# Higgs inflation and Quantum Gravity

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# Outline

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- Motivation

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- New physics at gravitational level
- Strong quantum gravity effects

# Inflation

Isotropic, homogeneous universe $\Rightarrow$ FRW metric

$$g_{\mu\nu} = \text{diag}\left(1, -a^2, -a^2, -a^2\right) \;, \tag{1}$$

where $a = a(t)$ is a scale factor.

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Example: massive scalar field domination
Problem: $m \sim 10^{13}\,GeV$ - way beyond SM physics

# Inflation from the non-minimal coupling

The action in Jordan frame is of the form of

$$S = \int d^4x \sqrt{-g} \Big[ -U(\phi)R + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \Big] \, , \quad (3)$$

where $U = \frac{1}{2} + f(\phi)$ and $f(\phi)R$ is a non-minimal coupling to gravity. EOMs from Lagrange equations.

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For $\xi\phi^2 \gg 1$, $\lambda \gg \xi m^2$ and $\dot{\phi}^2 \ll V(\phi)$ one obtains

$$\epsilon \sim \eta \sim \frac{1}{\xi\phi^2} \ll 1 \ , \qquad \xi = 47000\sqrt{\lambda}$$

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One obtains accelerated expansion of space-time!

Inflation ends for $\phi \sim \xi^{-1/2} \ll M_{pl}$

# Loop Quantum Gravity corrections

One can describe gravity by the Hamiltonian and Hamilton equations. Let us define Ashtekar canonical variables $c, p$ and their Hamiltonian by

$$p = a^2 \;, \qquad c = \gamma \dot{a} \;, \qquad \mathcal{H} = -\frac{3}{\gamma^2} c^2 \sqrt{p} + \mathcal{H}_{matt} \;. \qquad (4)$$

Classical equations of motion can be obtained from

$$\dot{c} = \{c, \mathcal{H}\} \;, \qquad \dot{p} = \{p, \mathcal{H}\} \;, \qquad \{c, p\} = \frac{\gamma}{3} \qquad (5)$$

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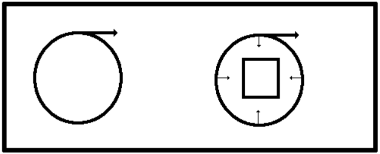
The loop correction



# Loop Quantum Gravity corrections

Assumption: only LQC holonomy correction is important. Then

$$\pi_\phi \to \pi_\phi \ , \qquad p \to p \ , \qquad c \to \frac{1}{\bar{\mu}} \sin\left(\bar{\mu} c\right) \ , \tag{6}$$

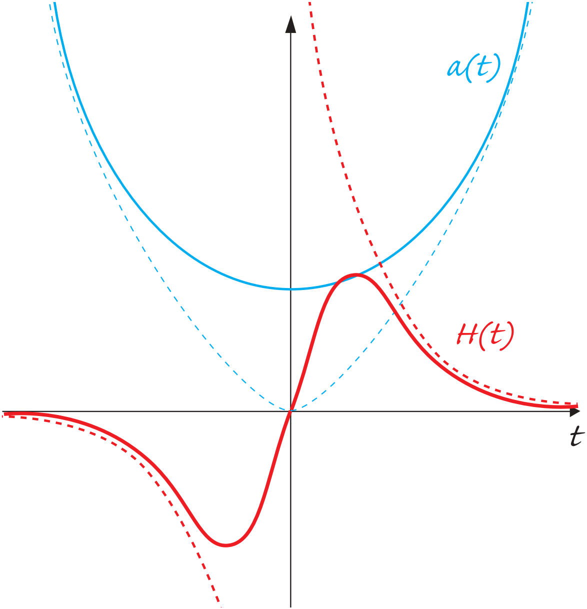
where $\bar{\mu} = \sqrt{\Delta / p}$ and $\Delta$ is a minimal value of area operator. This gives Friedmann equations of the form of

$$3H^2 = \rho \left(1 - \frac{\rho}{\rho_{cr}}\right) \ , \qquad 2\dot{H} = -(\rho + P)\left(1 - 2\frac{\rho}{\rho_{cr}}\right) \ , \tag{7}$$

where $\rho_{cr} = \frac{3}{\Delta\gamma^2}$ is a critical (maximal) energy density.

This leads to the Big Bounce for $\rho = \rho_{cr}$

# LQC in FRW Universe



# Einstein frame

Let us redefine $g_{\mu\nu}$ and $\phi(t)$, so new scalar field would be canonical and minimally coupled

$$\tilde{g}_{\mu\nu} = 2U(\phi)g_{\mu\nu} \,, \quad \left(\frac{d\tilde{\phi}}{d\phi}\right)^2 = \frac{1}{2}\frac{U + 3U'^2}{U^2} \,, \quad \tilde{V}\left(\tilde{\phi}\right) = \frac{1}{4}\frac{V(\phi)}{U^2(\phi)} \,. \tag{8}$$

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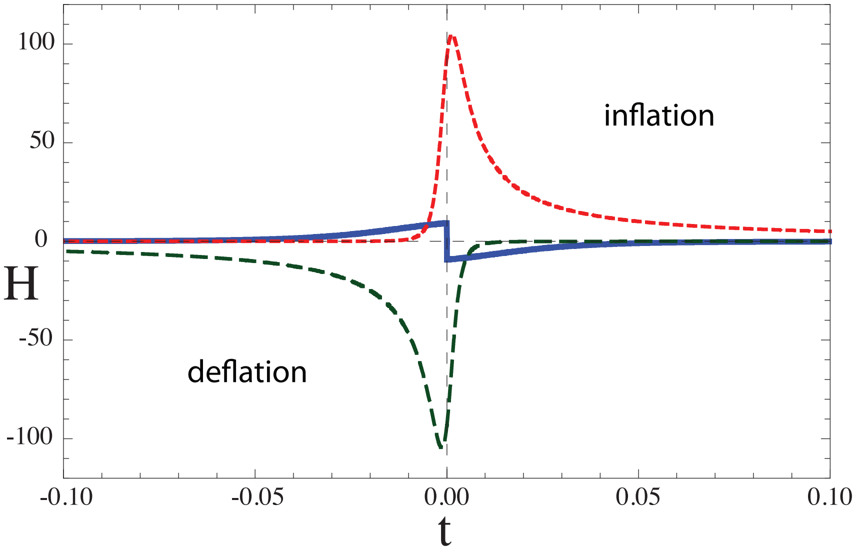
Together with LQC holonomy correction one finds

$$3\tilde{H}^2 = \tilde{\rho}\left(1 - \frac{\tilde{\rho}}{\rho_{cr}}\right) \ , \qquad 2\frac{d\tilde{H}}{d\tilde{t}} = -\left(\tilde{\rho} + \tilde{P}\right)\left(1 - 2\frac{\tilde{\rho}}{\rho_{cr}}\right) \ . \tag{9}$$

Big Bounce for $\tilde{\rho} = \rho_{cr}$!

# Evolution of Hubble parameters



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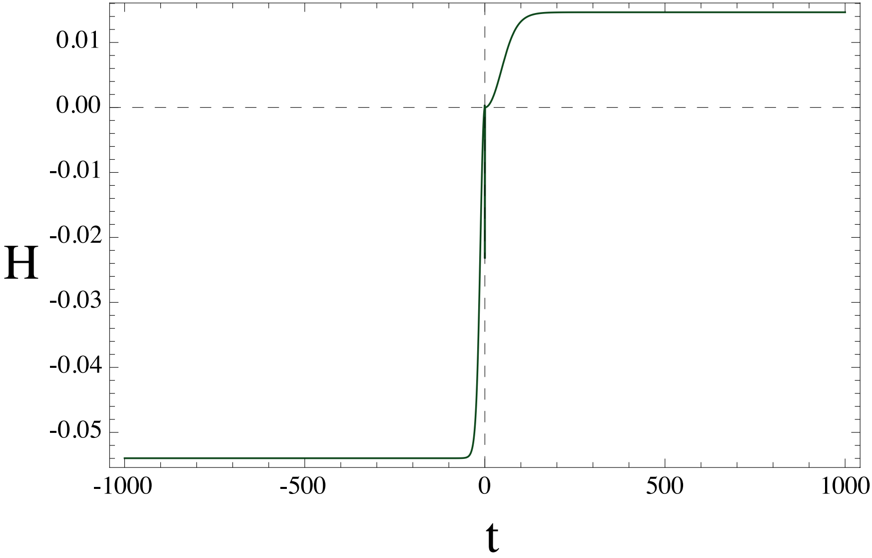
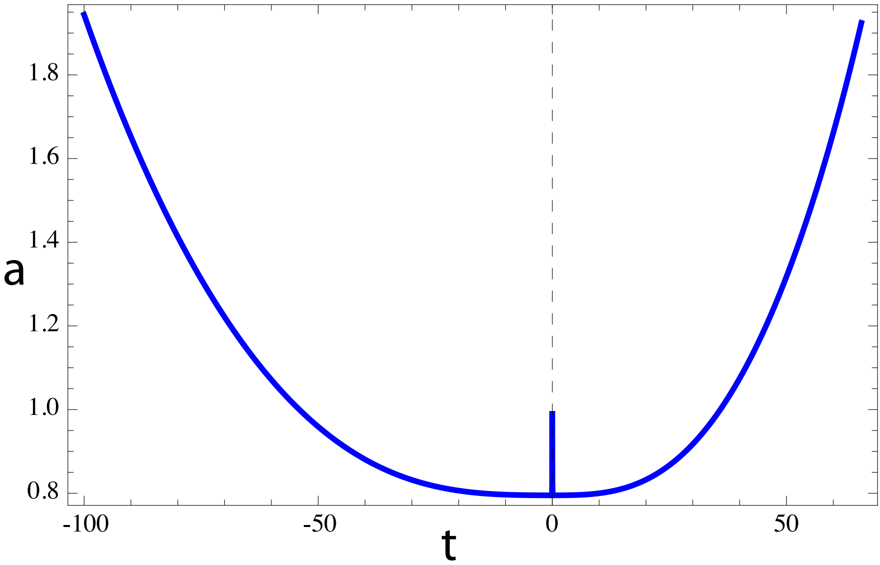
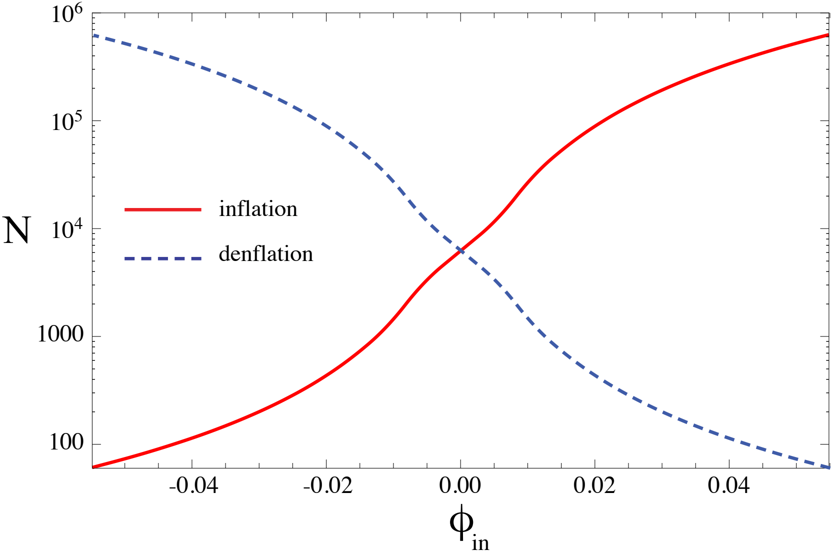


# Evolution of a scale factor



# e-folds



$10^6$
$10^5$
$10^4$
1000
100
N
inflation
denflation
-0.04
-0.02
0.00
0.02
0.04
$\phi_{in}$

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- Solution: non-minimal coupling of Higgs field
- Very small $\epsilon, \eta$ - long and stable inflation
- Strong quantum gravity effects: multiple bounces and anadiabatic evolution
- Natural initial conditions for inflation