

# More on the Tensor Response of the QCD Vacuum to an External Magnetic Field

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# Outline

- 1 Magnetic Susceptibility of the Chiral Condensate
  - Different Approaches to  $\chi$
  - Possible Alternative Derivations of  $\chi$
- 2 Holographic Model with a Tensor Field
  - The Setup of Holographic QCD
  - The Holographic Action
  - Classical Equations of Motion and Their Solution
  - Magnetization and Susceptibility: Results and Discussion
- 3 Summary

# Magnetic Susceptibility of the Chiral Condensate

- Introduced in the framework of QCD Sum Rules  
[B. L. Ioffe, A. V. Smilga]
- $\langle \bar{q} \sigma_{\mu\nu} q \rangle_F = \chi \langle \bar{q} q \rangle F_{\mu\nu}$ , where  $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ .
- Measures induced tensor current in the QCD vacuum

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- $\chi = -\frac{N_c}{4\pi^2 f_\pi^2} = -8.9 \text{ GeV}^{-2}$  - OPE of the  $\langle VVA \rangle$  correlator and pion dominance [A. Vainshtein]
- Sum rule fit  $\chi = -3.15 \pm 0.30 \text{ GeV}^{-2}$
- Vector dominance  $\chi = -(3.38 \div 5.67) \text{ GeV}^{-2}$  [Balitsky, Yung, Kogan ...]

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There is a significant discrepancy in the numerical results. It is natural to start looking elsewhere, e.g. holography, to identify the missing ingredients

- Holographic calculation of  $\langle VVA \rangle$ :  $\chi \sim -11.5 \text{ GeV}^{-2}$  [A. Gorsky, A. Krikun]. Vainshtein relation isn't exact, but fulfilled to good accuracy.
- Holographic Son–Yamamoto relations:  $\chi$  agrees with Vainshtein. Assumed to be valid at any momentum transfer. No field-theoretical derivation.
- A direct holographic calculation - motivated by enhanced AdS/QCD models [Cappiello, Cata, D'Ambrosio; Domokos, Harvey, Royston; Alvares, Hoyos, Karch] that take into account the  $1^{+-}$  mesons.
- Check them for self-consistency.

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# The Setup

According to the holographic prescription:

Quantum Field Theory	Classical Gravity in 5D
Source $J(x_\mu)$ of an operator $\mathcal{O}$	Boundary value $\Phi(x_\mu, 0)$ of a 5D field $\Phi(x_\mu, z)$
Effective action with sources	Action on classical trajectories
Asymptotic freedom at small distances and confinement at large distances	AdS near the boundary, an IR hard wall far from the boundary

- KK-decompose all the fields and integrate out the  $z$ -axis, get an effective action for mesons - a chiral Lagrangian.
- An "expansion" of  $\chi$ PT

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# The Action

$$\begin{aligned}
 S_{5D} = & \int d^5x \sqrt{-g} \operatorname{Tr} \left\{ -\frac{1}{4g_5^2} (F_L^2 + F_R^2) + g_X^2 (|DX|^2 - m_X^2 |X|^2) \right. \\
 & + \frac{\lambda}{2} (X^+ F_L B + B F_R X^+ + \text{c.c.}) \\
 & \left. - 2g_B \left( \frac{i \epsilon^{MNPQR}}{6 \sqrt{-g}} (B_{MN} H_{PQR}^+ - B_{MN}^+ H_{PQR}) + m_B |B|^2 \right) \right\}.
 \end{aligned}$$

$$H = DB = dB - iL \wedge B + iB \wedge R,$$

$$\bar{q}_{R\bar{i}} q_L^f \leftrightarrow X_{\bar{i}}^f,$$

$$\bar{q}_{R\bar{g}} \gamma_\mu q_R^{\bar{f}} \leftrightarrow R_{\mu\bar{g}}^{\bar{f}},$$

$$\bar{q}_{R\bar{i}} \sigma_{\mu\nu} q_L^f \leftrightarrow B_{\mu\nu\bar{i}}^f,$$

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## Rearranging the Degrees of Freedom

- In 4D,  $\bar{q}\sigma^{\mu\nu}\gamma_5 q = \frac{i}{2}\epsilon^{\mu\nu}_{\lambda\rho}\bar{q}\sigma^{\lambda\rho}q$
- From the holographic point of view, this condition is ensured by the fact that the kinetic term for  $B_{\mu\nu}$  is of the first order in derivatives, which leads to its complex self-duality.
- The “double counting” of the degrees of freedom that arises after we have introduced a complex tensor field is compensated by constraints imposed on half of them.

$$\bar{q}q \leftrightarrow X_+,$$

$$\frac{1}{\sqrt{2}}\bar{q}\sigma_{\mu\nu}q \leftrightarrow B_{+\mu\nu},$$

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# Equations of Motion

According to the prescription, we have to solve the classical E.o.M.'s

$$\left( \partial_z^2 + \frac{1}{z} \partial_z - \frac{1}{z^2} - \partial_\mu \partial^\mu \right) (B_\pm)_{12} = -\frac{\lambda}{8g_B} \frac{1}{z^2} X_\pm (F_V)_{12},$$

$$\left( \partial_z^2 - \frac{3}{z} \partial_z + \frac{3}{z^2} - \partial_\mu \partial^\mu \right) X_\pm = -\frac{2\lambda}{g_X^2} z^2 (F_V)_{12} (B_\pm)_{12}.$$

and calculate

$$\langle \bar{q}q \rangle \propto \left. \frac{\delta \mathcal{S}}{\delta X_+} \right|_{z=0} \quad \langle \bar{q} \sigma_{\mu\nu} q \rangle \propto \left. \frac{\delta \mathcal{S}}{\delta B_{+\mu\nu}} \right|_{z=0}$$

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## The Solution

- A most general property – the scalar and tensor degrees of freedom  $X_+$ ,  $B_{+12}$  decouple from the pseudoscalar and pseudotensor  $X_-$ ,  $B_{-12}$ , thus forming two independent sectors.
- The solutions for  $X$  and  $B$  are expressed in terms of Bessel and Neumann functions of  $|\mathbf{B}|/q^2$  and  $qz$  (where  $q = \sqrt{q_\mu q^\mu}$  is the momentum and  $|\mathbf{B}| = (F_V)_{12}$  is the magnetic field) or their analytical continuations into the complex plane.

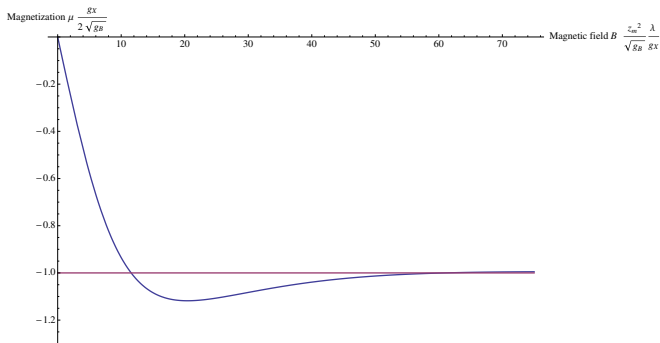
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# Magnetization

We are able to determine the magnetization  $\mu(\mathbf{B}) = \frac{\langle \bar{q}\sigma_{12}q \rangle}{\langle \bar{q}q \rangle}$ :



**Figure:** Magnetization of the chiral condensate  $\mu(\mathbf{B})$  as a function of the magnetic field (blue) vs its strong field asymptotics (red).

# Magnetization:

- is linear in  $|\mathbf{B}|$  when the field is weak,
- becomes a negative constant at  $\mathbf{B} \sim z_m^{-2} \sim \Lambda_{QCD}^2$ .
- Constant asymptotic is to be expected. In large magnetic fields 4D reduces to 2D and the tensor chiral condensate is kinematically reduced to a scalar one.
- The result points to the fact that not only the lowest Landau level plays a significant role:  $\lim_{\mathbf{B} \rightarrow \infty} \mu(\mathbf{B}) \neq -1$ .

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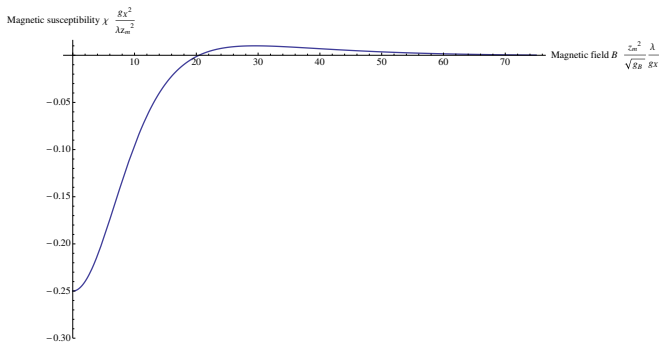
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# Magnetic Susceptibility

... and the magnetic susceptibility  $\chi(\mathbf{B}) = \frac{d}{d\mathbf{B}} \mu(\mathbf{B})$  :



**Figure:** Magnetic susceptibility of the chiral condensate  $\mu(\mathbf{B})$  as a function of the magnetic field.

# Susceptibility:

- possesses a quadratic behavior  $\chi \sim -(const - \mathcal{O}(|\mathbf{B}|^2))$  when the field is weak,
- tends to 0 at  $\mathbf{B} \sim z_m^{-2} \sim \Lambda_{QCD}^2$ .
- Parametrically  $\chi(|\mathbf{B}| = 0)$  is reasonable ( $\sim m_\rho^{-2}$ ), but numerically drastically differs from previous results.
- Some additional unknown factors may contribute, requires more investigation.



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# Summary

- A non-perturbative calculation of  $\mu(\mathbf{B})$  and  $\chi(\mathbf{B})$  to all orders in the magnetic field has been carried out.
- It has been performed in a holographic model enhanced by the inclusion of a tensor field – allows for a direct calculation.
- Our results reproduce the general properties both of the susceptibility and of the magnetization – the weak-field expansion of the former and the negative constant asymptotic of the latter.
- The numerical discrepancy may indicate the incompleteness of the model.

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