How QCD teaches us to look at Cosmological Large Scale Structure
(example of simplicity in the midst of complexity)

John Joseph M. Carrasco
Stanford Institute for Theoretical Physics

based on arXiv:1206.2926 with
Mark Hertzberg (KIPAC/SITP),
Leonardo Senatore* (KIPAC/SITP/SLAC)
Full disclosure: I’m not a professional cosmologist...

Day job is fairly formal high energy theory: Amplitudes

4-loop, 4-pt
N=4 sYM &
N=8 SUGRA
(2012)

Bern, JJMC, Dixon, Johansson, Roiban

3-loop, 5-pt
N=4 sYM &
N=8 SUGRA

JJMC, Johansson

\[ M_{n}^{GR} \propto \int \sum_{G \in \text{cubic}} \frac{n(G)\tilde{n}(G)}{D(G)} \]

\[ A_{n}^{YM} \propto \int \sum_{G \in \text{cubic}} \frac{n(G)c(G)}{D(G)} \]

Bern, JJMC, Johansson
today I’m going to talk about:

...in a classical (stochastic) field theory... relevant on scales > 10 megaparsecs

\(< 10^{-30} \text{eV} \)

Let me explain why...
Occasion for HEP Triumphalism

There will be the opportunity to parameterize our ignorance and be left with a predictive theory in a data-driven PHYSICAL PROBLEM

Get to apply our HEP intuition and perturbative techniques using the language of EFT -- long used in HEP & CM
cutoff, renormalization, running are concepts that will appear and prove useful
PHYSICAL PROBLEM of cosmological matter distribution

Large Scale Structure surveys promise tremendous amount of information in the IR

UV theory under control and predictive through simulations

No rigorous calculable IR theory.

SPT assumes pressureless fluids to all scales \(\leftarrow\) uncontrolled approximation.

complex & expensive

theory under control & predictive

UV
How to arrive at a rigorous prediction of large scale matter distribution?
How to arrive at a rigorous prediction of large scale matter distribution?
How to arrive at a rigorous prediction of large scale matter distribution?

Chiral Lagrangian Effective Theory

IR weakly interacting pions

\[ 4\pi F_\pi \approx 1 \text{ GeV} \]

strongly coupled

UV

QCD
Pions strongly coupled

weakly interacting pions

UV
IR

Chiral Lagrangian
Effective Theory

$4 \pi F_\pi \approx 1 \text{ GeV}$

QCD

Fluid description

Nonlinear Scale

$\mathbf{k}_{NL} \approx .1 \text{ 1/Mpc}$

IR
UV

small matter fluctuations

EFT coupling: $\frac{\mathbf{k}}{\mathbf{k}_{NL}} \sim \left( \frac{\delta \rho}{\rho} \right)^{1/2}$

Grav. collapse overtakes expansion

JJMC, Hertzberg, Senatore
Must be well described by rigorous perturbative methods - predictive after input from UV physics

**Effective Fluid description**

- speed of sound, viscosity
- small matter fluctuations
- GR effects

**IR Chiral Lagrangian Effective Theory**

- weakly interacting pions

**UV Lattice calculations**

- $4\pi F_\pi \approx 1$ GeV

**QCD**

**Effective N-body simulations**
Effective cutoff!

Smoothing with length scale $\Lambda^{-1}$

**Effective fluid continuity & Euler eqns** with stress tensor ($[\tau^{ij}]_{\Lambda}$) sourced by short (UV) modes

**BUT we want EFT params!!**

\[ \dot{\rho}_l + 3H\rho_l + \frac{1}{a}\partial_i (\rho_l v^i_l) = 0 , \]
\[ \dot{v}_l^i + H v^i_l + \frac{1}{a} v^j_l \partial_j v^i_l + \frac{1}{a} \partial_i \phi_l = -\frac{1}{a\rho_l} \partial_j [\tau^{ij}]_{\Lambda} \]

Stochastic description by Boltzmann eqn for NR matter in expanding FRW background

\[ k_{NL} \approx 0.1 \text{ 1/Mpc} \]
effective fluid continuity &
Euler eqns with stress
tensor \( [\mathcal{T}^{ij}]_{\Lambda} \) sourced by
short (UV) modes

take expectation value on long
wavelength background & Taylor expand
in long mode fluctuations \( \delta_l \)

\[
\langle [\mathcal{T}^{ij}]_{\Lambda} \rangle_{\delta_l} = \langle [\mathcal{T}^{ij}]_{\Lambda} \rangle_0 + \frac{\partial \langle [\mathcal{T}^{ij}]_{\Lambda} \rangle_{\delta_l}}{\partial \delta_l} \bigg|_0 \delta_l + \ldots
\]

parameterize UV physics
dependence: \( C_s^2, C_{sv}^2, C_{bu}^2 \)
...can measure in
two ways...
Fluid parameters: $C_s^2, C_{sv}^2, C_{bv}^2$

measurable from output of simulations by taking appropriate correlations of honest analytic fields defined by smoothing positions and velocities

$$W_{\Lambda}(\vec{x}) = \left( \frac{\Lambda}{\sqrt{2\pi}} \right)^3 e^{-\frac{1}{2}\Lambda^2 x^2}$$

$$W_{\Lambda}(k) = e^{-\frac{1}{2} \frac{k^2}{\Lambda^2}}$$

$\rho_l = \Lambda-$Gaussian smoothed particle positions

$\phi_l = $ Newtonian grav potential sourced by $\rho_l$
Fluid parameters: $C_s^2, C_{sv}^2, C_{bv}^2$

measurable from output of simulations by taking appropriate correlations of honest analytic fields defined by smoothing positions and velocities

$$W_\Lambda(\vec{x}) = \left( \frac{\Lambda}{\sqrt{2\pi}} \right)^3 e^{-\frac{1}{2} \Lambda^2 x^2}$$

$$W_\Lambda(k) = e^{-\frac{1}{2} \frac{k^2}{\Lambda^2}}$$

$\rho_l = \Lambda$–Gaussian smoothed particle positions

$\phi_l = $ Newtonian grav potential sourced by $\rho_l$

Consuelo:

$10^9$ particles, in $(420 \text{ Mpc})^3$

Parameter measurement efficiently parallelizable from a $10^6$ particle downsample in the UV (<20 Mpc)

Consuelo data from Busha & Wechsler / LASDAMAS Collab (McBride, et. al. 2012 in prep)
GO STRAIGHT TO OBSERVATION, using the EFT perturbative calculation

can measure the physical parameter \( c^2_{\text{comb}} \) through observations of powerspectrum at some scale
-- all other scales predictive

\[
P^{1\text{-loop}}_{\delta\delta} = P_{11}(k, a) + \left( P_{22}(k, a) + P_{13}(k, a) + P_{13, c^2_{\text{comb}}} \right) \]

(linear)

\[
P_{11}(k, a_0) = \langle \delta(\vec{k}, a_0) \delta(\vec{q}, a_0) \rangle', \]
\[
P_{22}(k, a_0) = \langle \delta^{(2)}(\vec{k}, a_0) \delta^{(2)}(\vec{q}, a_0) \rangle', \]
\[
P_{13}(k, a_0) = 2\langle \delta^{(3)}(\vec{k}, a_0) \delta^{(1)}(\vec{q}, a_0) \rangle', \]
\[
P_{13, c^2_{\text{comb}}}(k, a_0) = 2\langle \delta^{(3)}_{c_{\text{comb}}}(\vec{k}, a_0) \delta^{(1)}(\vec{q}, a_0) \rangle
\]

\[
\delta(a, k) = \lambda \delta^{(1)} + \lambda^2 \delta^{(2)} \ldots
\]
\[
\theta(a, k) = \lambda \theta^{(1)} + \lambda^2 \theta^{(2)} \ldots
\]
\( P_{11}(k, a_0) = \langle \delta(\vec{k}, a_0)\delta(\vec{q}, a_0) \rangle' \),
\( P_{22}(k, a_0) = \langle \delta^{(2)}(\vec{k}, a_0)\delta^{(2)}(\vec{q}, a_0) \rangle' \),
\( P_{13}(k, a_0) = 2\langle \delta^{(3)}(\vec{k}, a_0)\delta^{(1)}(\vec{q}, a_0) \rangle' \),
\( P_{13, c_{\text{comb}}^2}(k, a_0) = 2\langle \delta^{(3)}_{c_{\text{comb}}^2}(\vec{k}, a_0)\delta^{(1)}(\vec{q}, a_0) \rangle' \)

But this cutoff dependence cancels for physical observables -- the **counterterm** exists to eat up the UV cutoff dependence of \( P_{13} \)

Can measure params at finite \( \Lambda \) and renormalize by taking \( \Lambda \rightarrow \infty \)
Measuring fluid parameters

Running of $c^2_{\text{comb}}(\Lambda)$ at $k_{\text{ext}} = 0.01$, $a = 1$

- $k_{\text{ren}} = 0.1 \, h \, \text{Mpc}^{-1}$ (CAMB)
- $k_{\text{ren}} = 0.18 \, h \, \text{Mpc}^{-1}$ (CAMB)
- Running from Consuelo at $\Lambda = 1/3 \, (h/Mpc)$

Consuelo data from Busha & Wechsler / LASDAMAS Collab (McBride, et. al. 2012 in prep)

Saturday, June 30, 12
Even linear does great for much of this k-range ...

but note: log-log plot
Comparison with nonlinear power spectrum (CAMB):
nonlinear power spectrum normalized.
The punchline: "1-loop" prediction

Comparison with nonlinear power spectrum (CAMB):
nonlinear power spectrum normalized.

![Graph showing comparison between different power spectrum predictions](image)

- **SPT**
- **Linear**
- **Non-linear (CAMB)**
- **EFT \( K_{\text{ren}} = 0.16 \ h \ Mpc^{-1} \)**

**Figure 6:**
The order 4 prediction from our EFT is compared with the CAMB non-linear output in the top, and to the no-wiggle power spectrum in the bottom, as well with the linear theory and Standard Perturbation Theory (SPT). The results from the EFT agree at percent level with the non-linear theory up to \( k = 0.24 h \ Mpc^{-1} \), when some high scale power seems to be missing. Results should improve already by going to 5th order. The results are remarkably better than using SPT. The no-wiggle power spectrum we use is given by:

\[
P_{\delta_i,\delta_i}^{\text{No Wiggle}} = 5.1 \cdot 10^6 \log_2 \left( \frac{13 + 2 e}{54 q^2 (14 + 731/(457 q + 1))} + \log(13q + 2e) \right)
\]

- JJMC, Hertzberg, Senatore
- Saturday, June 30, 12
Natural next steps:

- Include stochastic $k^4$ counterterm for P22
- Other 2pt observables like $\langle \pi \pi \rangle$
- Higher multiplicity functions
- Higher loops
- Precision measurements in simulations
- Resummation (ala Cosmological RPT)
There's a very physical story here -- and this is the way we do science, finding the right description for the scale of interest
High Energy Physics important and relevant not only for crucial science from the sub-nuclear scale to the Planck scale...

But also for developing ways of thinking and intuition applicable to all scales in the universe

Teaching us how to snatch SIMPLICITY from the JAWS of COMPLEXITY
Figure 9: The numerator and denominators of $c_{2,\text{comb}}$ as measured with smoothing parameter $\sigma = 1/3$ (left) and $\sigma = 1/6$ (right), scaled to similar heights. This allows us to choose a convenient region of measurement to avoid zero over zero contamination. Precision calculations in the future should extend measurements farther into the IR.

Figure 10: Prediction of the non-linear power spectrum without the addition of higher derivative terms, as we send $\sigma \to 1$, normalized to the non-linear power spectrum. We see that if we keep $\sigma$ finite, non-included higher derivative terms that scale as powers of $k/\sigma$ are important. Indeed the results improve as $\sigma = 1$, which is the correct procedure.
\[ f(\vec{x}, \vec{p}) = \sum_n \delta^{(3)}(\vec{x} - \vec{x}_n) \delta^{(3)}(\vec{p} - m \alpha \vec{v}_n) \]

\[ \left[ \frac{Df}{Dt} \right]_\Lambda = \frac{\partial f_i}{\partial t} + \frac{\vec{p}}{ma^2} \cdot \frac{\partial f_i}{\partial \vec{x}} - m \sum_{n, \bar{n}, n\neq \bar{n}} \int d^3x' W_\Lambda(\vec{x} - \vec{x}') \frac{\partial \phi_n}{\partial \vec{x}''}(\vec{x}') \cdot \frac{\partial f_{\bar{n}}}{\partial \vec{p}} \]

\[ \int d^3p \ p^{i_1} \ldots p^{i_n} \left[ \frac{Df}{Dt} \right]_\Lambda (\vec{x}, \vec{p}) = 0 \]

\[ \dot{\rho}_l + 3H \rho_l + \frac{1}{a} \partial_i (\rho_l \nu^i_l) = 0 \]

\[ \nu^i_l(x) \equiv \frac{\pi^i_l(x)}{\rho_l(x)} = \frac{\sum_n \nu^i_n W_\Lambda(x - x_n)}{\sum_n W_\Lambda(x - x_n)} \]

\[ \dot{\nu}^i_l + H \nu^i_l + \frac{1}{a} \nu^j_l \partial_j \nu^i_l + \frac{1}{a} \partial_i \phi_l = -\frac{\rho_b}{a \rho_l} J^i_l \]

\[ J^i_l = \frac{1}{\rho_b} \left( \partial_j (\sigma^{ij}_l - \rho_l \nu^i_l \nu^j_l) + \sum_{n \neq \bar{n}} [\rho_n \partial_i \phi_{\bar{n}}]_\Lambda - \rho_l \partial_i \phi_l \right) \]

\[ [\rho_n \partial_i \phi_{\bar{n}}]_\Lambda = \frac{m^2 G(x^i_n - x^i_{\bar{n}})}{a^4 |x_n - x_{\bar{n}}|^3} (1 + \lambda |x_n - x_{\bar{n}}|) e^{-\lambda |x_n - x_{\bar{n}}|} W_\Lambda(x - x_n) \]
\[
J_l^i = \frac{1}{\rho_b} \left( \partial_j (\sigma_l^{ij} - \rho_l v_i^l v_j^l) + \sum_{n \neq \bar{n}} [\rho_n \partial_i \phi_n]_\Lambda - \rho_l \partial_i \phi_l \right)
\]

\[
[rho_n \partial_i \phi_n]_\Lambda = \frac{m^2 G (x_n^i - x_{\bar{n}}^i)}{a^4 |x_n - x_{\bar{n}}|^3} (1 + \lambda |x_n - x_{\bar{n}}|) e^{-\lambda |x_n - x_{\bar{n}}|} W_\Lambda (x - x_n)
\]

\[
[\tau^{ij}]_\Lambda = \kappa_l^{ij} + \Phi_l^{ij}
\]

\[
\kappa_l^{ij} = \sigma_l^{ij} - \rho_l v_i^l v_j^l
\]

\[
\Phi_l^{ij} = - \frac{w_l^{kk} \delta^{ij} - 2w_l^{ij}}{8\pi G a^2} + \frac{\partial_k \phi_l \partial_k \phi_l \delta^{ij} - 2\partial_i \phi_l \partial_j \phi_l}{8\pi G a^2}
\]

\[
\omega_l^{ij} (x) = \int d^3 x' W_\Lambda (x - x') \left( \partial_i \phi (x') \partial_j \phi (x') - \sum_n \partial_i \phi_n (x') \partial_j \phi_n (x') \right)
\]
\[
\left[ \tau^{ij} \right] \Delta \delta_l = p_b \delta^{ij} + \rho_b \left[ c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha} \delta^{ij} \partial_k v^k_l \right. \\
- \frac{3}{4} \frac{c_{sv}^2}{Ha} \left( \partial^j v^i_l + \partial^i v^j_l - \frac{2}{3} \delta^{ij} \partial_k v^k_l \right) \\
+ \Delta \tau^{ij} + \cdots
\]

Fluid parameters: \( c_s^2, c_{sv}^2, c_{bv}^2 \)

Long fields: over-density \( \delta_l \)
velocity \( \mathbf{v}_l \)
\[ J^i_l = \frac{1}{\rho_b} \partial_j [\tau^{ij}]_\Lambda \]
\[ A^{ki}_l = \frac{1}{\rho_b} \partial_k \partial_j [\tau^{ij}]_\Lambda = \partial_k J^i_l \]
\[ A^i_l = \frac{1}{\rho_b} \partial_i \partial_j [\tau^{ij}]_\Lambda = \partial_i J^i_l \]
\[ B_l = \frac{1}{\rho_b} \left( \partial_i \partial_j - \frac{\delta^{ij}}{3} \partial^2 \right) [\tau^{ij}]_\Lambda \]

**Definitions**

\[ J^i_l = c_s^2 \partial_i \delta_l + \frac{3}{4} c_{sv}^2 \partial_j \Theta^j_l + \left( \frac{1}{4} c_{sv}^2 + c_{bv}^2 \right) \partial_i \Theta_l \]
\[ A^{ki}_l = c_s^2 \partial_k \partial_i \delta_l + \frac{3}{4} c_{sv}^2 \partial_k \partial_j \Theta^j_l + \left( \frac{1}{4} c_{sv}^2 + c_{bv}^2 \right) \partial_k \partial_i \Theta_l \]
\[ A_l = c_s^2 \partial^2 \delta_l + (c_{sv}^2 + c_{bv}^2) \partial^2 \Theta_l \]
\[ B_l = c_{sv}^2 \partial^2 \Theta_l, \]

**Expression in EFT params**
\[ P_{A\delta}(x) = \langle A_l(\vec{x}' + \vec{x})\delta_l(\vec{x}') \rangle, \]
\[ P_{A\Theta}(x) = \langle A_l(\vec{x}' + \vec{x})\Theta_l(\vec{x}') \rangle, \]
\[ P^{ki}_{A\kappa_i}(x) = \langle A^{ki}_l(\vec{x}' + \vec{x})\Theta_{l\kappa_i}(\vec{x}') \rangle, \]
\[ P_{B\Theta}(x) = \langle B_l(\vec{x}' + \vec{x})\Theta_l(\vec{x}') \rangle, \]
\[ P_{\delta\delta}(x) = \langle \delta_l(\vec{x}' + \vec{x})\delta_l(\vec{x}') \rangle, \]
\[ P_{\delta\Theta}(x) = \langle \delta_l(\vec{x}' + \vec{x})\Theta_l(\vec{x}') \rangle, \]
\[ P_{\Theta\Theta}(x) = \langle \Theta_l(\vec{x}' + \vec{x})\Theta_l(\vec{x}') \rangle, \]
\[ P^{ji}_{\Theta\kappa_i}(x) = \langle \Theta^{ji}_l(\vec{x}' + \vec{x})\Theta^k_{l\kappa_i}(\vec{x}') \rangle, \]

Figure 9: Numerator and Denominator of \( c^2_{\text{comb}} \) for \( \Lambda = 1/6 \)

\[
\Lambda = \frac{1}{6} \text{ measurement of } c^2_{\text{comb}}
\]

\[
c^2_{s} = a^2 \frac{P_{A\Theta}(x)\partial^2 P_{\delta\Theta}(x) - P_{A\delta}(x)\partial^2 P_{\Theta\Theta}(x)}{(\partial^2 P_{\Theta\Theta}(x))^2 - \partial^2 P_{\delta\delta}(x)\partial^2 P_{\Theta\Theta}(x)}
\]
\[
c^2_{v} = a^2 \frac{P_{A\delta}(x)\partial^2 P_{\delta\Theta}(x) - P_{A\Theta}(x)\partial^2 P_{\delta\delta}(x)}{(\partial^2 P_{\delta\Theta}(x))^2 - \partial^2 P_{\delta\delta}(x)\partial^2 P_{\Theta\Theta}(x)}
\]
Stochastic eqns of motion

\[ a\mathcal{H}\delta' + \theta_l = -\int \frac{d^3q}{(2\pi)^3} \alpha(\vec{q}, \vec{k} - \vec{q})\delta_l(\vec{k} - \vec{q})\theta_l(\vec{q}), \]

\[ a\mathcal{H}\theta'_l + \mathcal{H}\theta_l + \frac{3\mathcal{H}^2\Omega}{a} \delta - c_s^2k^2\delta_l + \frac{c_v^2k^2}{\mathcal{H}}\theta_l = -\int \frac{d^3q}{(2\pi)^3} \beta(\vec{q}, \vec{k} - \vec{q})\theta_l(\vec{k} - \vec{q})\theta_l(\vec{q}) \]

\[ \delta(a, k) = \lambda\delta^{(1)} + \lambda^2\delta^{(2)} \ldots \]

\[ \theta(a, k) = \lambda\theta^{(1)} + \lambda^2\theta^{(2)} \ldots \]

\[ c_{\text{comb}}^2 = c_s^2 + f(a)\{c_v^2 \equiv c_{\text{sv}}^2 + c_{\text{bf}}\} \]

First nonlinear correction to powerspectrum (1-loop):

\[ (\lambda^2) P_{11}(k, a_0) = \langle \delta(\vec{k}, a_0)\delta(\vec{q}, a_0) \rangle' , \]

\[ (\lambda^4) P_{22}(k, a_0) = \langle \delta^{(2)}(\vec{k}, a_0)\delta^{(2)}(\vec{q}, a_0) \rangle' , \]

\[ (\lambda^4) P_{13}(k, a_0) = 2\langle \delta^{(3)}(\vec{k}, a_0)\delta^{(1)}(\vec{q}, a_0) \rangle' , \]

\[ (\lambda^4) P_{13, c_{\text{comb}}^2}(k, a_0) = 2\langle \delta^{(3)}_{c_{\text{comb}}}(\vec{k}, a_0)\delta^{(1)}(\vec{q}, a_0) \rangle' \]
Continuous green lines represent Green’s functions, red dashed lines represent free fields, and red crosses circled by a dotted blue line represent correlation among free fields.
Continuous green lines represent Green’s functions, red dashed lines represent free fields, and red crosses circled by a dotted blue line represent correlation among free fields.