BFKL pomeron in the external field of the nucleus in (2 + 1)-dimensional QCD

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QCD at high energies

Strong interaction at high energies is mediated by the exchange of BFKL pomerons.

In the limit $N_c \rightarrow \infty$ they split and fuse by triple pomeron vertex.

For *hadron-nucleus* scattering the relevant tree (fan) diagrams are summed by the Balitsky-Kovchegov (BK) evolution equation.
Fan diagrams

BK approximation can be justified if \( \gamma = \lambda \exp \Delta_P y \) is small

For a large nuclear target, such that \( A^{1/3} \gamma \sim 1 \), the tree diagrams indeed give the dominant contribution and loops can be dropped.

For supercritical BFKL pomeron \( \Delta_P > 0 \). Parameter \( \gamma \) grows with rapidity.

1) Loop contribution becomes not small
2) One can not apply the perturbative approach
Loop diagrams

With the growth of energy the role of pomeron loops becomes important

On has to search for methods to take them into accounts

Calculations of loops may become easier if one starts with the perturbative approach inside the nucleus from the start

The nuclear field transforms the supercritical pomeron into a subcritical one with the intercept smaller than unity. Parameter \( \gamma \) vanishes with rapidity growth. Perturbative treatment becomes possible
Local reggeon field theory

\[ \mathcal{L} = \phi^\dagger S\phi + \lambda\phi^\dagger\phi(\phi + \phi^\dagger) + g\rho\phi \]

To go beyond the classical approximation we make a shift in the quantum field

\[ \phi^\dagger(y, b) = \phi_1^\dagger(y, b) + \xi(y, b) \]

\[ \mathcal{L} = \phi_1^\dagger (S + 2\lambda\xi)\phi + \lambda\xi\phi^2 + \lambda\phi_1^\dagger\phi(\phi + \phi_1^\dagger) \]
Local reggeon field theory

Propagator in the external field:

\[
\frac{\partial P(y, b|y'b')}{\partial y} = (\epsilon - \alpha' \nabla_b^2)P(y, b|y', b') + 2\lambda \xi(y, b)P(y, b|y', b')
\]

with boundary conditions

\[
P(y, b|y', b') = 0, \quad y - y' < 0, \quad P(y', b|y', b') = \delta^2(b - b')
\]

At large \( y \) the Green function behaves as the free Green function with sign opposite to \( \epsilon \) and so vanishes at \( y \to \infty \)
BFKL pomeron

We use effective non-local field theory with \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I + \mathcal{L}_E \)

\[
\mathcal{L}_0 = \int d^2 r_1 d^2 r_2 \Phi^\dagger \nabla^2_1 \nabla^2_2 \left( \frac{\partial}{\partial y} + H_{BFKL} \right) \Phi
\]

BFKL pomeron propagator

interaction with the nuclear target

\[
\mathcal{L}_E = -\int d^2 r_1 d^2 r_2 \Phi^\dagger J
\]

BFKL pomeron vertex

\[
\mathcal{L}_I = \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2 r_1 d^2 r_2 d^2 r_3}{r_{12}^2 r_{23}^2 r_{13}^2} \Phi^\dagger(y, r_1, r_2) \Phi^\dagger(y, r_2, r_3) K_{31} \Phi(y, r_3, r_1) + (\Phi \leftrightarrow \Phi^\dagger)
\]
BFKL pomeron

Propagator in the external field:

\[
\frac{\partial P(y, r_1, r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi} \int d^2 r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2} \left( P(y, r_1, r_3) + P(y, r_2, r_3) - P(y, r_1, r_2) - \Phi(y, r_1, r_3) P(y, r_2, r_3) - \Phi(y, r_2, r_3) P(y, r_1, r_3) \right)
\]

with boundary conditions

\[
P(y = y', r_1, r_2; y', r'_1, r'_2) = \nabla_1^{-2} \nabla_2^{-2} \delta^2(r_1 - r'_1) \delta^2(r_2 - r'_2)
\]

Since the study is only possible numerically, to avoid using the singular initial condition, we shall consider a convolution with an arbitrary function

\[
P(y, r_1, r_2) = \int d^2 r'_1 d^2 r'_2 P(y, r_1, r_2; y', r'_1, r'_2) \nabla_1^2 \nabla_2^2 \psi(r'_1, r'_2)
\]
BFKL pomeron

With the chosen set of initial conditions, the convoluted BFKL pomeron propagator vanishes at large rapidity distances

\[ P(y = 0, r_1, r_2) = 1 - e^{-c_1 r_2} e^{-b^2/c_2} \]

\[ c_1 = 10; \quad c_2 = 2 \]

Calculation is possible only numerically
BFKL pomeron in (2+1) dimensional QCD

Propagator in the external field:

\[
\frac{\partial P(y, r_1, r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi} \int d^2 r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2} \left( P(y, r_1, r_3) + P(y, r_2, r_3) - P(y, r_1, r_2) - \Phi(y, r_1, r_3) P(y, r_2, r_3) - \Phi(y, r_2, r_3) P(y, r_1, r_3) \right)
\]

\[
2\alpha_s N_c \theta(r_{2,1}^{max} - r_3) \theta(r_3 - r_{2,1}^{min})
\]

We expect to find analytical solution in (2+1) dimensional QCD

BFKL pomeron in (2+1) dimensional QCD

In terms of $S$-matrix $S_{r_2r_1}(y) = 1 - \Phi_{r_2r_1}(y)$

**BK equation**

$$\frac{\partial S_{r_2r_1}}{\partial y} = \int_{r_1}^{r_2} dr_0 \left(S_{r_2r_0}S_{r_0r_1} - S_{r_2r_1}\right)$$

$$\Psi_{r_2r_1}(y) = e^{r_21y}S_{r_2r_1}$$

$$\frac{\partial \Psi(y)}{\partial y} = \Psi^2(y)$$

$$\Psi(y) = \Psi(0)[1 - y\Psi(0)]^{-1}$$

**Pomeron propagator**

$$\frac{\partial P_{r_2r_1}}{\partial y} = \int_{r_1}^{r_2} dr_0 \left(S_{r_2r_0}P_{r_0r_1} + P_{r_2r_0}S_{r_0r_1} - P_{r_2r_1}\right)$$

$$Q_{r_2r_1}(y) = e^{r_21y}P_{r_2r_1}(y)$$

$$\frac{\partial Q(y)}{\partial y} = \left\{ Q(y), \Psi(y) \right\}$$

$$Q(y, y') = \Psi^{-1}(y')\Psi(y)Q(y', y')\Psi(y)\Psi^{-1}(y')$$

$$\equiv T(y, y')Q(y'y')T(y, y')$$

**Solution was found in matrix notation**
BFKL pomeron in (2+1) dimensional QCD

The initial condition for the BFKL pomeron propagator in the nuclear field may be chosen as:

\[ g_{r_2 r_1 | r_2' r_1'} (y, y') = (r_2 - r_2') \theta (r_2 - r_2') (r_1' - r_1) \theta (r_1' - r_1) \]

The solution factorizes:

\[ g(y, y') = T(y, y') g(y' y') T(y, y') \]

\[ g_{r_2 r_1 | r_2' r_1'} (y, y') = e^{-r_{21}(y-y')} U_{r_22', (y, y')} U_{r_1'1} (y, y') \]

\[ U_r(y, y') = \int_0^r dr' (r - r') T_{r'} (y, y') e^{-y' r'} \]

\[ g_{r_2 r_1 | r_2' r_1'} (y, y') \neq 0 \text{ only if } r_1 < r_1' < r_2' < r_2 \]
BFKL pomeron in (2+1) dimensional QCD

The pomeron self-mass:

\[
\Sigma(y, r_2, r_1 | y', r'_2, r'_1) = -(8\pi\alpha_s^2 N_c)^2 \int_{\min\{r_2, r_1\}}^{\max\{r_2, r_1\}} \int_{\min\{r'_2, r'_1\}}^{\max\{r'_2, r'_1\}} dr_3 dr'_3 g(y, r_2, r_3 | y', r'_2, r'_3) g(y, r_3, r_1 | y', r'_3, r'_1)
\]

It is trivial to see that this expression is zero due to the properties of the pomeron propagator.

Pomeron cannot form loops in the nuclear field. This implies that the BK equation gives the complete solution to the quantum field theory of interacting pomeron in the nuclear field in 2 + 1 dimensions.