# H1 QCD Fits

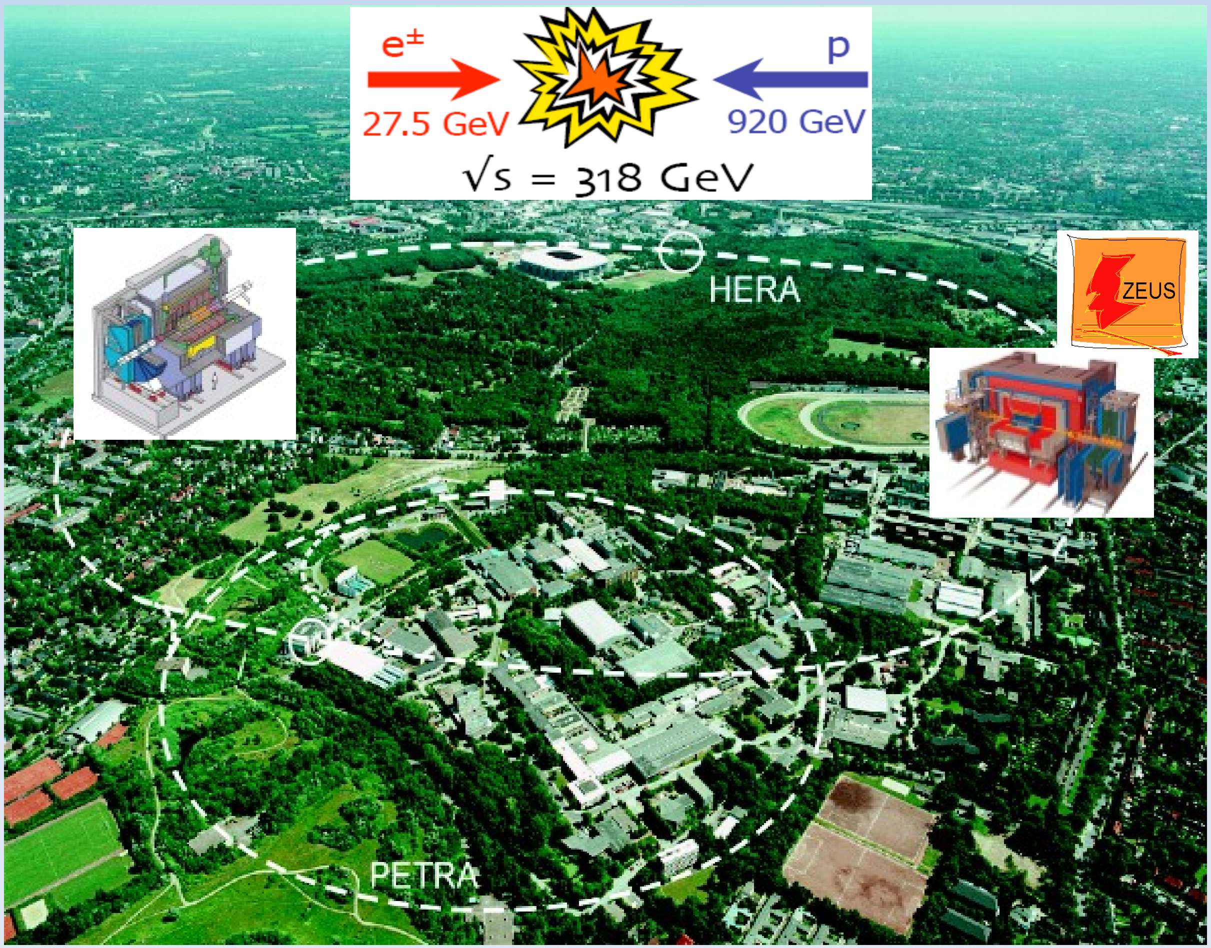
*Hayk Pirumov*
*PI Heidelberg*

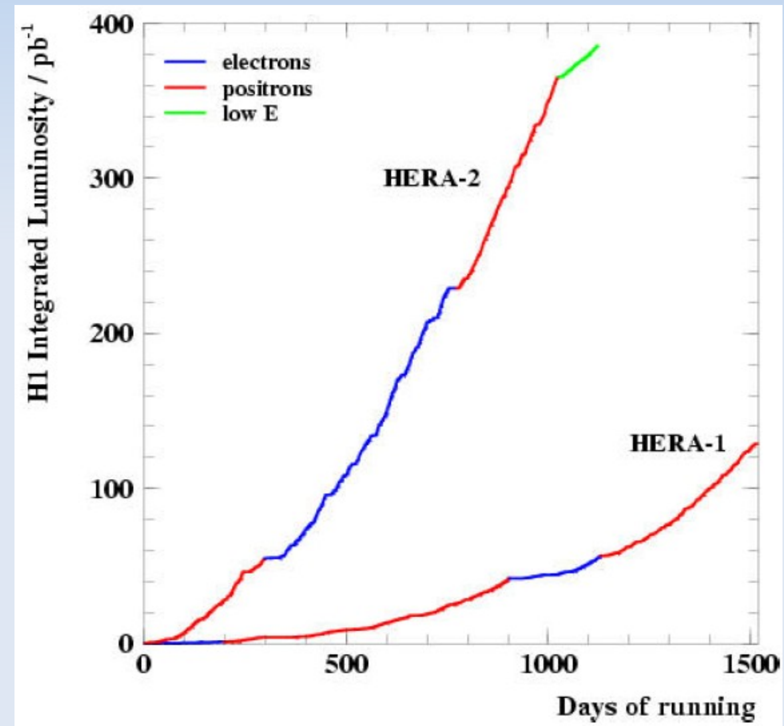
Outline:

- DIS at HERA with H1
- QCD Fit Analysis
  - Data and settings
  - Uncertainties
- Summary

# The HERA Collider

- World's only electron proton collider, at DESY, Hamburg.
- HERA I (1992 – 2000): ~ mainly $e^+p$.
- HERA II (2003 – 2007): lumi upgrade, longitudinally polarised lepton.

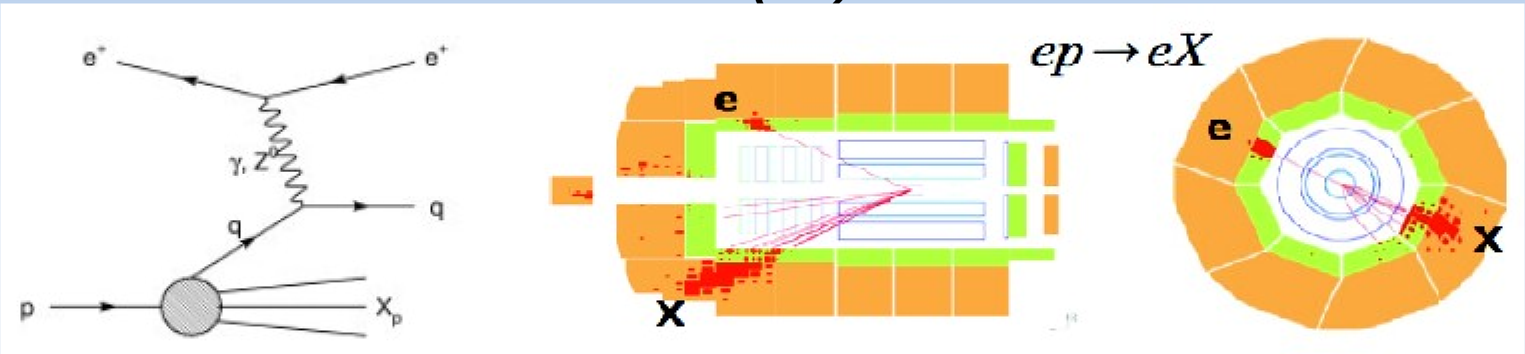


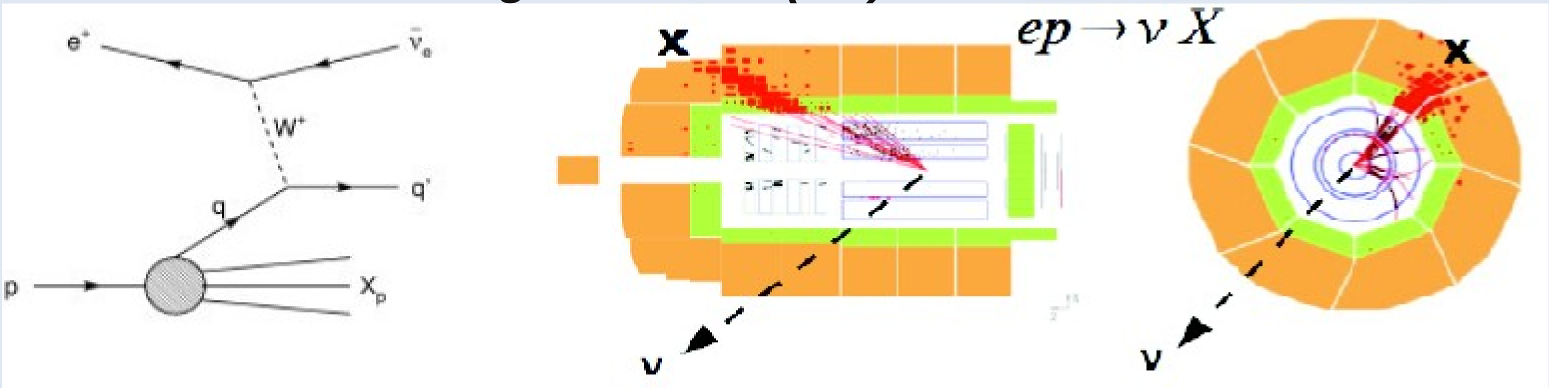

- Two multipurpose detectors: H1 and ZEUS
- About $0.5\ fb^{-1}$ of data collected by each of the experiments.

# Deep Inelastic ep Scattering

- HERA gives an opportunity to study the structure of the proton via DIS processes:
  - ***Neutral Current (NC)***




  - ***Charged Current (CC)***




- Kinematic variables:

$$Q^2 = -q^2 = -(k - k')$$
$$x = Q^2 / 2(P \cdot q)$$
$$y = (P \cdot q)/(P \cdot q)$$

$Q^2$ is the square of the 4-momentum transferred
$x$ is the Bjorken scaling variable
$y$ is the inelasticity of the process

# Deep Inelastic ep Scattering

- NC cross section in terms of generalized structure functions:

$$\frac{d^2\sigma_{NC}}{dx\,dQ^2}(ep\rightarrow eX)=\frac{2\pi\alpha^2}{x\,Q^4}[Y_-\tilde{F}_2 \mp Y_+ x\tilde{F}_3 - y^2\tilde{F}_L]$$

  - $F_2$ is dominating — $\tilde{F}_2 \propto \sum (q_i+\bar{q}_i)$
  - $xF_3$ is sensitive at large $Q^2$ — $x\tilde{F}_3 \propto \sum (q_i-\bar{q}_i)$
  - $F_L$ is sensitive at low $Q^2$ and high y — $\tilde{F}_L \propto g$

- CC cross section in terms of generalized structure functions:

$$\frac{d^2\sigma_{CC}}{dx\,dQ^2}(ep\rightarrow \nu X)=\frac{G_F^2}{2\pi x}\left(\frac{M_W^2}{M_W^2+Q^2}\right)[Y_+W_2 \mp Y_- xW_3 - y^2 W_L]$$

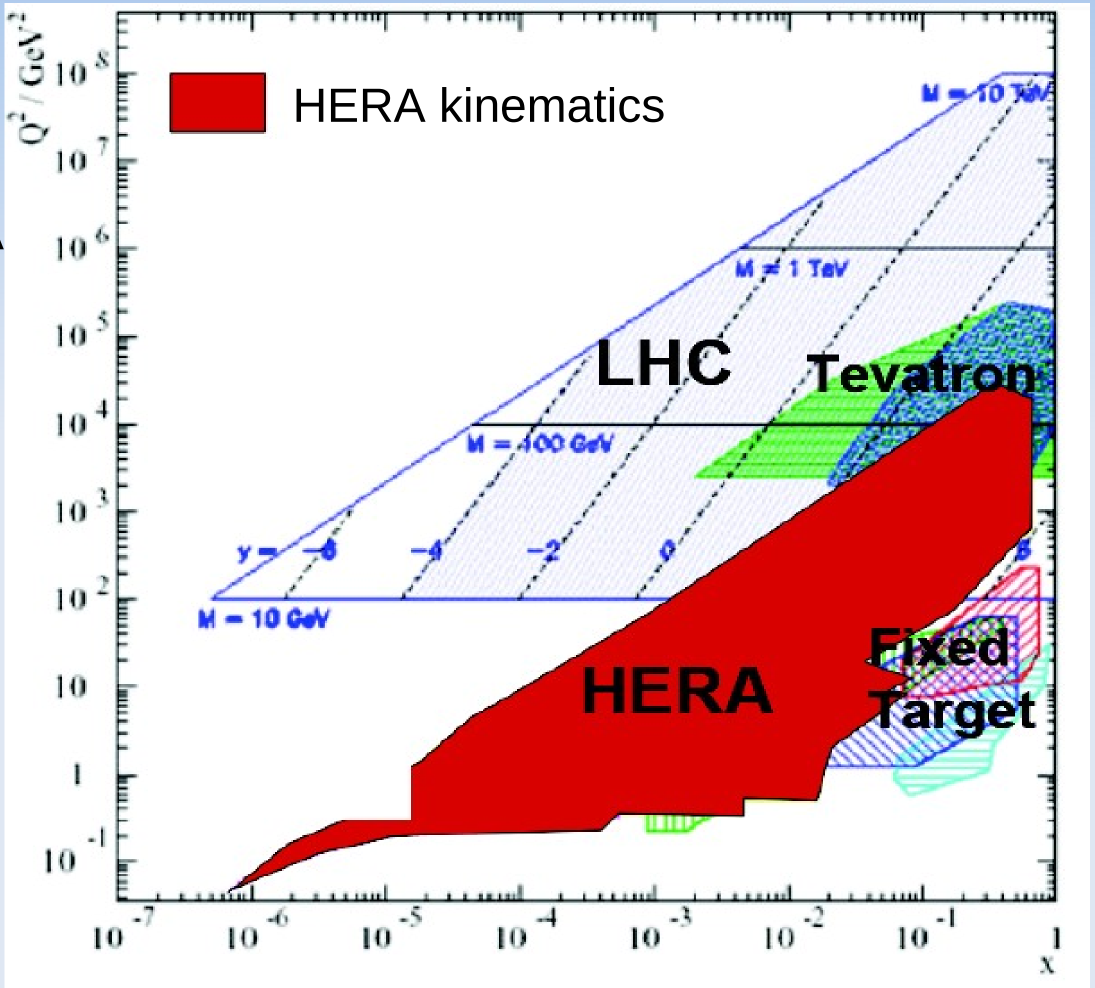
$$W_2^+=x(\bar{U}+D),\qquad W_2^-=x(U+\bar{D})$$
$$xW_3^+=x(D-\bar{U}),\qquad xW_3^-=x(U-\bar{D})$$

- Where U is the sum of up-type and D the sum of down-type quarks

# Deep Inelastic ep Scattering

- A wide kinematic plane is covered by HERA.
- The kinematic range accessible by HERA compared to other experiments is shown at the plot.
- The H1 kinematic plane is:

$$
\begin{array}{l}
0.00004 \leq x \leq 0.65 \\
0.5 \leq Q^2 \leq 50000
\end{array}
$$



# QCD Fit – PDF Determination

***Model:***

| | | |
|---|---|---|
| PDFs parametrization starting scale | : | $Q^2_0$ = 1.9 $GeV^2$ |
| PDFs evolution | : | evaluated at NLO |
| Heavy Quark Coefficients | : | Robert Thorne's scheme |
| $M_c$ | : | 1.4 GeV |
| $M_b$ | : | 4.75 GeV |
| $f_s$ at $Q^2_0$ | : | 0.31 |
| $\alpha_s$ | : | 0.1176 |
| $Q^2_{min}$ of data | : | 3.5 $GeV^2$ |

# QCD Fit – PDF Determination

- Fit using general functional form: $xf(x) = A \cdot x^{B} \cdot (1-x)^{C} \cdot (1 + Dx + Ex^{2})$
- Find the set of parameters giving the best solution (minimal $\chi^2$)

- Constrains:
  - Momentum sum rules:

$$\int_0^1 dx \cdot (xu_v + xd_v + x\,\bar{U} + x\,\bar{D} + xg) = 1$$

$$\int_0^1 dx \cdot u_v = 2, \quad \int_0^1 dx \cdot d_v = 1$$

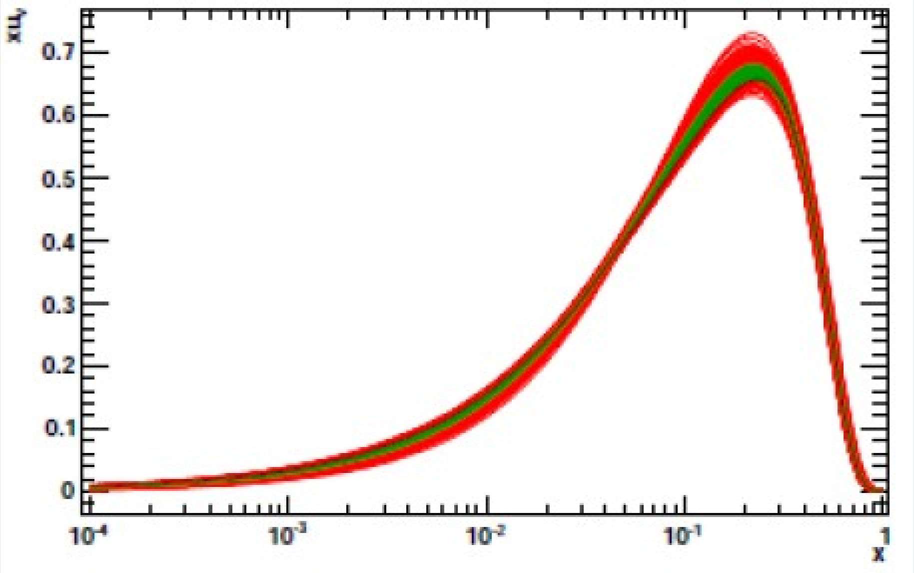
  - $B_{\bar{U}} = B_{\bar{D}}, \quad A_{\bar{U}} = A_{\bar{D}}(1 - f_s)$

# QCD Fit – PDF Determination / Uncertainties

- **Experimental Uncertainties**
- Model Uncertainties
- Parametrisation Uncertainties

# QCD Fit – PDF Determination / Uncertainties

- The estimation of the experimental uncertainties is based on toy experiments.
- N replica (N is about 400 for the presented analysis) of data sets are prepared by fluctuating the central values of the cross sections within their systematic and statistical uncertainties.
- For each replica a fit is performed and PDFs are extracted.
- Uncertainties are estimated using the RMS and the mean value.




- Example plots for u valence. The PDFs extracted using the toys (red lines) are compared to the result from data (black line).
- The green band corresponds to the RMS.

# QCD Fit – PDF Determination / Uncertainties

- Experimental Uncertainties
- **Model Uncertainties**
- Parametrization Uncertainties

# QCD Fit – PDF Determination / Uncertainties

- Model uncertainties are evaluated by varying the input assumptions:

| Variation | Central Value | Lower Limit | Upper Limit |
|---|---|---|---|
| $f_s$ | 0.31 | 0.23 | 0.38 |
| $m_c$ (GeV) | 1.4 | 1.35 (for $Q_0^2 = 1.8$ GeV) | 1.65 |
| $m_b$ (GeV) | 4.75 | 4.3 | 5.0 |
| $Q^2_{\text{min}}$ (GeV$^2$) | 3.5 | 2.5 | 5.0 |

- The difference between the central fit and the fits corresponding to model variations are added in quadrature, separately for positive and negative deviations.
- The dominant effect is due to the variation of the $Q^2_{min}$ of data.

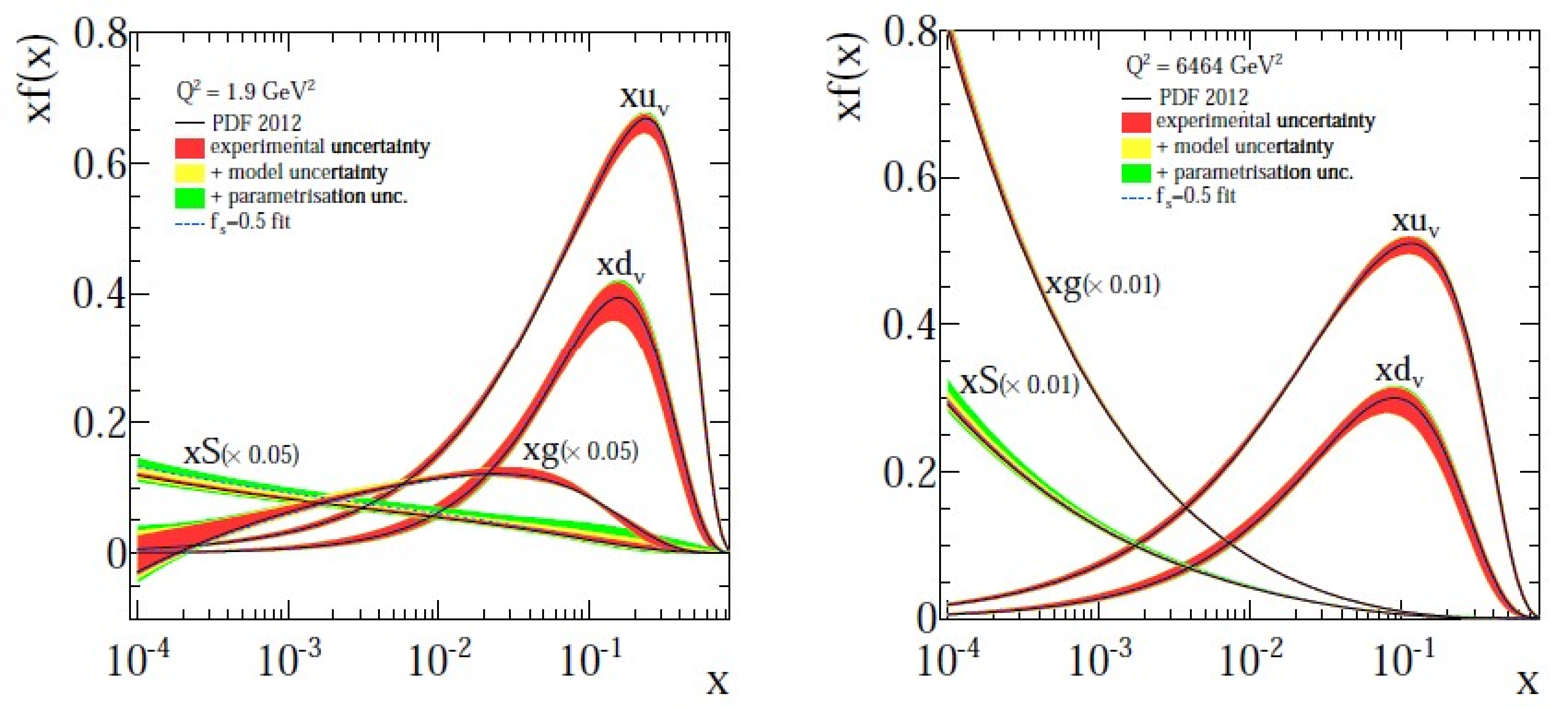
# QCD Fit – PDF Determination / Uncertainties

- Experimental Uncertainties
- Model Uncertainties
- **Parametrization Uncertainties**

# QCD Fit – PDF Determination / Uncertainties

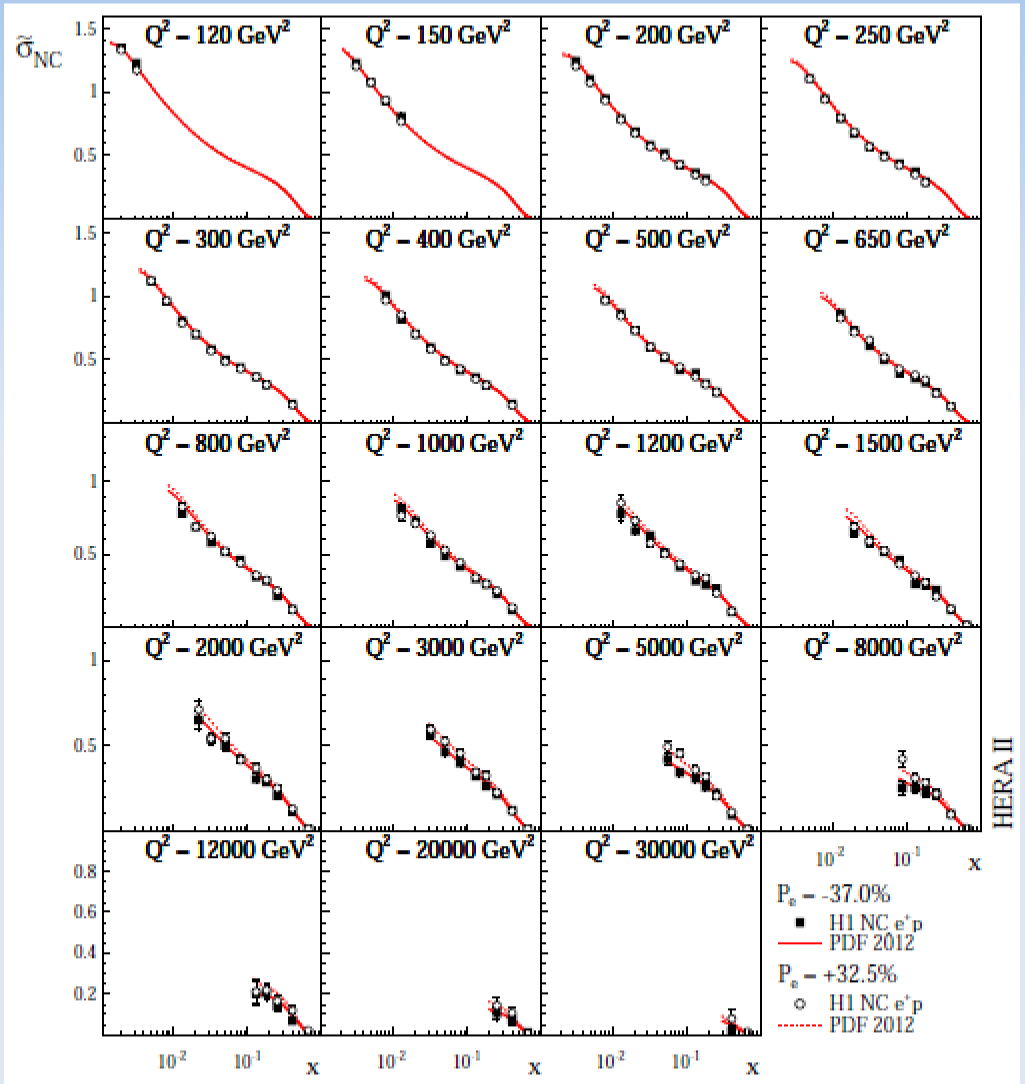
- Variation of the starting scale $Q^2_0$
- Variations of central + 1 parameter fits
- The parametrization uncertainty is then constructed as an envelope, built from maximal deviation at each x value from the central fit.

# Results




- The PDFs determined from the fit to the full amount of H1 data are presented for *$Q^2$* of *1.9* and *6464 GeV²*.
- The black line represents the central fit. The red band corresponds to the experimental uncertainty. The yellow and green bands correspond to the model and parametrization uncertainties added in quadrature to the experimental onces.

# Results




- Total $\chi^2$ / ndof for the central fit is 1569/1461.
- The fit is in a good agreement with the data.
- The NC e+p double differential cross sections measured by the H1 as well as the prediction from the fit are shown at the plot.

# Summary

- *NLO QCD fit shows good description of the H1 NC, CC cross section measurements.*
- *The data are well described by the QCD fit over the full range of* $Q^2 > 3.5$.

# Backup

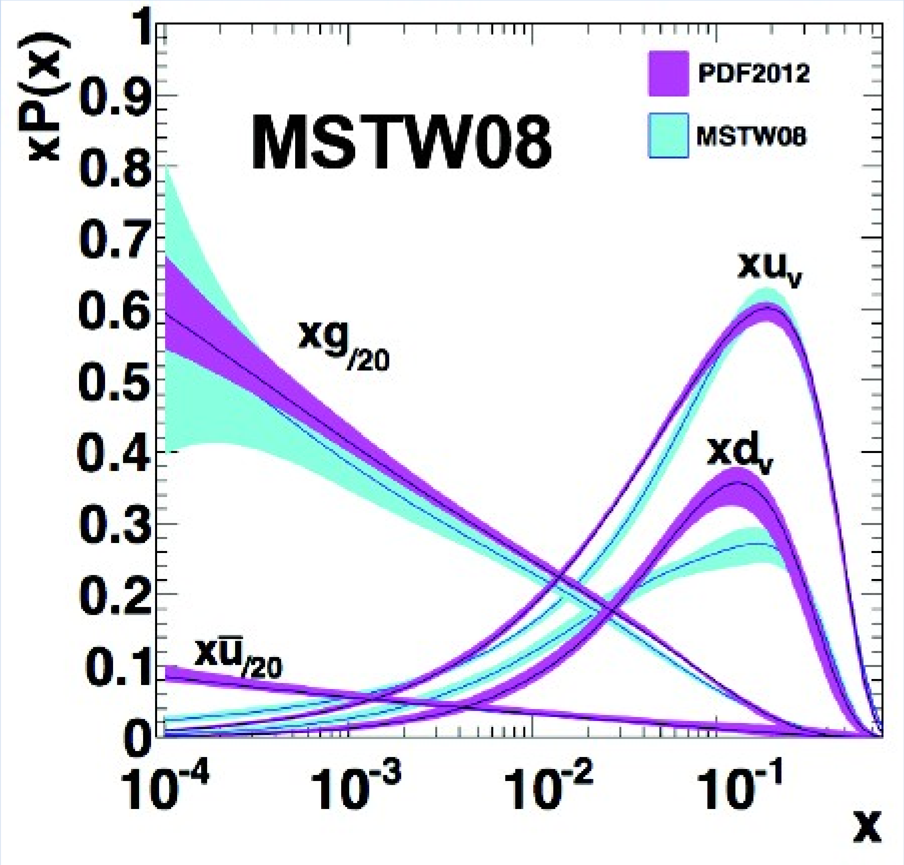
# QCD Fit – PDF Determination

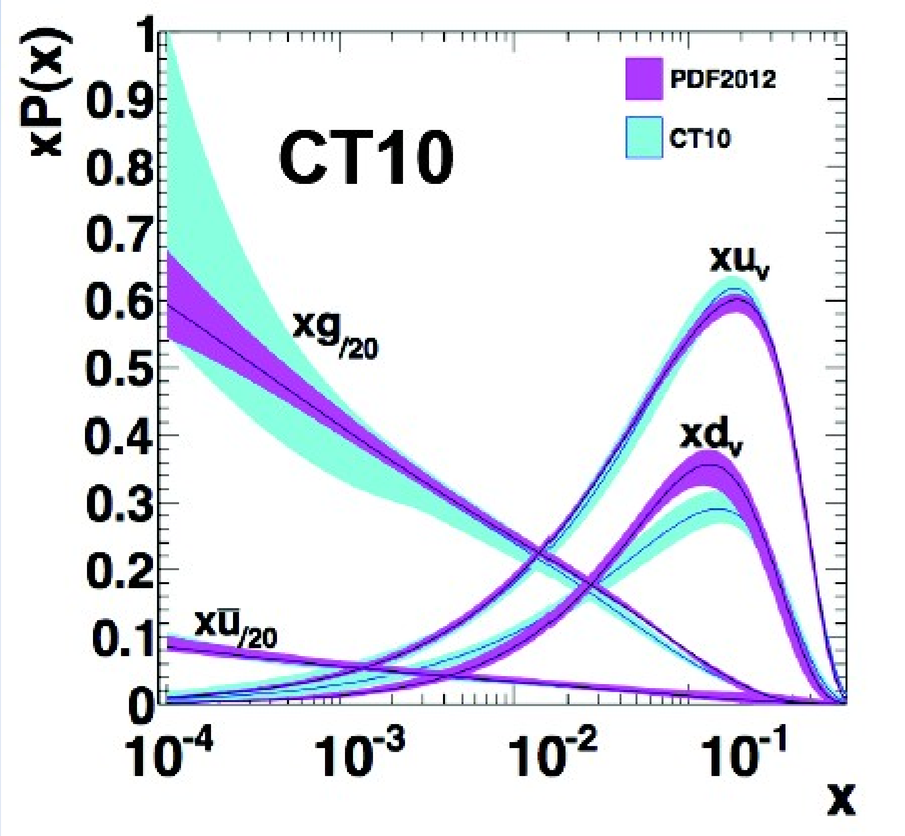
- The analysis uses the full amount of NC and CC measurements of the H1 for both HERA I and HERA II periods.
- The kinematic ranges for each of the separate data sets are presented in the table.

| Data set | $x_{\min}$ | $x_{\max}$ | $Q^2_{\min}$ (GeV$^2$) | $Q^2_{\max}$ (GeV$^2$) |
|---|---|---|---|---|
| Combined low $Q^2$ | 0.00004 | 0.20 | 0.5 | 150 |
| Combined low $E_p$ | 0.00003 | 0.003 | 1.5 | 90 |
| NC 94 – 97 | 0.0032 | 0.65 | 150 | 30 000 |
| CC 94 – 97 | 0.013 | 0.40 | 300 | 15 000 |
| NC 98 – 99 | 0.0032 | 0.65 | 150 | 30 000 |
| CC 98 – 99 | 0.013 | 0.40 | 300 | 15 000 |
| NC 98 – 99 | 0.00131 | 0.0105 | 100 | 800 |
| NC 99 – 00 | 0.0032 | 0.65 | 150 | 30 000 |
| CC 99 – 00 | 0.013 | 0.40 | 300 | 15 000 |
| NC *high* $y$ | 0.0008 | 0.0105 | 60 | 800 |
| NC *high* $y$ | 0.0008 | 0.0105 | 60 | 800 |
| NC $L$ | 0.002 | 0.65 | 120 | 30 000 |
| CC $L$ | 0.008 | 0.40 | 300 | 15 000 |
| NC $R$ | 0.002 | 0.65 | 120 | 30 000 |
| CC $R$ | 0.008 | 0.40 | 300 | 15 000 |
| NC $L$ | 0.002 | 0.65 | 120 | 50 000 |
| CC $L$ | 0.008 | 0.40 | 300 | 30 000 |
| NC $R$ | 0.002 | 0.65 | 120 | 30 000 |
| CC $R$ | 0.008 | 0.40 | 300 | 15 000 |

# Results / Comparisons

- PDFs determined from the fit to the full amount of H1 NC and CC data compared to the MSTW08 and CT10 PDFs. The plots are presented for the value of $Q^2$ of *10 GeV* $^2$*.* The bands represent total uncertainties.



# QCD Fit – PDF Determination / Parametrization

- Fit using general functional form: $xf(x) = A \cdot x^{B} \cdot (1-x)^{C}$
- Modify *xf(x)*: $xf(x) = A \cdot x^{B} \cdot (1-x)^{C} \cdot (1 + Dx + Ex^{2})$

  find *D* and *E* that give the best 10 parameter fit => $\boldsymbol{E_{u_v}}$
- Repeat for more parameters => $\boldsymbol{B_{d_v}}$, $\boldsymbol{A'_g}$, $\boldsymbol{B'_g}$
- Find that $\chi^2$ saturates at 14 parameters => stick to 13 parameters fit:

$$
\begin{aligned}
xf_g(x) &= A_g \cdot x^{B_g} \cdot (1-x)^{C_g} - A'_g \cdot x^{B'_g} \cdot (1-x)^{25} \\
xf_{u_v}(x) &= A_{u_v} \cdot x^{B_{u_v}} \cdot (1-x)^{C_{u_v}} \cdot (1 + E_{u_v} x^{2}) \\
xf_{d_v}(x) &= A_{d_v} \cdot x^{B_{d_v}} \cdot (1-x)^{C_{d_v}} \\
xf_{\bar{U}}(x) &= A_{\bar{U}} \cdot x^{B_{\bar{U}}} \cdot (1-x)^{C_{\bar{U}}} \\
xf_{\bar{D}}(x) &= A_{\bar{D}} \cdot x^{B_{\bar{D}}} \cdot (1-x)^{C_{\bar{D}}}
\end{aligned}
$$

# $\chi^2$ functional form

- The following definition $\chi^2$ is used in the fit:

$$\chi^2=\sum_i \frac{\left(\sigma_i^{\exp}-\sigma_i^{theo}\left[1-\sum_k \Delta_{ik}^{corr}(\epsilon_k)\right]\right)^2}{\left(\delta_{i,stat}^2\,\sigma_i^{\exp}\,\sigma_i^{theo}\left[1-\sum_k \Delta_{ik}^{corr}(\epsilon_k)\right]+(\delta_{i,uncorr}^2\,\sigma_i^{theo})^2\right)}+\sum_k \epsilon_k^2$$

| | |
|---|---|
| $\sigma_i^{\exp}$ | *experimental cross section in* $Q^2$ *bin i* |
| $\sigma_i^{theo}$ | *theoretical cross section* |
| $\Delta_{ik}(\epsilon_k)$ | *effect due to correlated error k for bin i* |
| $\delta_{i,stat}$ | *relative statistical error* |
| $\delta_{i,uncorr}$ | *relative uncorrelated error* |
| $\epsilon_k$ | *shift of the* $k^{th}$ *source of correlated uncertainty* |