

# On calculation of cross sections in Lorentz-violating QED

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based on [hep-ph/1204.5782](https://arxiv.org/abs/hep-ph/1204.5782) + further work

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# Outline

- Motivation of LV
- Model, modified Feynman rules
- Particle reactions:
  - Perturbative calculations:  
 $\gamma \rightarrow e^+e^-$ ,  $e^- \rightarrow e^-\gamma$ ,  $\gamma\gamma \rightarrow e^+e^-$ ,  $\gamma N \rightarrow Ne^+e^-$
  - Semiclassical calculation:  
 $\gamma \rightarrow e^+e^-$  in a weak magnetic field.
- Conclusions for experiments.

# Motivation

- Several approaches to quantum gravity predict Lorentz-invariance violation (LIV) at high energies (e.g. Horava-Lifshitz).

*Horava, 2009*

*Blas, Pujolas, Sibiryakov, 2010*

- Theories with violation of the Lorentz Symmetry as an effective theory

*Kostelecky, Colladay, 1998*

*Coleman, Glashow, 1999*

- Lorentz invariance as an emergent phenomena at low energies

*Chadha, Nielsen, 1982*

# Model

Assumptions:  $C$ -,  $P$ -,  $T$ -parity, rotational symmetry, LV operators of dimension up to 6.

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} +$$
$$+ i\kappa \bar{\psi} \gamma^i D_i \psi + \frac{ig}{M^2} D_j \bar{\psi} \gamma^i D_i D_j \psi + \frac{\xi}{4M^2} F_{kj} \partial_i^2 F^{kj}$$

$\Downarrow$

Dispersion relations:

$$E_\gamma^2 = k^2 + \frac{\xi k^4}{M^2}, \quad E_e^2 = m^2 + p^2 (1 + 2\kappa) + \frac{2gp^4}{M^2}.$$

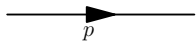
## Modified spin sums and propagators

$$\sum_{a=1,2} \varepsilon_{\mu}^{(a)} \varepsilon_{\nu}^{(a)} = \text{diag} \left( -1 - \frac{\xi^2 k^2}{M^2}, 1, 1, 1 \right)$$

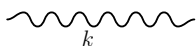
$$\sum_{s=1,2} u^s(p) \bar{u}^s(p) = \gamma^{\mu} \tilde{p}_{\mu} + m, \quad \sum_{s=1,2} v^s(p) \bar{v}^s(p) = \gamma^{\mu} \tilde{p}_{\mu} - m$$

$$\tilde{p}^0 = E, \quad \tilde{p}^i = p^i \left( 1 + \varkappa + \frac{gp^2}{M^2} \right)$$

### Propagators

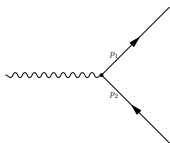


$$= \frac{i (\gamma^{\mu} \tilde{p}_{\mu} + m)}{\tilde{p}^{\mu} \tilde{p}_{\mu} - m^2 + i\epsilon}$$

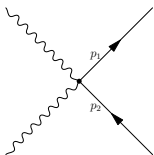


$$= \frac{i}{E^2 - k^2 \left( 1 + \frac{\xi k^2}{M^2} \right) + i\epsilon} \text{diag} \left( -1 - \frac{\xi k^2}{M^2}, 1, 1, 1 \right)$$

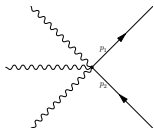
# Modified vertices



$$= -ie\gamma^\mu - ie\delta_i^\mu \left[ \not{x}\gamma^i + \frac{g}{M^2} (p_1^i(p_1 \cdot \gamma) + p_2^i(p_2 \cdot \gamma) - (p_1 \cdot p_2)\gamma^i) \right]$$



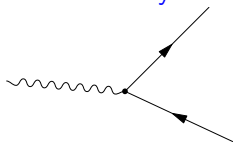
$$= \frac{ige^2}{M^2} \left[ \gamma^i(p_2 - p_1)^j + \gamma^j(p_2 - p_1)^i + \delta^{ij} ((p_2 - p_1) \cdot \gamma) \right] \delta_i^\mu \delta_j^\nu$$



$$= -\frac{2ige^3}{M^2} \left[ \delta_i^\mu \delta_j^\nu \delta_j^\lambda + \delta_j^\mu \delta_i^\nu \delta_j^\lambda + \delta_j^\mu \delta_j^\nu \delta_i^\lambda \right]$$

# Photon decay $\gamma \rightarrow e^+e^-$ and Cerenkov radiation $e^- \rightarrow e^-\gamma$

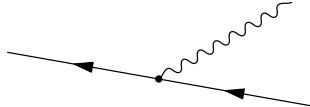
## Photon decay



If  $0 \leq 2g \leq \xi_*$  or  $0 < g, g/2 \leq \xi_*$ ,  
 ( $\xi_* = \xi - 2M^2\kappa/k^2$ ).

$$\Gamma = \alpha k \left[ -\frac{2\kappa}{3} + \frac{k^2}{M^2} \left( \frac{\xi}{3} - \frac{11g}{30} \right) \right] \quad \frac{dE}{dt} = -\alpha p^2 \left[ \frac{7\kappa}{12} + \frac{p^2}{M^2} \left( \frac{11g}{12} - \frac{2\xi}{15} \right) \right]$$

## Cerenkov radiation



The case  $0 \leq \kappa, 0 \leq \xi \leq 2g$ .

$$\Gamma = \alpha p \left[ \frac{4\kappa}{3} + \frac{p^2}{M^2} \left( \frac{157g}{60} - \frac{11\xi}{60} \right) \right]$$

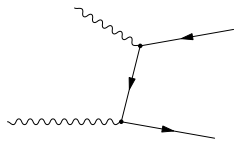
The case of dim 4 operators ( $\xi = g = 0$ ) —

*Klinkhamer, Schreck, 2008*

*Hohensee et.al., 2008*

Effects of LIV in matrix element and in phase volume are of the same order!

# Pair production on the soft photon $\gamma\gamma \mapsto e^+e^-$



$$\omega_{LV} = -\varkappa k + \left(\frac{\xi}{2} - g\right) \frac{k^3}{M^2}.$$

If  $k(2\omega + \omega_{LV}) \gg m^2$ ,

$$\sigma = \frac{\alpha^2 \pi}{2k\omega} \left[ 1 + \left( 1 + \frac{\omega_{LV}}{\omega} \right)^2 \right] \ln \left[ \frac{k(2\omega + \omega_{LV})}{m^2} \right]$$

LI case:

$$\sigma = \frac{\alpha^2 \pi}{2k\omega} \ln \left[ \frac{2k\omega}{m^2} \right]$$

**BUT:**  $\omega_{LV} > 0$

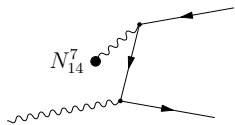
$\longrightarrow$

photon decay  $\gamma \rightarrow e^+e^-$

*thresholds — Galaverni, Sigl, 2008*



# Pair production on a nuclei $\gamma N \mapsto e^+e^-N$



$$\omega_{LV} = -\varkappa k + \left(\frac{\xi}{2} - g\right) \frac{k^3}{M^2}, \quad k \gg m$$

$$k |\omega_{LV}| \ll m^2 \quad \longrightarrow \quad \sigma = \frac{28}{9} \frac{z^2 \alpha^3}{m^2} \ln \frac{2k}{m}$$

*Bethe, Heitler, 1934*

$$k |\omega_{LV}| \gg \frac{m^2}{\alpha^2 Z^{2/3}} \quad \longrightarrow \quad \sigma = \frac{4Z^2 \alpha^3}{3} \frac{1}{k |\omega_{LV}|} \left( \ln \frac{k |\omega_{LV}|}{m^2} \right)^2$$

$$m^2 \ll k |\omega_{LV}| \ll \frac{m^2}{\alpha^2 Z^{2/3}} \quad \longrightarrow \quad \sigma = \frac{8Z^2 \alpha^3}{3} \frac{1}{k |\omega_{LV}|} \ln \frac{2k |\omega_{LV}|}{m^2} \ln \frac{1}{\alpha Z^{1/3}}$$

# Pair production in a weak magnetic field. LI case.

Semiclassical method: used to compute Schwinger effect (pp in electric field)

*Affleck, Alvarez, Manton, 1982*

*Monin, Voloshin, 2008*

$$H = (H, 0, 0); \quad k_\mu = (\omega, 0, \omega, 0)$$

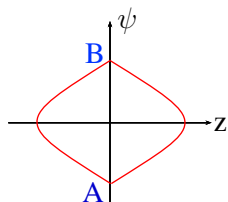
$$\Gamma \approx \int Dx_\mu e^{-S[x_\mu]}, \quad S = \oint d\phi \left( m\sqrt{\dot{x}_\mu^2} - ieA_\mu \dot{x}_\mu \right) - \omega T + \omega Y,$$

$$\text{EOM: } \frac{m\ddot{x}_\mu}{\sqrt{\dot{x}_\mu^2}} = ieF_{\mu\nu}\dot{x}_\nu$$

**Solution** (the right arc):

$$t = \frac{\sqrt{p^2 + m^2}}{eH} \left( \phi - \frac{\theta}{2} \right), \quad y = -i\psi = -i\frac{p}{eH} \text{sh} \left( \phi - \frac{\theta}{2} \right),$$

$$x = 0, \quad z = -\frac{p}{eH} \left[ \text{ch} \left( \phi - \frac{\theta}{2} \right) - \text{ch} \frac{\theta}{2} \right]$$



## Pair production in a weak magnetic field.

LI case

$$S = \frac{8}{3} \frac{m^3}{\omega e H}, \quad \Gamma \propto e^{-\frac{8}{3} \frac{m^3}{\omega e H}}$$

*Erber, 1966*

LIV case

$$\beta = \kappa - \frac{\xi}{2} \frac{k^2}{M^2} + \frac{2m^2}{\omega^2}$$

$$S = \frac{\omega^2}{eH} \frac{2\sqrt{2}}{3} \beta^{3/2}, \quad \Gamma \propto e^{-S}.$$

$$\beta > \frac{2m^2}{\omega^2} \quad \longrightarrow \quad \text{strong suppression}$$

$$0 < \beta < \frac{2m^2}{\omega^2} \quad \longrightarrow \quad \Gamma \text{ grows — fine-tuning!} \quad \left( \frac{2m^2}{\omega^2} \sim 10^{-26} \right)$$

$$\beta < 0 \quad \longrightarrow \quad \text{photon decay}$$

## Bounds on LV parameters

$$E_\gamma^2 = k^2 + \frac{\xi k^4}{M^2},$$

$$E_e^2 = m^2 + p^2 (1 + 2\kappa) + \frac{2gp^4}{M^2}.$$

- $|\kappa| < 10^{-15}$  — absence of anomalously synchrotron losses at LEP  
*Altshul, 2009*
- $-5 \cdot 10^{13} < \xi < 10^{11}$  — photons with energies 50 TeV have been detected  
*HEGRA, CANGAROO, HESS*
- $-10^{11} < g < 10^6$  — photon spectrum of Crab nebula and AGNs.(assumption — SSC model)  
*Altshul, 2006*

# Conclusions

- Effects of kinematical and dynamical aspects of LIV are of the same order
- Even small ( $\varkappa \sim 10^{-26}$  or  $\xi \sim g \sim 10^{-6}$ ) LIV strongly suppresses pair production (both in an electric field of a nuclei or in a weak magnetic field)
- Subplankian LIV is NOT experimentally closed yed.
- $\omega_{LV} \geq m^2/k^2$  — no detected photons due to photon decay  
 $\omega_{LV} \leq -\#m^2/k^2$  — no detected photons due to non-detection
- If even one photon with energy  $10^{18} - 10^{19}$  eV will be detected, subplankian Lorentz violation will be strongly restricted.

Thank you for your attention!