Emergent Lorentz-invariance: holographic description.

Grigory Bednik
in collaboration with Sergey Sibiryakov

Institute for Nuclear Research of the Russian Academy of Sciences

June 2012

Publication is in preparation
Historical background. A model of Chadha and Nielsen.

- A toy model with Lorentz-invariance violation (Chadha, Nielsen 1982):

\[ L = \frac{F_{0i}^2}{2} - \frac{c_\gamma^2 F_{ij}^2}{4} + \bar{\psi} \left( i\gamma^0 D_0 + i c_e \gamma^i D_i - m \right) \psi \]

- RG evolution. \( c_\gamma - c_e \) vanishes in the IR. It means that Lorentz-invariance emerges at low energies.
- LI emergence is more efficient at strong coupling
- This property holds in more general theories.
  - Yengo, Russo, Serone (2009)
  - Giudice, Raidal, Strumia (2010)
- At strong coupling it is natural to use holography.
Our goal is to provide the holographic description of the Lorentz-invariance at low energies.

We consider \((d + 1)\) - dimensional space interpolating between AdS in the bulk and Lifshitz near the boundary (proposed by Kachru, Liu, Mulligan, 2008)

We calculate correlators of the scalar field in this space

and dispersion relations of discrete excitations of the field (if the space is surrounded by branes)
Interpolating solution

- The space interpolates between Anti-de Sitter and Lifshitz.
- This space is formed by massive vector $A_M$ interacting with gravity.
- \[ ds^2 = L^2 \left( -\frac{f^2}{u^2} dt^2 + \frac{g^2}{u^2} du^2 + \frac{1}{u^2} \sum_{i=1}^{d-1} dx_i^2 \right), \]
  
  \[ u \to 0 \quad u \to \infty : \]
  
  \[ f \to u^{1-z} \quad f \to 1 \]
  \[ g \to \text{const} \quad g \to 1 \]
- Equations allow two Lifshitz fixed points and only one of them is stable.
  \[ 1 < z < d - 1 \quad d - 1 < z < (d - 1)^2 \]
Correlator dual to the scalar field at small $w, k$

- Lorentz-invariance emerges up to local terms.
  - Massless case:
    \[
    \langle O_\phi(k, w), O_\phi(-k, -w) \rangle =
    \begin{dcases}
      L^2 g_0^2 (w^2 + k^2)^{3/2}, & d = 3; \\
      -\frac{L^3 g_0^3 (w^2 + k^2)^2}{4} \ln(g_0 u_2 \sqrt{w^2 + k^2}), & d = 4.
    \end{dcases}
    \]
  - Massive case:
    \[
    \langle O(k, w, u)O(-k, -w, u) \rangle \propto \Gamma(-\nu + 1)(\sqrt{w^2 + k^2})^{2\nu}
    \]
    \[
    \nu = \sqrt{\frac{d^2}{4} + M^2 L^2}
    \]
Correlator dual to the scalar field at large $w, k$

- Lorentz-invariance does not emerge;
- at $z = 2$ behavior is the same as in Lifshitz:

$$\langle O_\phi(k, w), O_\phi(-k, -w) \rangle = w^\gamma G\left(\frac{k^2}{w}\right); \quad \gamma = \begin{cases} 2, & d = 3 \\ 5/2, & d = 4 \end{cases}$$

- Covariance under anisotropic scale transformations:

$$k \to k/\lambda,$$

$$w \to w/\lambda^2,$$
The brane problem at small $w, k$

- Let us assume that the space is bounded by the brane at large $u = u_0$. Therefore the dual theory contains particles and their $w, k$ correspond to the modes of the scalar field. Do they obey the Lorentz-symmetry?

- At small $w, k$ Lorentz-invariance emerges if Dirichlet boundary conditions (at $u = 0$) are satisfied. The equation for $w, k$ (non-zero modes only) in the case $d = 3$:

$$g_0^2 (w^2 - k^2)^{3/2} \left( \int_{u_1}^{u_2} du \frac{u^2 g}{f} - \frac{g_0 u_2^3}{3} \right) = - \tan(g_0 u_0 \sqrt{w^2 - k^2})$$

- In the case of small $w, k$ and Neumann boundary conditions Lorentz-invariance does not emerge.

$$w^2 = k^2 \frac{\int_{u_1}^{u_0} du \frac{fg}{u^{d-1}}}{\int_{u_1}^{u_0} du \frac{g}{fu^{d-1}}}$$

(1)
The brane problem at large $w$ and small $k$

At large $w$, small $k$ and Dirichlet boundary conditions the dispersion relation is:

$$w = w_0 + \frac{k^2}{2w_0} \left(1 + O\left(\frac{1}{u_0}\right)\right)$$

(2)

We see low-energy form of Lorentz-invariance
Conclusions

- We studied the solutions for gravitational and field equations describing space interpolating between $AdS$ and Lifshitz at general $d$, $z$, found the conditions of stability of the solutions;
- explored correlator of scalar field in this space:
  - At small $w$, $k$ the correlator is Lorentz-invariant,
  - at large $w$, $k$ the correlator is the same as in pure Lifshitz,
- If the space is bounded by the brane on the $AdS$ side, the excitations are Lorentz-invariant at
  - small $w$, $k$;
  - large $w$ and small $k$.
- future directions:
  - generalization for the case of gauge field,
  - search for massless Lorentz-invariant mode.