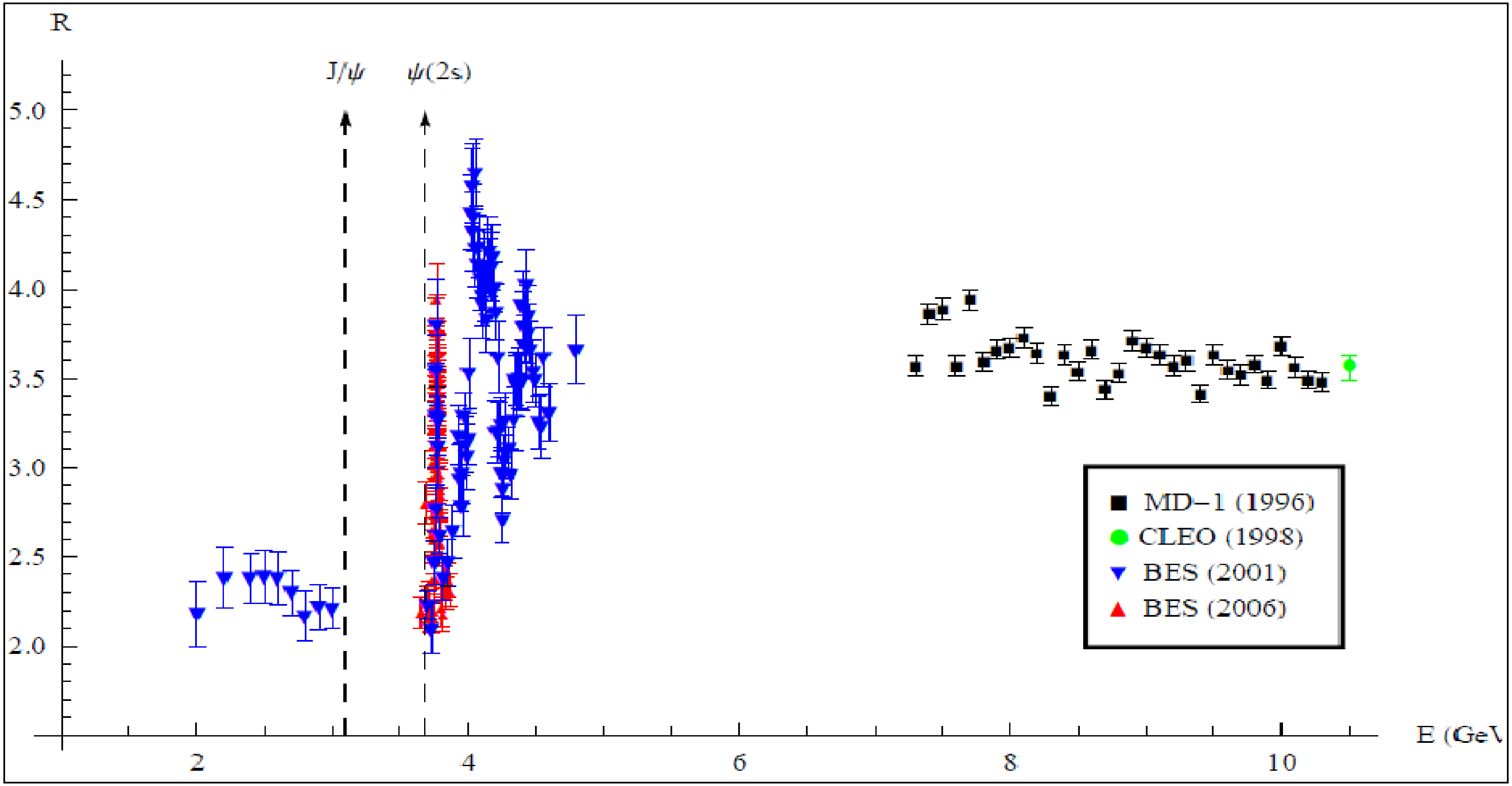
K. Chetyrkin[a], J.H. Kühn[a‡], A. Maier[a], P. Maierhöfer[b], P. Marquard[a], M. Steinhauser[a] and C. Sturm[c]

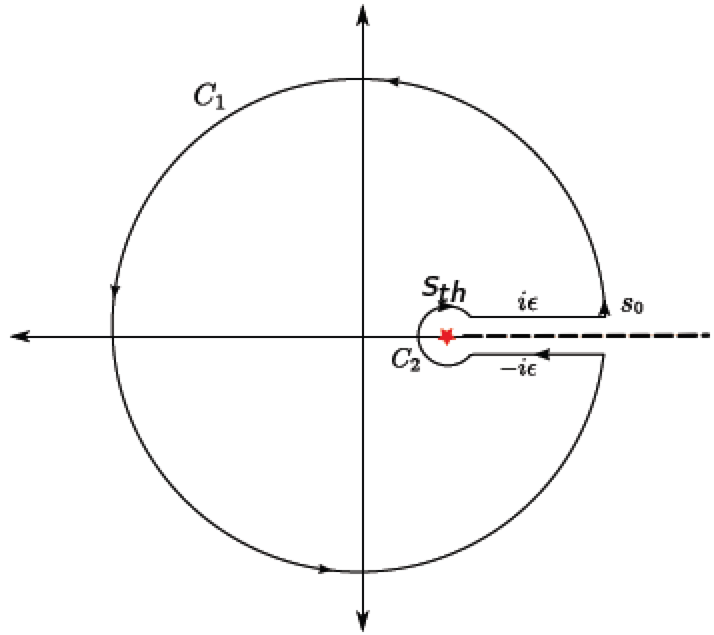
Other recent work:

A. H. Hoang, B. Dehnadi, V. Mateu, S. Mohammad

$e^+ e^- \to$ hadrons

charm-quark region





$$\int_{s_{th}}^{s_0} p(s)\, R_c(s)\, \mathrm{d}s = 6\pi i e_c^2 \int_{|s|=s_0} \Pi(s)\, p(s)\, \mathrm{d}s$$
$$+ 12\pi^2 e_c^2\, \mathrm{Res}[\Pi(s)\, p(s), s = 0]$$

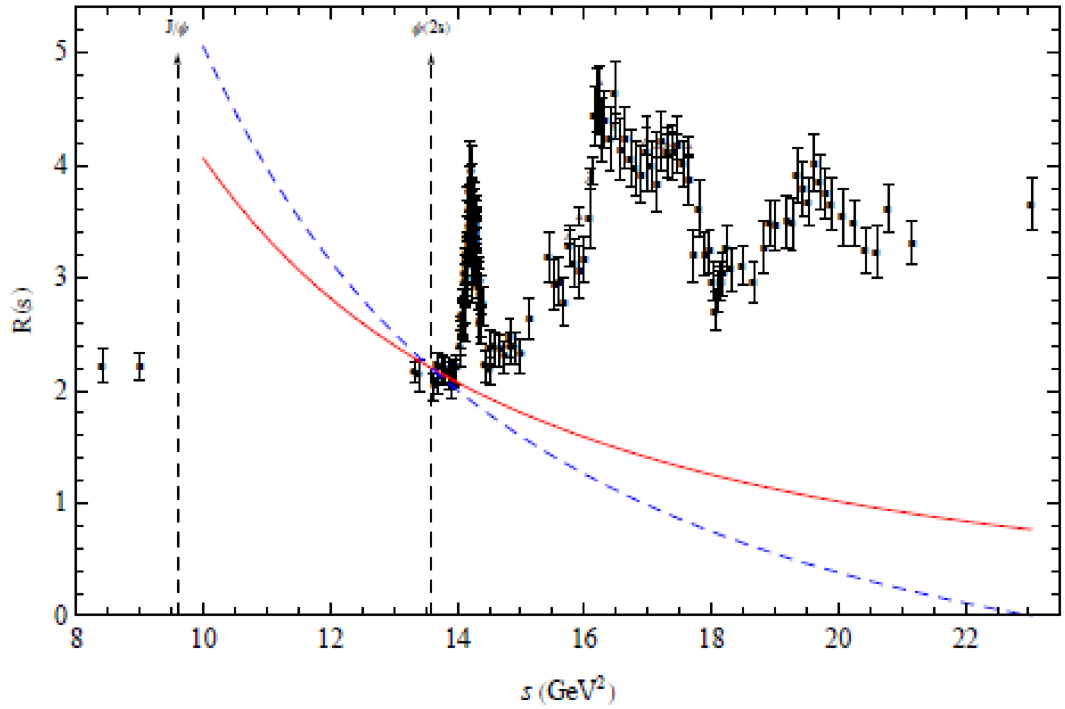


Figure: Red line is $p(s) = s^{-2}$ and blue dashed is $p(s) = 1 - \frac{s_0^2}{s^2}$.

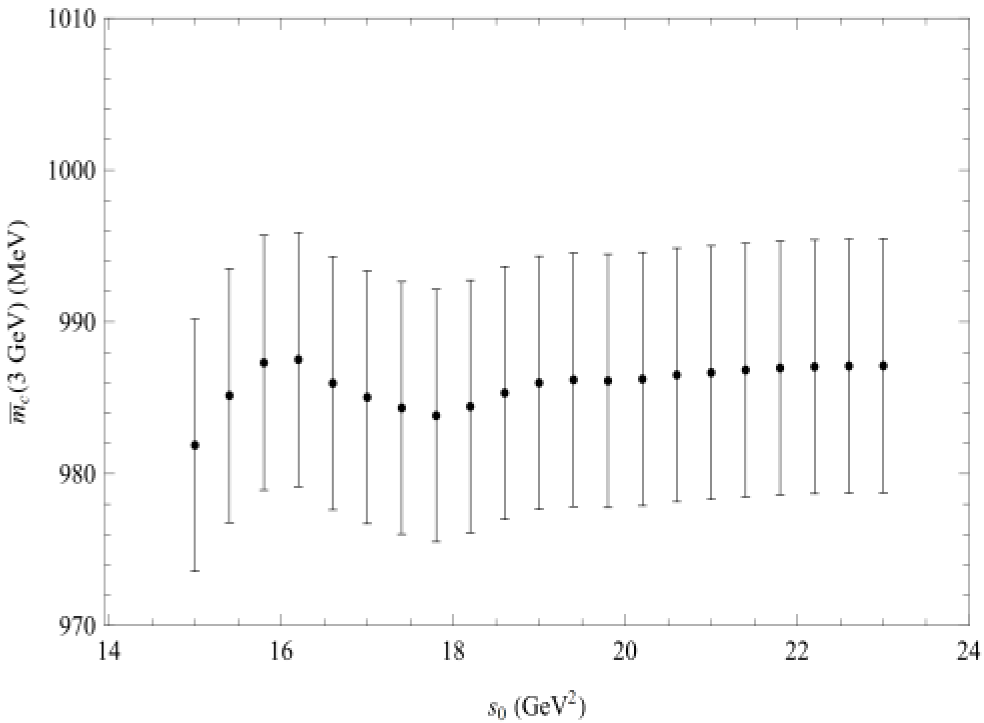


Figure: $\bar{m}_c(3\,\mathrm{GeV})$ using $p(s) = 1 - \frac{s_0^2}{s^2}$ for different values of s_0.

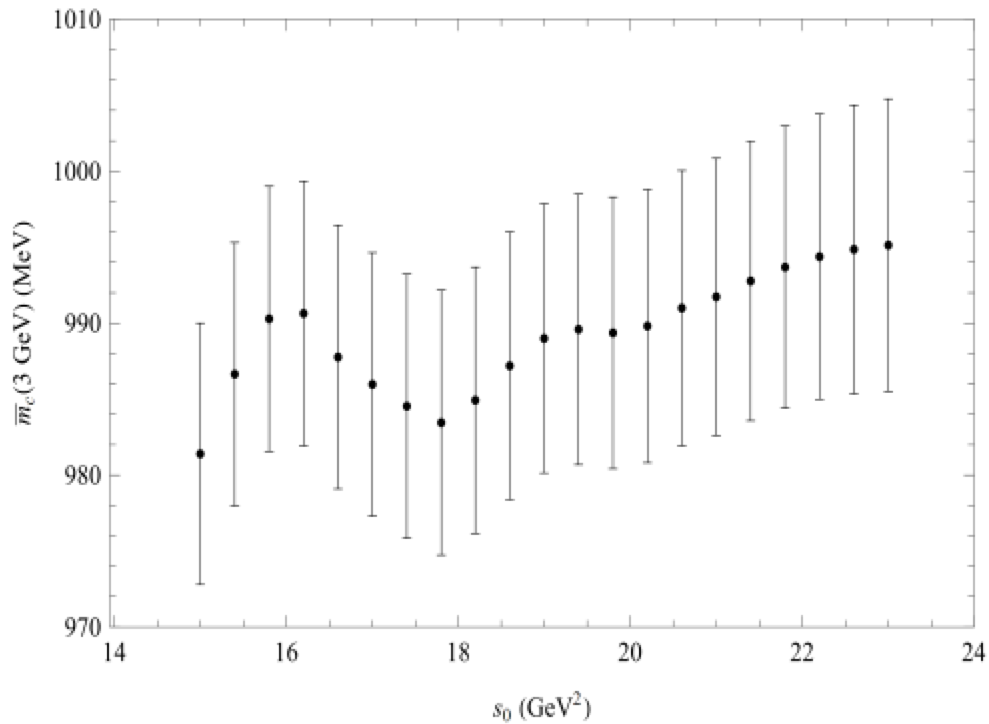


Figure: $\bar{m}_c(3\,\mathrm{GeV})$ using $p(s) = s^{-2}$ for different values of s_0.

Kernel	$\bar{m}_c(3\,\mathrm{GeV})$	Uncertainties (in MeV)					
		EXP	$\Delta\alpha_s$	$\Delta\mu$	NP	s_0	Total
s^{-2}	995	9	3	1	1	14	17
$1-(s_0/s)^2$	987	7	4	1	1	4	9

Figure: The various uncertainties due to the data (EXP), the value of α_s ($\Delta\alpha_s$), changes of $\pm 35\%$ in the renormalization scale around $\mu = 3 GeV (\Delta\mu)$, the value of the gluon condensate (NP), and due to variations in s_0.

S. Bodenstein, J. Bordes, CAD, J. Penarrocha, K.Schilcher

$m_c(3\text{ GeV}) = 1008 \pm 26\text{ MeV}$ (2010)

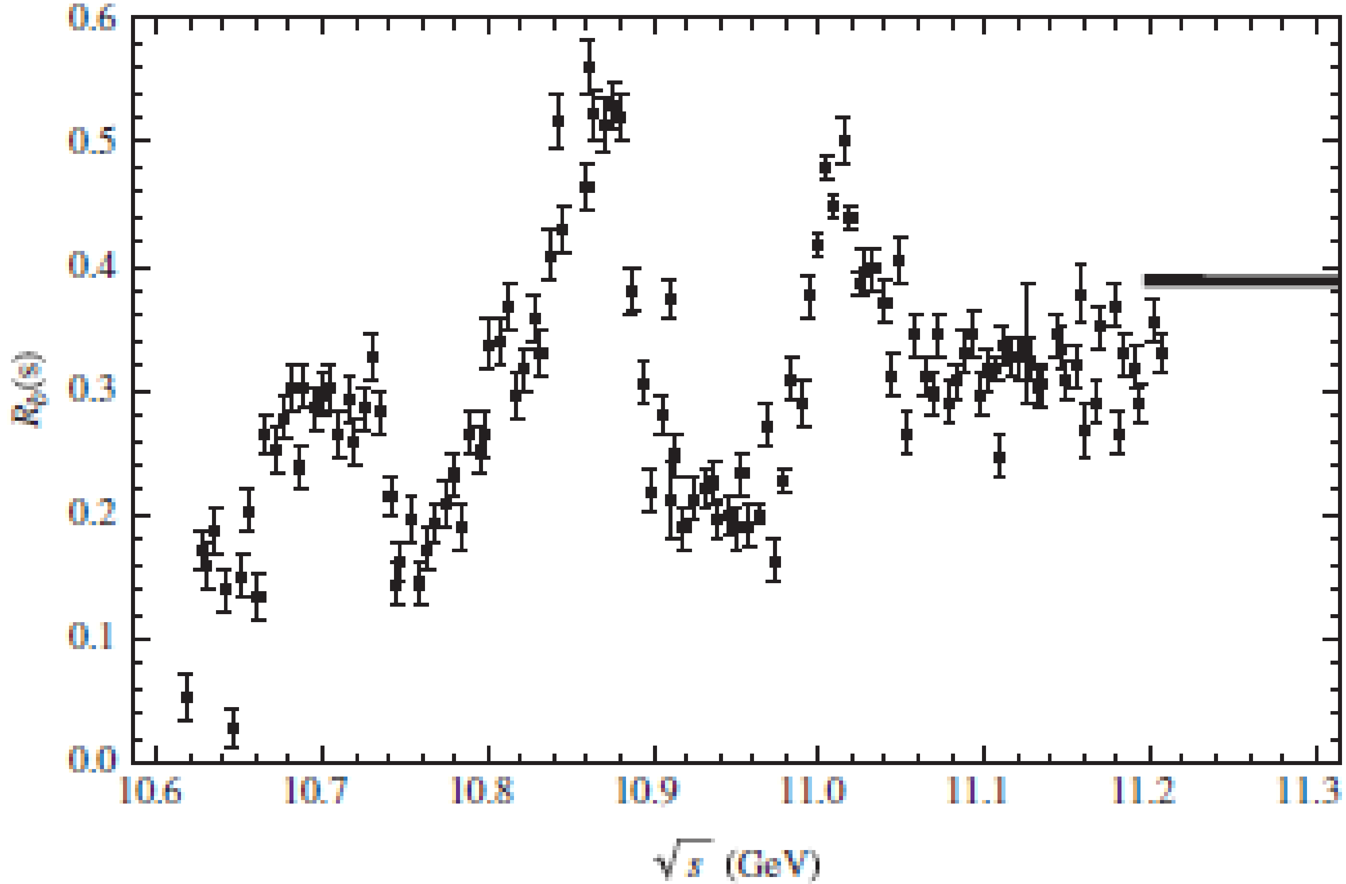
$m_c(3\text{ GeV}) = 987 \pm 9\text{ MeV}$ (2011)

$m_c(3\text{ GeV}) = 986 \pm 13\text{ MeV}$ (Karlsruhe group, K. Chetyrkin *et al.*)

$m_c(3\text{ GeV}) = 998 \pm 29\text{ MeV}$ (A. Hoang *et al.*)

$m_c(3\text{ GeV}) = 986 \pm 6\text{ MeV}$ (Lattice QCD C. McNeile *et al.*)

$m_b : m_b$ (10 GeV)



0.6
0.5
0.4
0.3
0.2
0.1
0.0
$R_b(s)$
10.6
10.7
10.8
10.9
11.0
11.1
11.2
11.3
$\sqrt{s}$ (GeV)

$m_b(10\ \text{GeV}) = 3623 \pm 9\ \text{MeV}$

S. Bodenstein, J. Bordes, CAD, J. Penarrocha, K.Schilcher (2012)

$m_b(10\ \text{GeV}) = 3610 \pm 16\ \text{MeV}$

$3619 \pm 18\ \text{MeV}$

(Karlsruhe group, K. Chetyrkin *et al.*) (2009-2010)

$m_b(10\ \text{GeV}) = 3617 \pm 25\ \text{MeV}$

Lattice QCD, C. Mc Neile et *al.* (2010)

THANK YOU FOR YOUR ATTENTION