Emergent Lorentz-invariance: holographic description.

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Historical background. A model of Chadha and Nielsen.

- A toy model with Lorentz-invariance violation (Chadha, Nielsen 1982):

$$L = \frac{F_{0i}^2}{2} - \frac{c_\gamma^2 F_{ij}^2}{4} + \bar{\psi}\left(i\gamma^0 D_0 + i c_e \gamma^i D_i - m\right)\psi$$

- RG evolution. $c_\gamma - c_e$ vanishes in the IR. It means that Lorentz-invariance emerges at low energies.
- LI emergence is more efficient at strong coupling
- This property holds in morte general theories.
  - Yengo, Russo, Serone (2009)
  - Giudice, Raidal, Strumia (2010)
- At strong coupling it is natural to use holography.

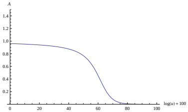
Overview.

- Our goal is to provide the holographic description of the Lorentz-invariance at low energies.
- We consider $(d+1)$ - dimensional space interpolating between AdS in the bulk and Lifshitz near the boundary (proposed by Kachru, Liu, Mulligan, 2008)
- We calculate correlators of the scalar field in this space
- and dispersion relations of discrete excitations of the field (if the space is surrounded by branes)

Interpolating solution

- The space interpolates between Anti-de Sitter and Lifshitz.
- This space is formed by massive vector A_M interacting with gravity.
- $ds^2 = L^2(-\frac{f^2 dt^2}{u^2} + \frac{g^2 du^2}{u^2} + \frac{1}{u^2}\sum_{i=1}^{d-1} dx_i^2),$

$u \to 0$	$u \to \infty$:
$f \to u^{1-z}$	$f \to 1$
$g \to \mathrm{const}$	$g \to 1$



- Equations allow two Lifshitz fixed points and only one of them is stable.

$1 < z < d-1$ $\qquad$ $d-1 < z < (d-1)^2$

Correlator dual to the scalar field at small w, k

- Lorentz-invariance emerges up to local terms.
  - Massless case:

$$\langle O_\phi(k,w), O_\phi(-k,-w)\rangle =$$

$$\begin{cases} L^2 g_0^2 (w^2+k^2)^{3/2}, & d=3; \\ -\frac{L^3 g_0^3 (w^2+k^2)^2}{4} \ln(g_0 u_2 \sqrt{w^2+k^2}), & d=4. \end{cases}$$

  - Massive case:

$$\langle O(k,w,u) O(-k,-w,u)\rangle \propto \Gamma(-\nu+1)(\sqrt{w^2+k^2})^{2\nu}$$

$$\nu = \sqrt{\frac{d^2}{4} + M^2 L^2}$$

Correlator dual to the scalar field at large w, k

- Lorentz-invariance does not emerge;
- at $z = 2$ behavior is the same as in Lifshitz:

$$\langle O_\phi(k,w), O_\phi(-k,-w)\rangle = w^\gamma G(\frac{k^2}{w}); \qquad \gamma = \begin{cases} 2, & d=3 \\ 5/2, & d=4 \end{cases}$$

- Covariance under anisotropic scale transformations:

$$k \to k/\lambda,$$
$$w \to w/\lambda^2,$$

The brane problem at small w, k

- Let us assume that the space is bounded by the brane at large $u = u_0$. Therefore the dual theory contains particles and their w, k correspond to the modes of the scalar field. Do they obey the Loretz-symmetry?
- At small w, k Lorentz-invariance emerges if Dirichlet boundary conditions (at $u = 0$) are satisfied. The equation for w, k (non-zero modes only) in the case $d = 3$:

$$g_0^2(w^2 - k^2)^{3/2}\left(\int_{u_1}^{u_2} du \frac{u^2 g}{f} - \frac{g_0 u_2^3}{3}\right) = -\tan(g_0 u_0 \sqrt{w^2 - k^2})$$

- In the case of small w, k and Neumann boundary conditions Lorentz-invariance does not emerge.

$$w^2 = k^2 \frac{\int_{u_1}^{u_0} du \frac{fg}{u^{d-1}}}{\int_{u_1}^{u_0} du \frac{g}{fu^{d-1}}} \tag{1}$$

The brane problem at large w and small k

- At large w, small k and Dirichlet boundary conditions the dispersion relation is:

$$w = w_0 + \frac{k^2}{2w_0}\left(1 + O(\frac{1}{u_0})\right) \tag{2}$$

- We see low-energy form of Lorentz-invariance

Conclusions

- We studied the solutions for gravitational and field equations describing space interpolating between AdS and Lifshitz at general d, z, found the conditions of stability of the solutions;
- explored correlator of scalar field in this space:
  - At small w, k the correlator is Lorentz-invariant,
  - at large w, k the correlator is the same as in pure Lifshitz,
- If the space is bounded by the brane on the AdS side, the excitations are Lorentz-invariant at
  - small w, k;
  - large w and small k.
- future directions:
  - generalization for the case of gauge field,
  - search for massless Lorentz-invariant mode.