Vacuum stability in the SM and the three-loop β-function for the Higgs self-interaction

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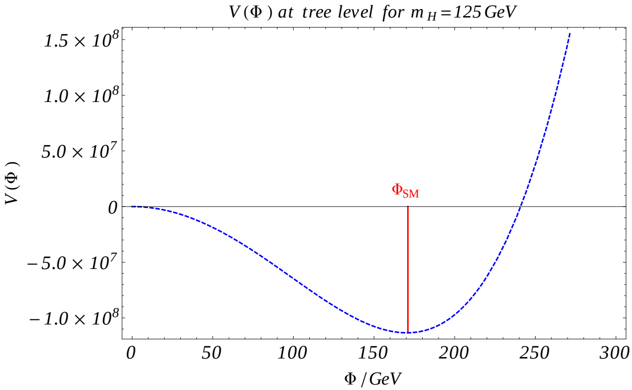
The Higgs potential

$$\mathcal{L}_\Phi = \partial_\mu \Phi^\dagger \partial^\mu \Phi - \underbrace{\left(m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2\right)}_{V(\Phi)}$$

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}$$



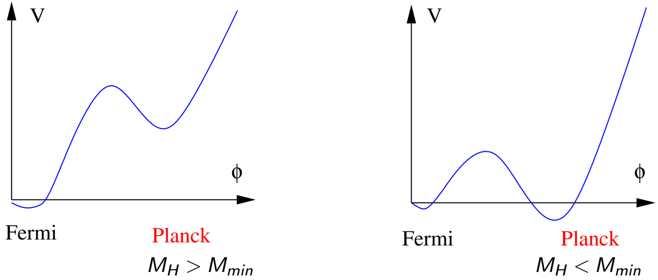
$|\Phi_{SM}| = \sqrt{\frac{-m^2}{2\lambda}} = \frac{v}{\sqrt{2}}$, $M_H^2 = -2m^2 = 2\lambda v^2$ at tree level

The effective Potential

include radiative corrections $\Rightarrow V_{eff}(\lambda(t), g_i(t), \Phi(t))$

with $t = \log\left(\frac{\Lambda}{\mu_0}\right)$ and $\Phi(t) = \Phi_{cl} \cdot \exp\left(\int\limits_0^t dt' \gamma_\Phi(\lambda(t'), g_i(t'))dt'\right)$ [Coleman, Weinberg]

(Λ: scale up to which the SM is valid, μ_0: starting point for running, e.g. $\mu_0 = M_Z$)



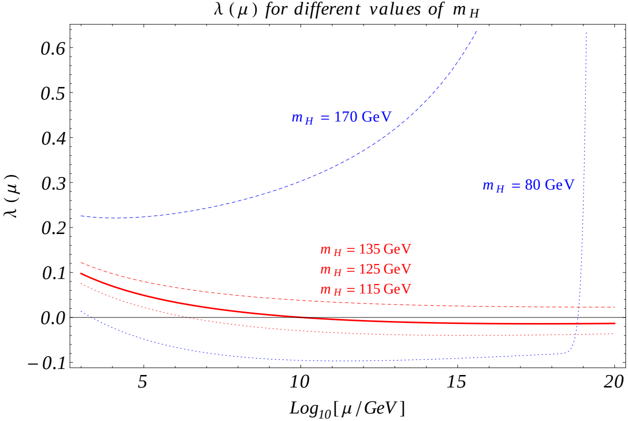
[Shaposhnikov,...]

$$\boxed{V_{eff}[\Phi] \approx \lambda(\Lambda)\Phi^4 + \mathcal{O}(\lambda^2(\Lambda), g_i^2(\Lambda))}$$ for $\Phi \sim \Lambda \gg \mu_0$ [Altarelli, Isidori]

Stability of SM vacuum $\Leftrightarrow \lambda(\Lambda) > 0$

Behaviour of $\lambda(\mu)$

Stability bound on the Higgs mass: $M_H > M_{min}$
Upper bound $M_H < M_{max}$: no Landau pole for $\mu \leq \Lambda$



for $\Lambda = M_{Planck}$: $M_{max} \approx 175$ GeV,
$M_{min} \approx 127$ GeV $\Rightarrow$ curious coincidence with recent data!

Main contributions to $\beta_\lambda(\lambda, y_t, g_i, \ldots) = \mu^2 \frac{d}{d\mu^2}\lambda(\mu)$

- β-functions for gauge couplings at 3 loop [Mihaila, Salomon, Steinhauser]
- β-functions for Yukawa-,Higgs-sector at 2 loop [Machacek, Vaughn; Luo, Xiao]

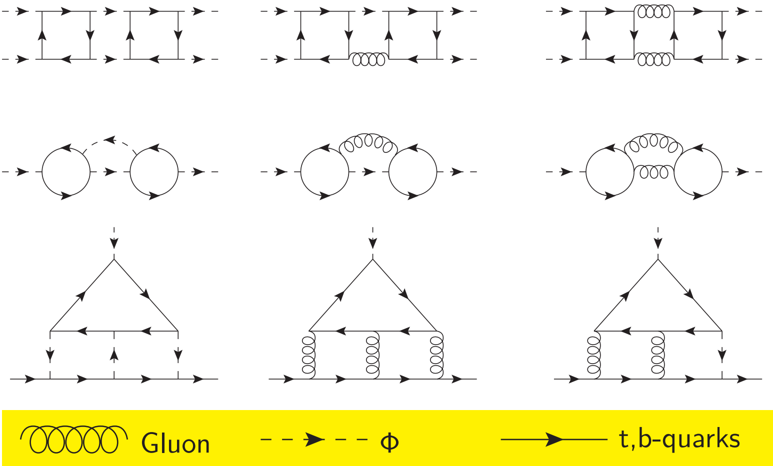
Values for SM couplings at $\mu = M_t = 172.9$ GeV

- strong coupling: $g_s \approx 1.17 \Rightarrow \frac{g_s^2}{4\pi} \approx 0.11$
- top-Yukawa coupling:$y_t \approx 0.93 \Rightarrow \frac{y_t^2}{4\pi} \approx 0.07$
- EW couplings:$g_2 \approx 0.65,\ g_1 \approx 0.36 \Rightarrow \frac{g_2^2}{4\pi} \approx 0.03,\ \frac{g_1^2}{4\pi} \approx 0.01$
- Possible Higgs around 125 GeV $\Longrightarrow \lambda(M_H) \approx 0.13 \Rightarrow \frac{\lambda}{4\pi} \approx 0.01$

Main 3 loop contributions to β_λ: $g_s,\ y_t,\ \lambda$

Model: QCD & top-Yukawa & Higgs sector of the SM in the unbroken phase.

Calculation (in collaboration with K. G. Chetyrkin)



Difficulties:

- γ_5 treatment ['t Hooft, Veltman]
- infrared divergences (auxiliary mass [Chetyrkin,...])

Results: $\mu^2 \frac{d}{d\mu^2}\lambda(\mu) = \beta_\lambda(g_s, y_t, \lambda) = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta_\lambda^{(n)}(g_s, y_t, \lambda)$

in the $\overline{\text{MS}}$-scheme

$$\beta_\lambda^{(1)} = 12\lambda^2 + 6y_t^2\lambda - 3y_t^4$$

$$\beta_\lambda^{(2)} = -156\lambda^3 - 72y_t^2\lambda^2 - \frac{3}{2}y_t^4\lambda + 15y_t^6 + 40g_s^2y_t^2\lambda - 16g_s^2y_t^4$$

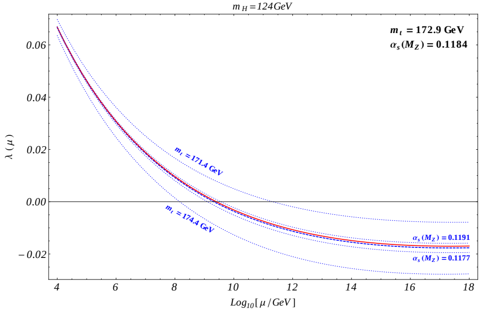
$$\begin{aligned}\beta_\lambda^{(3)} = &\lambda^4(3588 + 2016\zeta_3) + 873y_t^2\lambda^3 + y_t^4\lambda^2\left(\frac{1719}{2} + 756\zeta_3\right) \\ &+ y_t^6\lambda\left(\frac{117}{8} - 198\zeta_3\right) - y_t^8\left(\frac{1599}{8} + 36\zeta_3\right) \\ &+ g_s^2y_t^2\lambda^2(-1224 + 1152\zeta_3) + g_s^2y_t^4\lambda(895 - 1296\zeta_3) \\ &+ g_s^2y_t^6(-38 + 240\zeta_3) + g_s^4y_t^2\lambda\left(\frac{1820}{3} - 32n_f - 48\zeta_3\right) \\ &+ g_s^4y_t^4\left(-\frac{626}{3} + 20n_f + 32\zeta_3\right)\end{aligned}$$

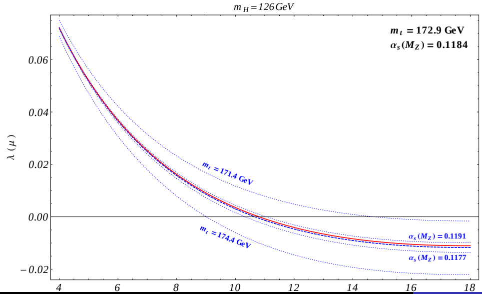
For $M_H \approx 125$ GeV at $\mu = M_Z$:

$$\frac{\beta_\lambda^{(3)}}{(16\pi^2)^3} = (\underbrace{+7.9}_{g_s^2y_t^6} \quad \underbrace{-4.8}_{y_t^8} \quad \underbrace{-3.1}_{g_s^2y_t^4\lambda} \quad \underbrace{-2.5}_{g_s^4y_t^4} \quad \underbrace{+2.6}_{g_s^4y_t^2\lambda}) \times 10^{-5}$$

2 loop: **1%**, 3 loop: **(-0.04)%** correction to 1 loop result.

$\lambda(\mu)$





3 loop

2 loop

3 loop running (incl. 2 loop EW),
1 loop matching for
on-shell $\rightarrow$ $\overline{\text{MS}}$-parameters
(2 loop matching for QCD)

Summary

- 3 loop correction to β_λ smaller than the α_s uncertainty.
- Individual contributions to β_λ much larger than overall effect!
- 3 loop result: small improvement of the stability of the SM vacuum.
- Main uncertainty for SM stability: M_t