Baryon as a dyonic instanton in holographic QCD

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Baryon

Baryon is a colorless object in quantum chromodynamics, carrying unit baryon charge:

$$\langle B|\hat{J}_0|B\rangle = 1$$

where J_μ is a current of vector $U(1)$ subgroup in chiral $U(N_f)_L \times U(N_f)_R$.

Baryons have a **mass** of the QCD scale Λ_{QCD} and carry the axial charge

$$g_A \simeq 1.2$$

Holographic Baryons

Chern-Simons term in holographic 5D action sources the baryon $U(1)$ current $\hat{A}_0$

$$S_{CS} = \int d^4xdz \ \frac{N_c}{24\pi^2} \ \hat{A}_0 \ \epsilon_{MNPQ} \ tr\big(F_{MN}F_{PQ}\big) + \ldots$$

And the baryon charge coincides with the topological charge of the field configuration in 4D spacelike slice of the bulk space.

Holographic Baryons

Holographic baryons were extensively studied and they appeared to have some problems:

- The holographic instantons tend to zero size, becoming singular.
- Due to the curvature of space they tend to fall on the IR boundary of the model.
- The axial charge of the baryon is hard to define.

Dyonic instanton

An interesting topological solution in flat 5D was found by Lambert and Tong, the dyonic instanton:

$$A_\mu = \frac{2}{g}\frac{\rho^2}{x^2(x^2+\rho^2)}\eta^a_{\mu\nu}x_\nu\frac{\sigma^a}{2}, \qquad \phi = v\frac{x^2}{x^2+\rho^2}\frac{\sigma^3}{2}$$

Thanks to the scalar field VEV it has an electric charge, which stabylizes its radus.

$$\rho^2 = \frac{1}{4\pi^2}\frac{\mathbf{Q}}{v}$$

Holographic model

In AdS/QCD the action contains two $SU(2)$ gauge fields and the bifundamental scalar field X, dual to the scalar quark current $\langle\bar{q}q\rangle$

$$S = \int d^3x\ dt\ dz \Big\{ \frac{1}{z}\left(-\frac{1}{4g_5^2}\right)(F_L^2 + F_R^2) + \Lambda^2 \left[\frac{1}{z^3}(DX)^2 + \frac{3}{z^5}\ |X|^2\right]\Big\}$$

One immediately recognizes the scalar with VEV, needed for the dyonic instanton solution

$$\langle X_0\rangle \sim \sigma z^3$$

Cylindrical ansatz

To study the solution with scalar field we adopt the "cylindrical ansatz" for gauge fields

$$A_j^a = \frac{1+\varphi_2}{r}\epsilon_{jak}\frac{x_k}{r} + \frac{\varphi_1}{r}\left(\delta_{ja} - \frac{x_j x_a}{r^2}\right) + A_r\frac{x_j x_a}{r^2}$$
$$A_z^a = A_z\frac{x_a}{r}$$

and for scalar field

$$X = \chi_1\frac{\mathbf{1}}{2} + i\chi_2\frac{\tau^a x^a}{r}$$

2D action

The action for complex fields

$$\boldsymbol{\varphi} = \varphi_1 + i\varphi_2 \equiv \varphi\ e^{i\alpha},$$
$$\boldsymbol{\chi} = \chi_0 + i\chi_1 \equiv \chi\ e^{i\beta},$$

looks like

$$S = \int dt\ 4\pi \int drdz \Big\{ -\frac{1}{2g_5^2}\Big[\frac{2}{z}|D_a\boldsymbol{\varphi}|^2 + \frac{1}{2}(F_{ab})^2 + \frac{1}{r^4}(1-|\boldsymbol{\varphi}|^2)^2\Big]$$
$$-\Lambda^2\Big[\frac{r^2}{2z^3}|D_a\boldsymbol{\chi}|^2 + \frac{1}{z^3}\chi^2\varphi^2\ \cos(\alpha-\beta)^2\Big]$$
$$+\Lambda^2\ \frac{3r^2}{2z^5}|\boldsymbol{\chi}|^2 \Big\},$$

and has two interesting potential terms

Topological charge

The first potential

$$\frac{1}{r^4}(1-|\varphi|^2)^2$$

defines the vacua

$$\varphi\Big|_{vac} = e^{i\alpha}.$$

The solution, interpolating between vacua with different α has a topological charge

$$B = \frac{1}{2\pi}\int dr \int_{\epsilon}^{z_m} dz\ \epsilon_{ab}\Big(\partial_a(-2i(\varphi^2\partial_b\alpha) + F_{ab}\Big).$$

Second charge

The second potential

$$\frac{1}{z^3}\chi^2\varphi^2 \ \cos(\alpha-\beta)^2$$

defines a discrete set of vacua

$$\gamma = \alpha - \beta - \frac{Pi}{2} = \pi n, \quad n \in \mathbb{Z}.$$

The solution, interpolating between vacua with different γ should have second topological charge.

Axial current

This charge is related to the phase β.
The X field is a source to the axial current, namely

$$J_\mu^A \sim i(\partial_\mu X X^\dagger - X \partial_\mu X^\dagger).$$

Or in the 2D fields notation

$$J_r^A \sim 2\chi^2 \, \partial_r \beta(r, z).$$

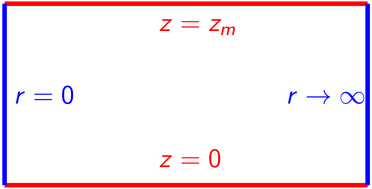
The solution with changing β couples to the axial current, thus it has an axial charge

$$g_A \neq 0.$$

Possibility of dyonic solution

In what fallows, we study the boundary values of the 2D fields, which respect two requirements:

- The action of the solution under consideration is finite
- The solutions for the fields on the boundaries are not singular

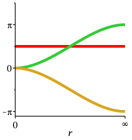


Boundary values: $r = 0,\ z = 0,\ z = z_m$

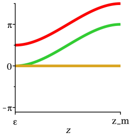
In order to have finite action one requires:

$$\varphi(r,z)\Big|_{z=0,r=0,z=z_m} = 1$$

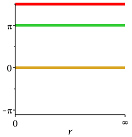
$z = 0 \quad \Rightarrow \quad \partial_r \boldsymbol{\alpha} = 0$



$r = 0 \quad \Rightarrow \quad \boldsymbol{\gamma} = 0$



$z = z_m \quad \Rightarrow \quad \partial_r \boldsymbol{\alpha} = \partial_r \boldsymbol{\beta} = \partial_r \boldsymbol{\gamma} = 0$



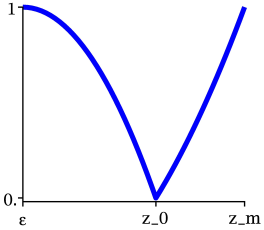
Boundary values: $r = \infty$

The $r = \infty$ is the most interesting because the potential

$$\frac{1}{r^4}(1 - |\varphi|^2)^2 \to 0.$$

And we can use nontrivial boundary value for φ.

$$\varphi(r,z)\Big|_{r\to\infty} = \theta(z_0 - z)\left(1 - \frac{z^2}{z_0^2}\right) + \theta(z - z_0)\left(\frac{z^2 - z_0^2}{1 - z_0^2}\right)$$



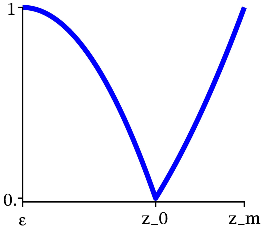
Boundary values: $r = \infty$

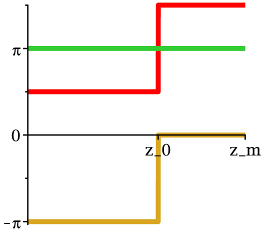
$$\frac{1}{z^3}\chi^2\,\varphi^2\,\sin(\gamma)^2 = 0$$

This solution allows γ to jump at certain point z_0, where φ vanishes.

$$\gamma(r,z)\Big|_{r\to\infty} = -\pi + \theta(z - z_0)\,\pi,$$

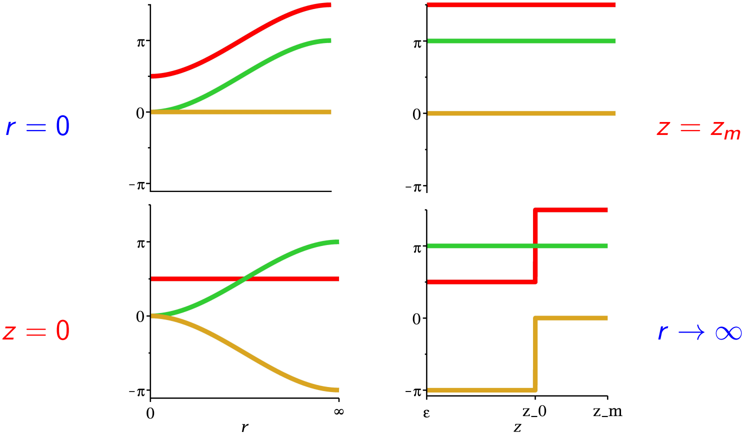
$$\beta(r,z)\Big|_{r\to\infty} = const$$





Overview of the solution

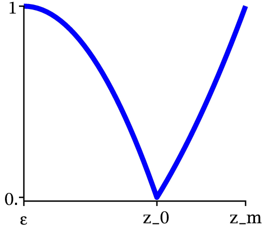
At the end of the day we got the nontrivial allowed boundary values for the solution, carrying two topological charges.



Overview of the solution

The solution can not fall on the IR wall

$$z_0 \neq z_m$$

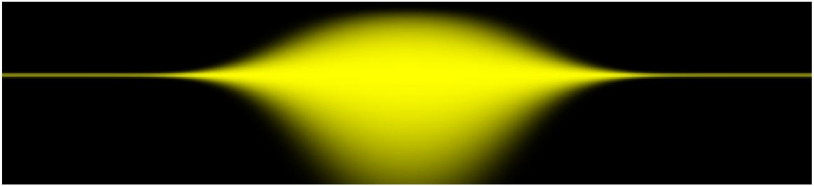


The position of the solution z_0 is governed by VEV σ dual to the chiral condensate. Hence:

$$M_B \sim \langle \bar{q} q \rangle^{1/3}$$

Overview of the solution

The discussed solution has a form of domain wall, whose thickness rises from radial infinity to the core of instanton.



Artist view: the energy density of holographic dyonic instanton

Conclusion

In this work we show a possibility to construct the "dyonic instanton" solution in AdS/QCD. The solution

- Has nonzero baryon charge
- Has nonzero axial charge
- Can not fall on the IR boundary
- Has a mass related to the chiral condensate
- Has a peculiar form of flying saucer