The chiral magnetic effect in strongly–coupled anisotropic plasma

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I.G., Tigran Kalaydzhyan, Ingo Kirsch arXiv:1203.4259

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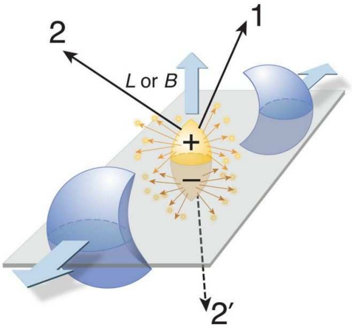
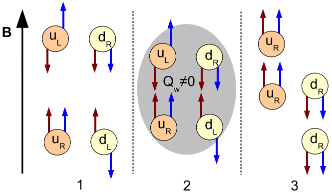
Motivation



- The chiral magnetic effect represents remarkable implications of anomalies in QFT
- The chiral magnetic effect is a good candidate for the explanation of an experimentally observed charge asymmetry in heavy-ion collisions at RHIC
- Lattice QCD results suggest the existence of the effect

Chiral magnetic effect



The CME is a hypothetical phenomenon which states that, in the presence of a magnetic field $\vec{B}$, an electric current is generated along $\vec{B}$ in the background of topologically nontrivial gluon fields.

$$\vec{j} = C\mu_5\vec{B}$$

[Kharzeev,Fukushima,Warringa,Mclerran]

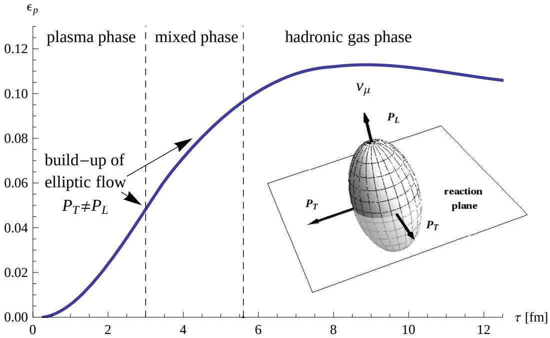
Basics of relativistic hydrodynamics

Hydrodynamics states is completely determined by conservation laws associated with some global symmetries

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu j^\mu = 0$$

In equilibrium, for ideal hydrodynamics

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$
$$j^\mu = \rho u^\mu$$



Why the anisotropic case interesting?

Experimental observation: The charge separation is proportional to the elliptic flow v_2.

Anisotropic fluid

Stress-energy tensor $T^{\mu\nu}$ and $U(1)$ currents j^μ:

$$T^{\mu\nu} = (\epsilon + P_T)u^\mu u^\nu + P_T g^{\mu\nu} - (P_T - P_L)v^\mu v^\nu + \tau^{\mu\nu}$$
$$j^\mu = \rho^{u\mu} + \nu^\mu$$

In the rest frame

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & P_L \end{pmatrix}$$

In presence of anomalies one need to modify the dissipative part of the current

$$j^\mu \to j^\mu + \xi\omega^\mu + \xi_B B^\mu$$

[Erdmenger,Haack,Kaminski,Yarom '08]
[Son & Surowka '09]

These transport coefficients can be computed exactly.

In presence of external fields

Now the energy-momentum tensor and current are not conserved

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda$$
$$\partial_\mu j^{a\mu} = C^{abc} E^b \cdot B^c$$

2nd law of thermodynamics

Positivity of the entropy production completely fixes new transport coefficients

$$\xi_\omega = C\left(\mu^2 - \frac{2}{3}\frac{\rho\mu^3}{\epsilon + P_T}\right) + \mathcal{O}(T^2)$$
$$\xi_B = C\left(\mu - \frac{1}{2}\frac{\rho\mu^2}{\epsilon + P_T}\right) + \mathcal{O}(T^2)$$

Two charge case

- Isotropic case [Sadofyev,Isachenkov '10]
- For an anisotropic case the conductivities

$$\kappa_\omega = 2C\mu_5\left(\mu - \frac{\rho}{\epsilon + P_T}\left[\mu^2 + \frac{\mu_5^2}{3}\right]\right)$$
$$\kappa_B = C\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P_T}\right)$$
$$\kappa_{5,B} = C\mu\left(1 - \frac{1}{2}\frac{\mu\rho}{\epsilon + P_T}\left[1 + \frac{\mu_5^2}{3\mu^2}\right]\right)$$

- We expand the CME-coefficient κ_B to linear order in ε_p

$$\boxed{\kappa_B \approx C\mu_5\left(1 - \frac{\mu\rho}{\epsilon + \bar{P}}\left[1 - \frac{\varepsilon_p}{16}\right]\right)}$$

Holographic principle

Holographic principle

Any theory of quantum gravity in $(d+1)$ – dimensions has a dual description in terms of a QFT without gravity in d – dimensions

In its most familiar example, the AdS/CFT correspondence, the quantum theory of gravity is string theory on asymptotically AdS space, and the theory on the boundary is a conformal field theory.

Hydrodynamic approximation

Duality between the long wavelength asymptotically AdS planar black hole solution of the Einstein–Maxwell theory with negative cosmological constant and the equations of charged fluid dynamics

Gravity computation

We start from a five-dimensional $U(1)^n$ Einstein-Maxwell theory in an asymptotic AdS space

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} [R - 2\Lambda - F^a_{MN} F^{aMN} + \underbrace{\frac{S_{abc}}{6\sqrt{-g}} \varepsilon^{PKLMN} A^a_P F^b_{KL} F^c_{MN}}_{\text{information about triangle anomalies}}]$$

Solution of the equations of motion: AdS black hole with $U(1)_V \times U(1)_A$

Gravity computation

An ansatz for an anisotropic AdS black hole solution

$$ds^2 = -f(r)dt^2 + 2drdt + r^2(w_T(r)dx^2 + w_T(r)dy^2 + w_L(r)dz^2)$$
$$A^a = -A_0^a(r)dt$$

An asymptotic solution ($r \to \infty$)

$$A_0^a(r) = \mu_\infty^a + \frac{\sqrt{3}q^a}{2r^2} + \mathcal{O}(r^{-8}) \qquad w_T(r) = 1 + \frac{w_T^{(4)}}{r^4} + \mathcal{O}(r^{-8})$$
$$f(r)/r^2 = 1 - \frac{m}{r^4} + \sum_a \frac{(q^a)^2}{r^6} + \mathcal{O}(r^{-8}) \quad w_L(r) = 1 + \frac{w_L^{(4)}}{r^4} + \mathcal{O}(r^{-8})$$

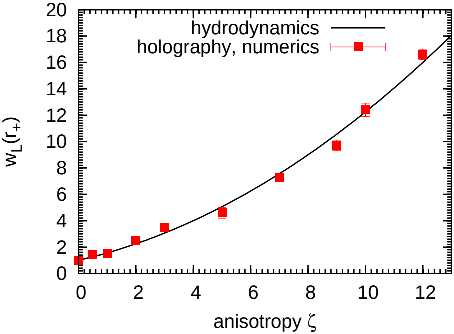
Transverse and longitudinal pressures

$$P_T = \frac{m - 4w_T^{(4)} - 4w_L^{(4)}}{16\pi G_5} = \frac{m(1+\zeta)}{16\pi G_5}$$
$$P_L = \frac{m - 8w_T^{(4)}}{16\pi G_5} = \frac{m(1-2\zeta)}{16\pi G_5}$$

A roadmap of computation

- Vary 4-velocity and background fields up to first order
- Solve equations of motion and find the first-order corrections
- Read off $U(1)$ currents from the near-boundary expansion of the first-order corrected background

Comparing apples to oranges



Comparing

$$\tilde{j}^{a\mu} = \frac{1}{16\pi G_5}\eta^{\mu\nu}Q^a_\nu(r_+) + r_+ A_0^{a\prime}(r_+)C_\nu$$

with the general expansion

$$\tilde{j}^{a\mu} = \xi^a_\omega \epsilon^{\nu\rho\sigma\mu}u_\nu\partial_\rho u_\sigma + \xi^{ab}_B \epsilon^{\nu\rho\sigma\mu}u_\nu\partial_\rho \mathcal{A}^b_\sigma$$

The transport coefficients from gravity

$$\xi_\omega^a = \frac{4}{16\pi G_5}\left(S^{abc}\mu^b\mu^c - \frac{2}{3}\frac{\rho^a}{\varepsilon + P_T}S^{bcd}\mu^b\mu^c\mu^d\right)$$

$$\xi_B^{ab} = \frac{4}{16\pi G_5}\left(S^{abc}\mu^c - \frac{1}{2}\frac{\rho^a}{\varepsilon + P_T}S^{bcd}\mu^c\mu^d\right)$$

Vi ringrazio per l'attenzione!