*Micro*canonical *jet*-fragmentation at *LEP* and *LHC* energies *and* *Tsallis Statistics*

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with

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Content

1, **The** ***Tsallis-distribution*** **pops up (nearly) everywhere** ***in*** **highenergy reactions**

Hadronspectra in ***heavy-ion, proton-proton*** **and** ***electron-positron*** **collisions**

2, **Three ways to obtain the Tsallis distribution**

a, ***Special entropy*** **formulas**

b, ***Special*** **N-body** ***interactions***

c, ***Super-Statistics***

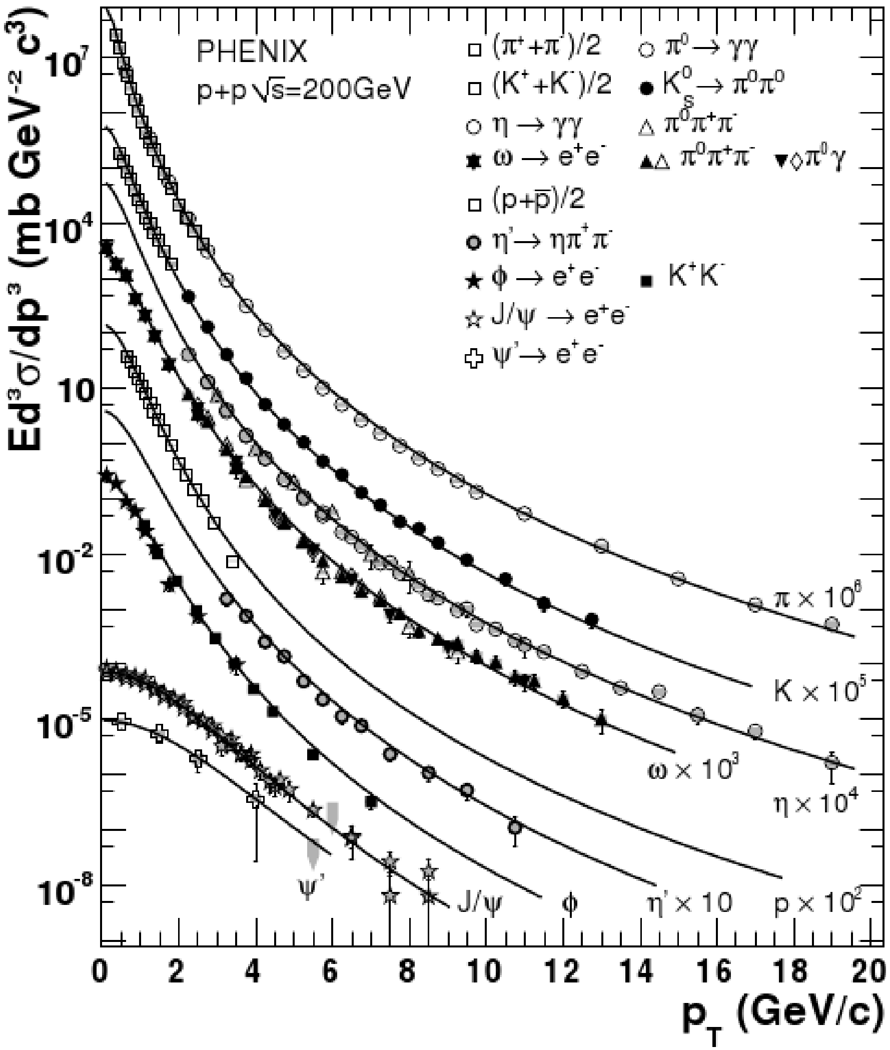
3, **My bet:** ***Super – Statistics***

a, Jet-fragmentation in proton-proton and electron-positron collisions

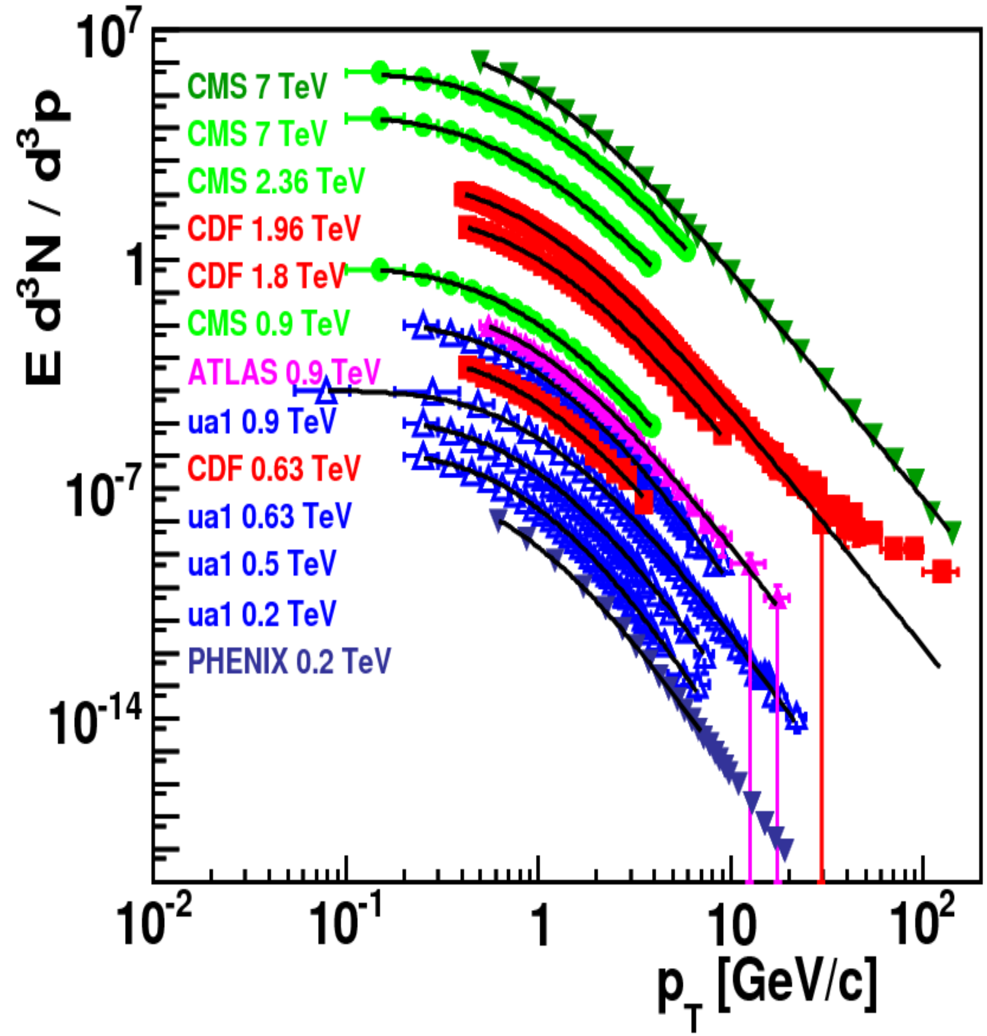
b, Multiplicity dependence of transverse π, K, p spectra from latest CMS results

Hadron spectra in proton-proton collisions

Phys. Rev. D 83: 052004, 2011



J. Phys.: Conf. Ser. 270 012008 (2011)

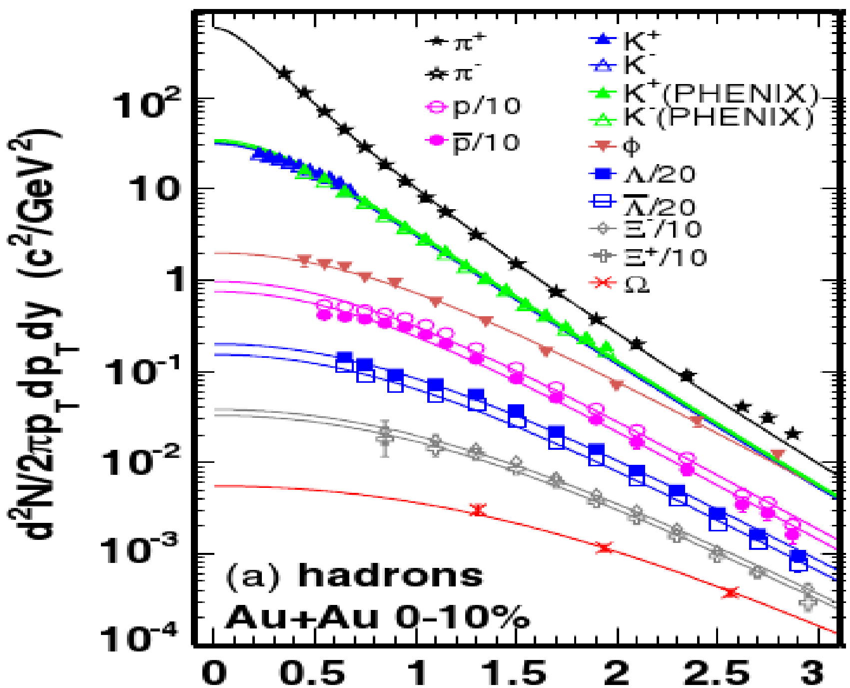


$$E\frac{d\sigma}{d^{٣}p} \propto \left(١+m_T/nT\right)^{-n}$$

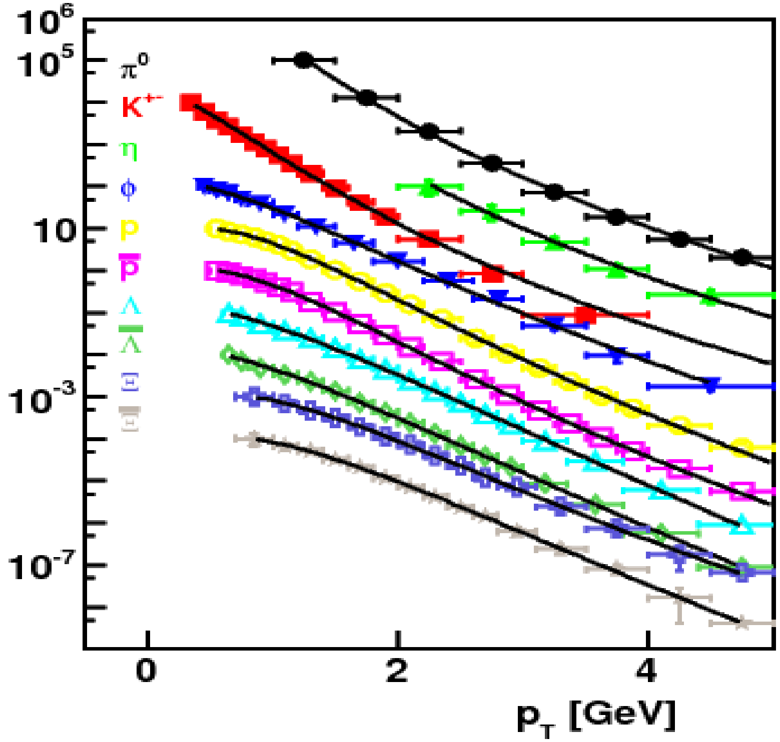
$$E\frac{d\sigma}{d^{3}p} \propto m_T\left(1+m_T/nT\right)^{-n}$$

Hadron spectra in heavy-ion collisions

J. Phys. G: Nucl. Part. Phys. 37 085104 (2010)



Phys. Lett. B, 689: 14-17, (2010)



Theoretical model: Recombination of Tsallis distributed thermal quarks + „*Blast Wave”* flow profile for the expanding QGP

How to Obtain Tsallis Distribution

Generalise the free-energy functional in the Maximum Entropy variational ansatz

$$S\left[f(\epsilon)\right]-\beta C\left[f(\epsilon)\right] = max$$

- ***Deform the entropy***, ***S[f]*** **(C. Tsallis, Eur. Phys. J. A, 40, 257-266 (2009))**

- **Introduce *special N-body interactions* of the type**

  $$E = E_1 + E_2 + \ldots\ a\,(E_1 E_2 + E_1 E_3 + \ldots) + \ldots + a^{N-1} E_1 * \ldots * E_N$$

  which is equivalent to

  $$L(E) = L(E_1) + L(E_2) + \ldots + L(E_N)$$

  with $L(E) = (1/a)\ \ln(1 + a\,E)$

  Urmossy et. al., Proc. of 'Hot and Cold Baryonic Matter 2010'
  T. S. Biro et. al., *J. Phys. G,* **36** 064044 (2009)
  T. S. Biro et. al., *Eur. Phys. J. A* **40** 325-340 (2009)

- ***Super - Statistics***

Entropy and Energy Functionals with the Corresponding Equilibrium Distributions

$$S\left[f(\epsilon)\right] - \beta C\left[f(\epsilon)\right] = max$$

$S[f]$	$C[f]$	$f_{eq}(\epsilon)$	
$-\int f \ln f$	$\int \epsilon f$	$A \exp\{-\beta\,\epsilon\}$	
$-\int f \ln f$	$\int L(\epsilon) f$	$A \exp\{-\beta L(\epsilon)\}$	T. S. Biro et. al., *Eur. Phys. J. A* **40** 325-340 (2009)
$\int f \ln_q f$	$\int \epsilon f$	$A\left[1 + (q-1)\beta\,\epsilon\right]^{-1/(q-1)}$	C. Tsallis
$\int f \ln_q f$	$\int L(\epsilon) f$	$A\left[1 + (q-1)\beta L(\epsilon)\right]^{-1/(q-1)}$	T.S. Biró et. al., Phys. Rev. E, 83, 061187, (2011)

Deformed logarith (C. Tsallis, Eur. Phys. J. A, 40, 257-266 (2009) ***):***

$$\ln_q(z) = \frac{z^{1-q} - 1}{q - 1}$$

Super-Statistics and Microcanonical Fragmentation

- **A *jet* is a bunch of hadrons flying *almost colinearly* *(quasi – 1 dimension!)*.**
  If the *cross-section* of the production of these hadrons is proportional to their phasespace *restricted only by energy conservation*, these hadrons form a *microcanonical ensemble*.
  Thisway, in a jet of *N* (massless) hadrons, hadrons have the *energy distribution*:

$$f_N(z) = A_N (1-z)^{N-2}, \qquad z = \frac{\epsilon_h}{E_{jet}}$$

- **The *number of hadrons* in a *jet fluctuates* as *(experimental observation)***

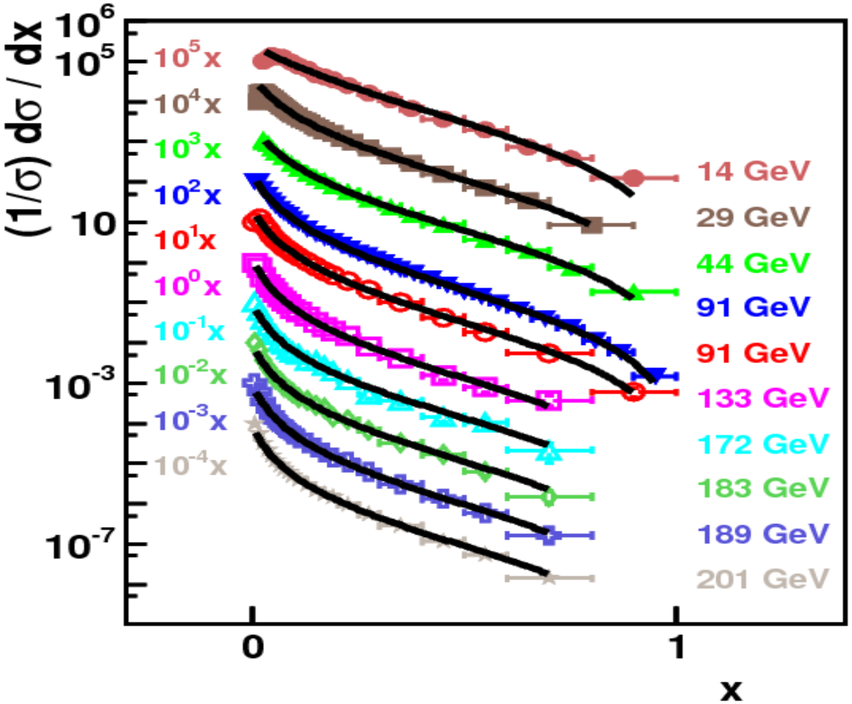
$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$

- **Thus, the *multiplicity averaged hadron distribution* (*fragmentation function*) becomes**

$$\frac{d\sigma}{dz} = \sum_{N=N_0}^{\infty} f_N(z)\, N\, p(N) \propto \frac{(1-z)^{\nu(N_0)}}{\left(1 - \frac{(q-1)}{T/E_{jet}} \ln(1-z)\right)^{1/(q-1)}}$$

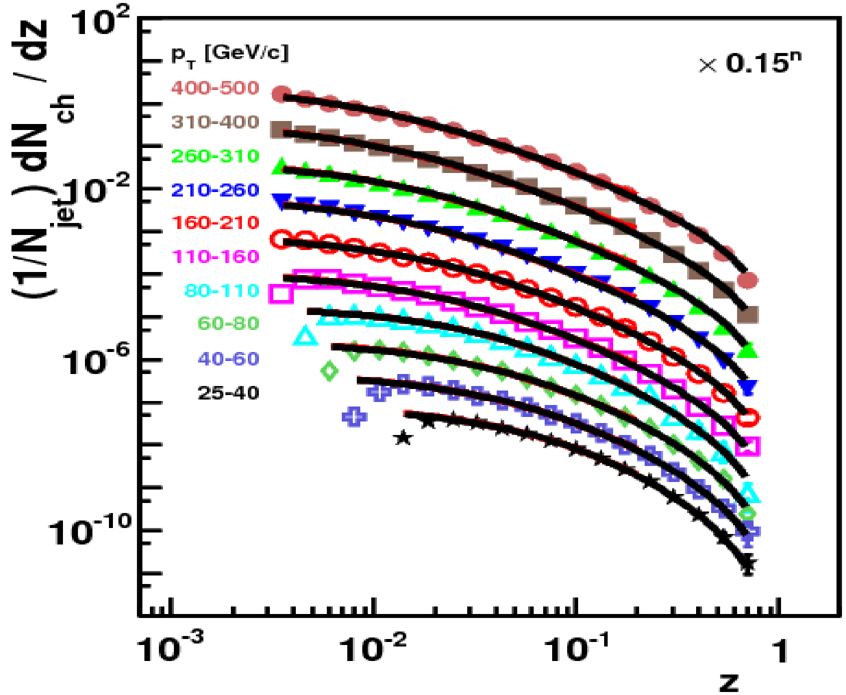
Confrontation with Measurements

e^+e^- annihilations @LEP ($\sqrt{s}$ = 14–200 GeV)



Urmossy et. al.,
Phys. Lett. B, 701, 111-116 (2011),
arXiv:1101.3023

proton-proton collisions @LHC (pT = 25–500 GeV/c)

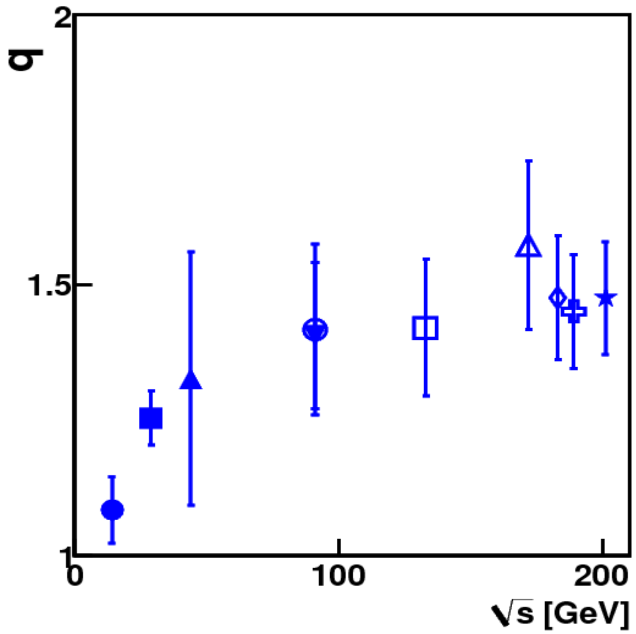


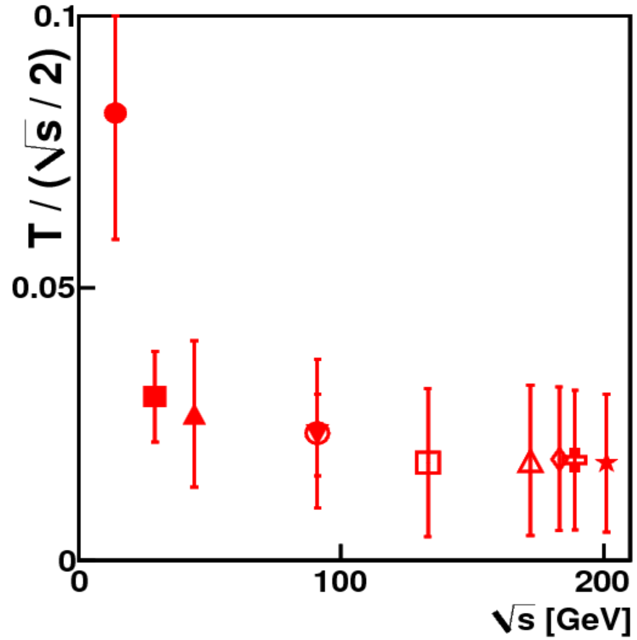
Urmossy et. al.,
arXiv:1204.1508v1

Scale-evollution of the Parameters

e^+e^- annihilations @LEP ($\sqrt{s}$ = 14–200 GeV)

$$\frac{d\sigma}{dz} \propto (1-z)^{\nu} \left(1 - \frac{(q-1)}{T/E_{jet}} \ln(1-z)\right)^{-1/(q-1)}$$



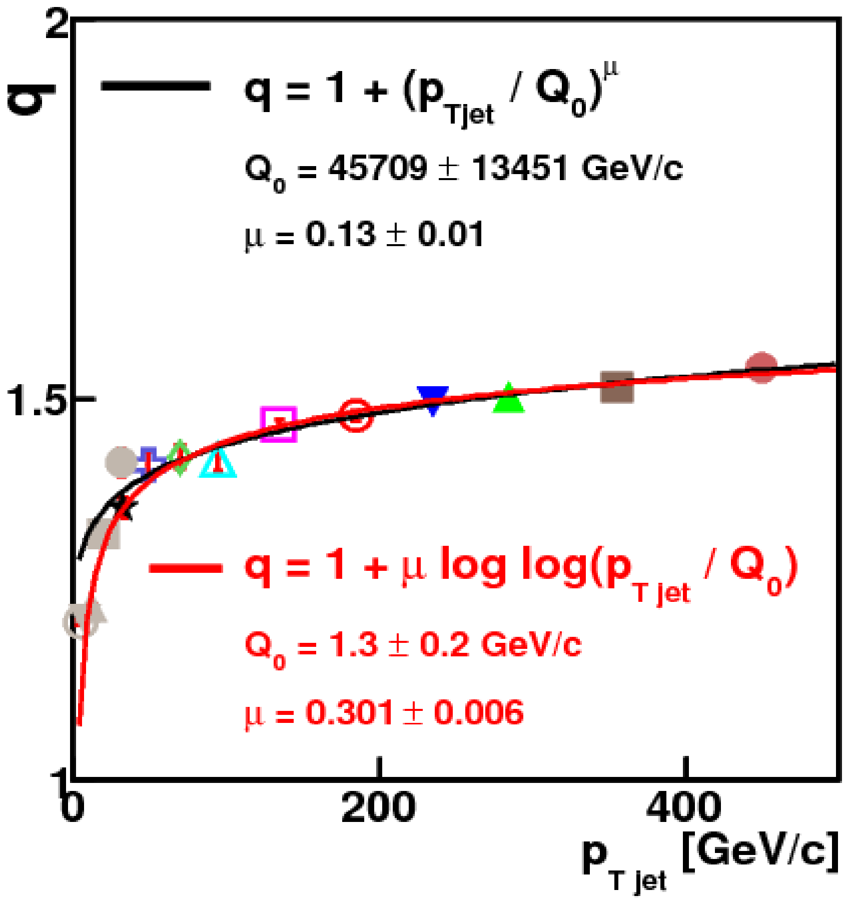


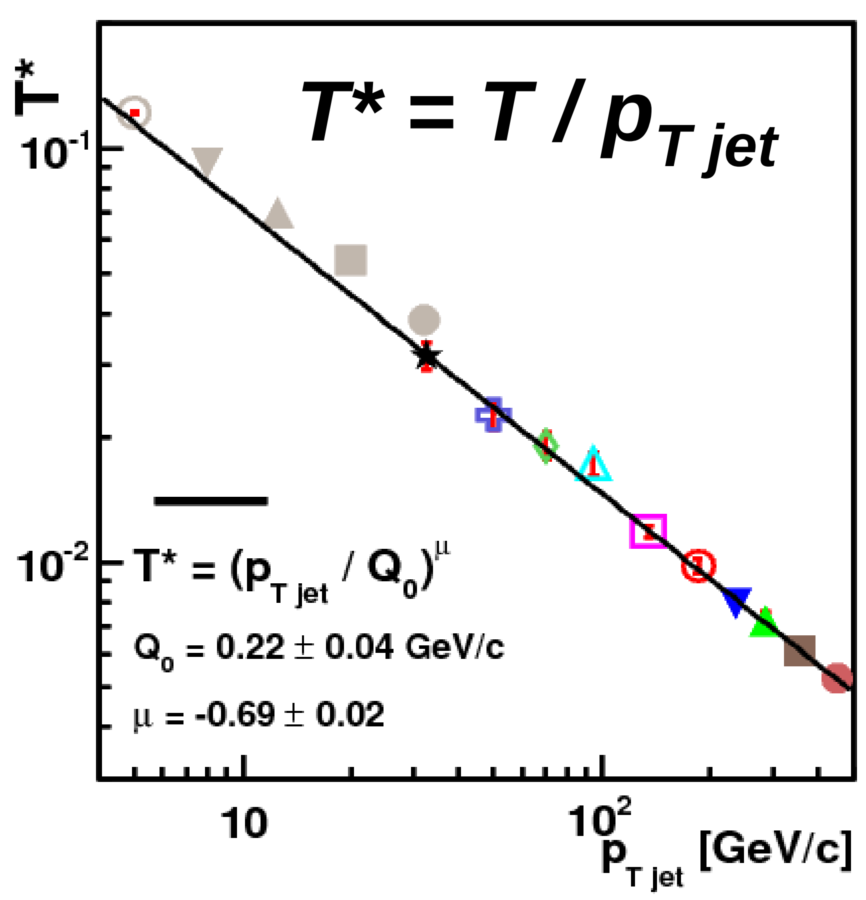
q rises, *T/√s falls as* the collision energy (√s) grows

Urmossy et. al., *Phys. Lett. B*, 701, 111-116 (2011), arXiv:1101.3023

Similar Scale-depencence in

proton-proton collisions @LHC (pT = 25–500 GeV/c)





Urmossy et. al., arXiv:1204.1508v1

Tsallis Distribution at low x

At low hadronenergy (x << 1): Microcanonical --> Canonical

The distribution of 1 hadron in a jet of N hadrons is Boltzmann-Gibbs:

$$f_N(x) \;\to\; \exp(-\beta x)$$

The temperature means mean energy per particle

$$T \;\propto\; E/N \;\to\; \beta \;\propto\; N$$

Thus, multiplicity fluctuations cause fluctuations of the temperature (Super-Statistics):

$$p(N) \;\to\; p(\beta) \;\propto\; (\beta)^{A-1} e^{-B\beta}$$

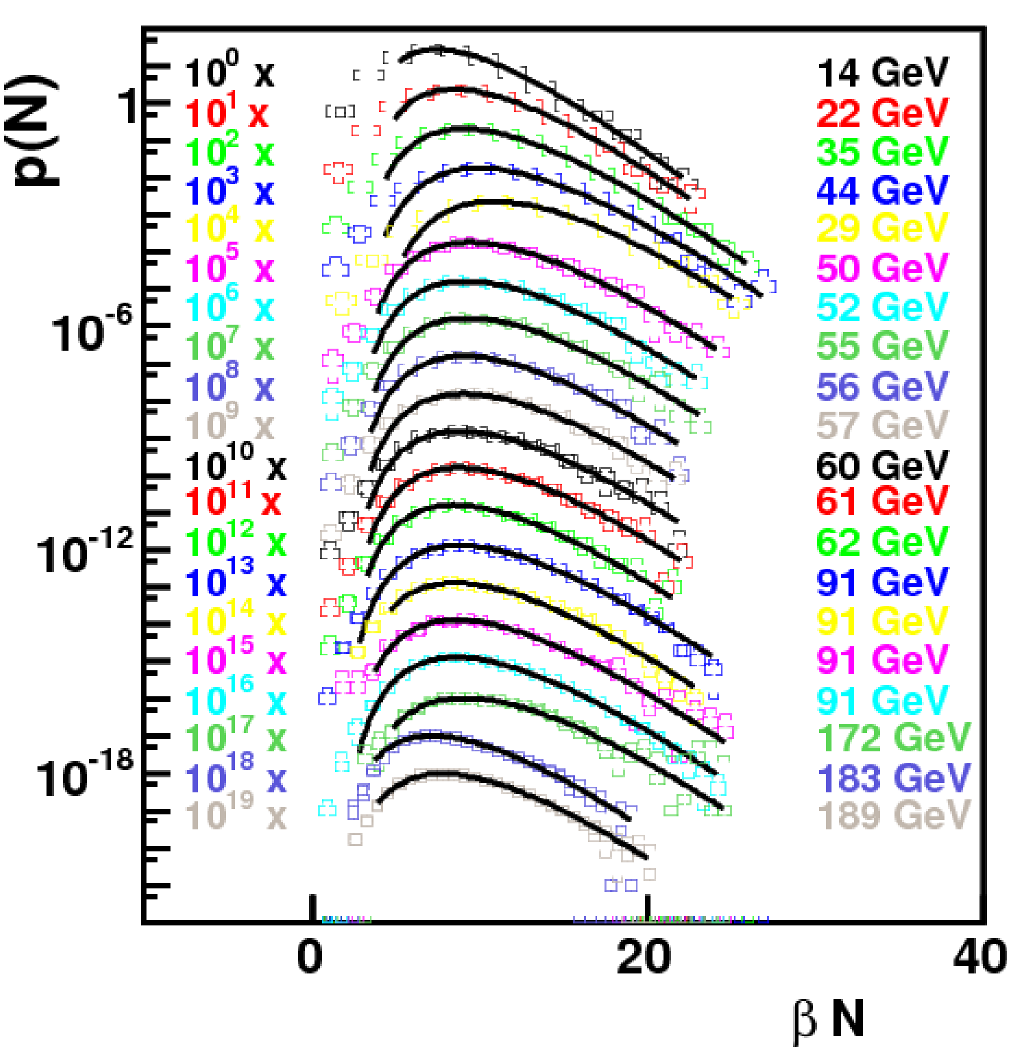
The multiplicity-averaged single-hadron distribution is the Tsallis-distribution:

$$f(x) = \int d\beta\, f_\beta(x)\, p(\beta) \;\propto\; \frac{1}{\left(1+\frac{q-1}{T/E_{jet}}x\right)^{1/(q-1)}}$$

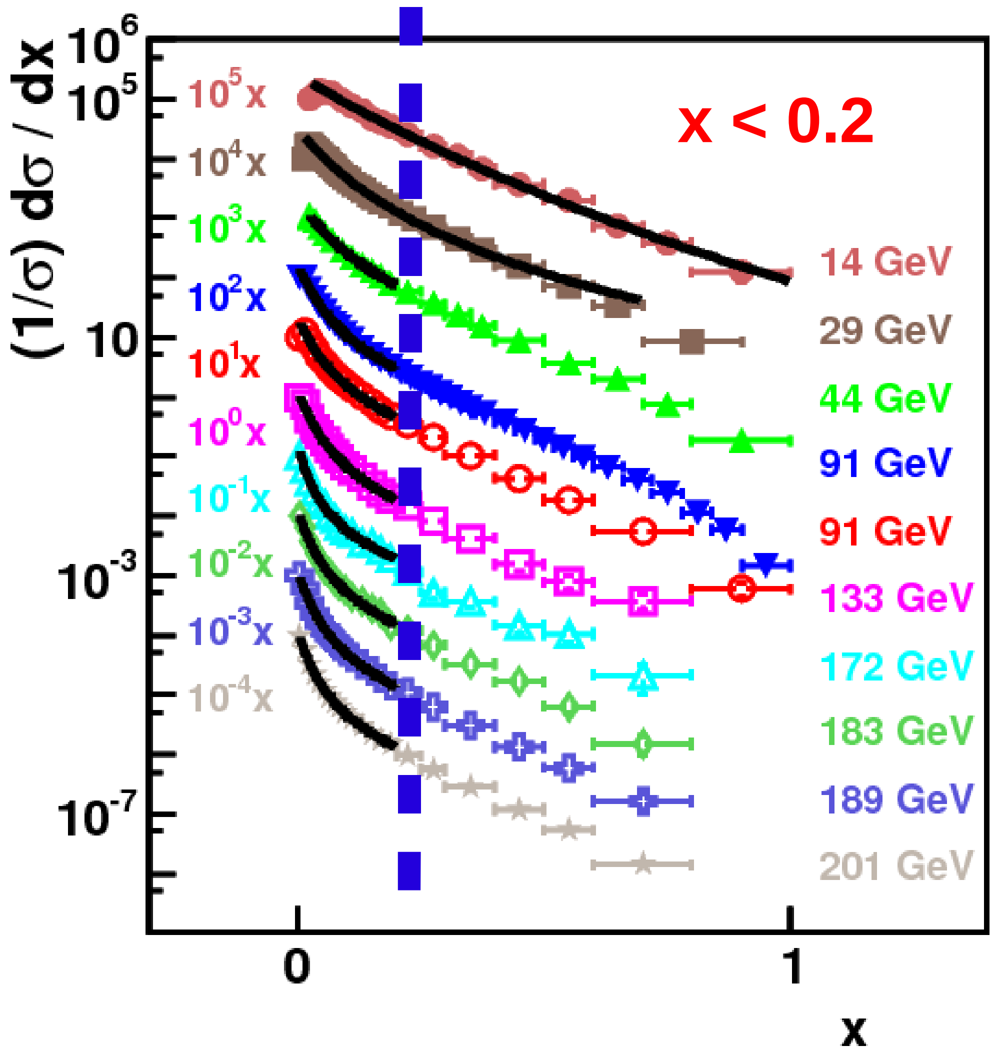
Talk of Murray Gell-Mann

e^+e^- --> $h^\pm$+X @ $\sqrt{s}$ = 14–200 GeV (fitted with Tsallis)

Multiplicity distributions



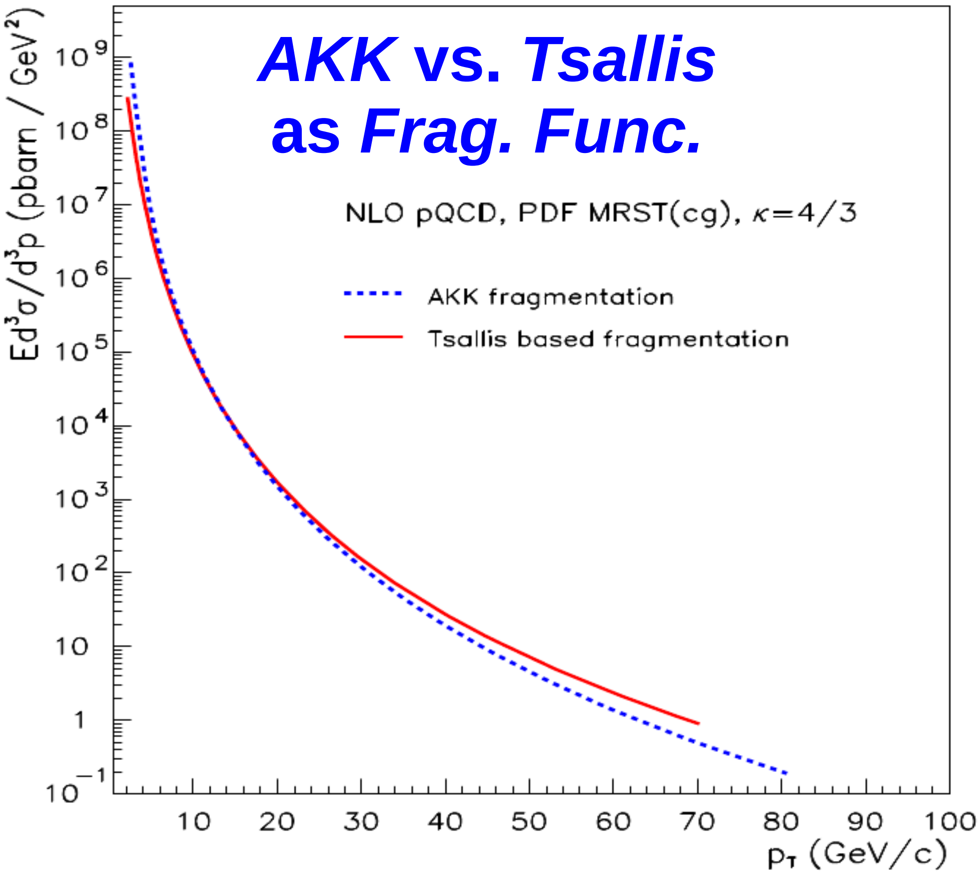
Fragmentation functions



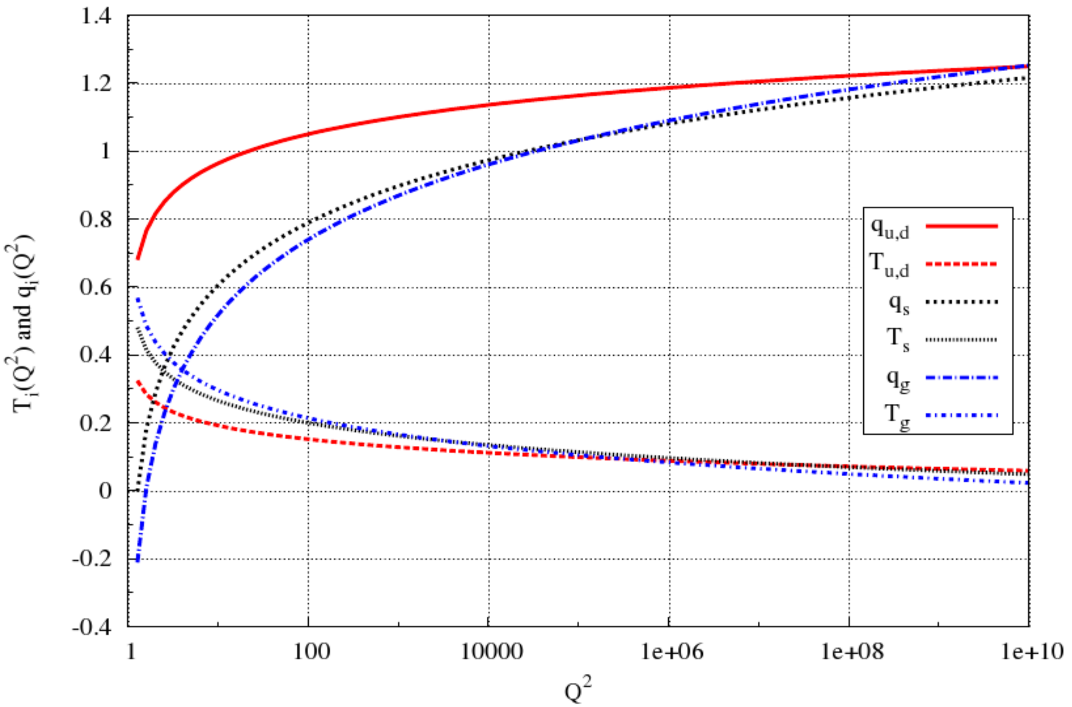
Urmossy et. al., Phys. Lett. B, 701: 111-116 (2011), arXiv:1101.3023

Why do we see such scale depencence?

π^+ spectrum in pp --> π^+X @ $\sqrt{s}$=7 TeV (NLO pQCD)



Scale dependence of *q* and *T*



$$D_{p_i}^{\pi^+}(z) \sim (1+(q_i-1)z/T_i)^{-1/(q_i-1)}$$

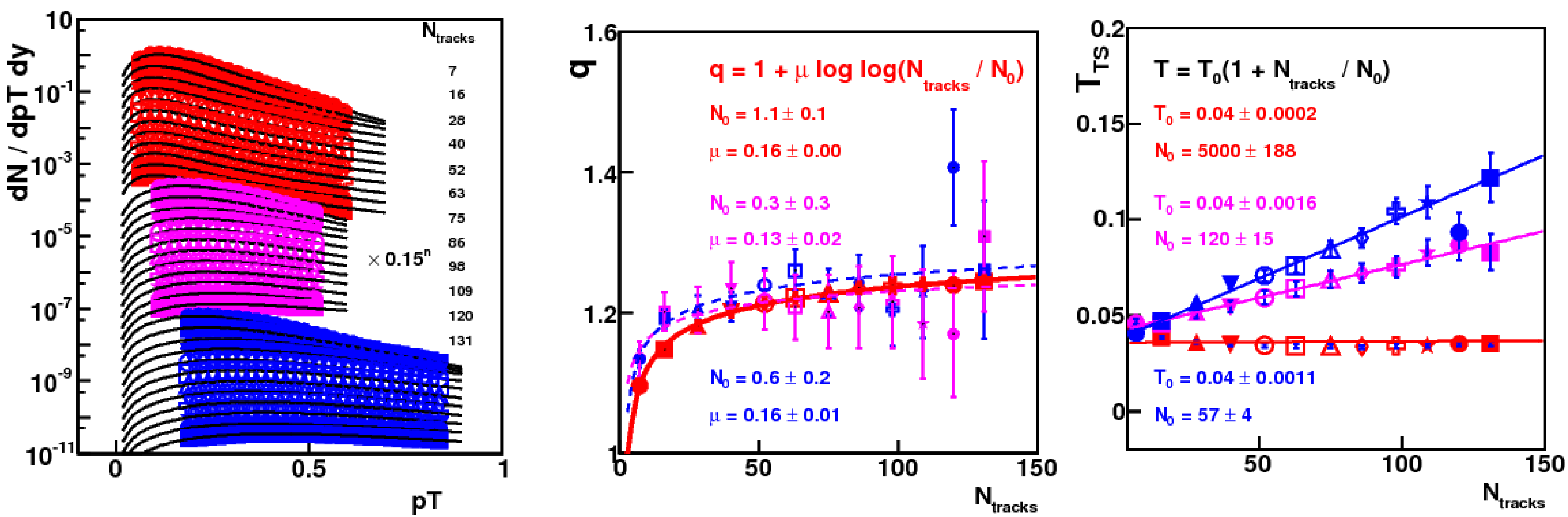
$$q_i = q_{0i} + q_{1i} \ln(\ln(Q^2))$$

$$T_i = T_{0i} + T_{1i} \ln(\ln(Q^2))$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

Multiplicity Dependence of Transverse π, K, p Spectra In pp --> hX @ 7 TeV

$$\frac{dN}{d\,pT\,dy} \propto \left(1+\frac{(q-1)}{T}(m_T-m)\right)^{-1/(q-1)}$$



Based on CMS Preliminary Results

Take Home Message:

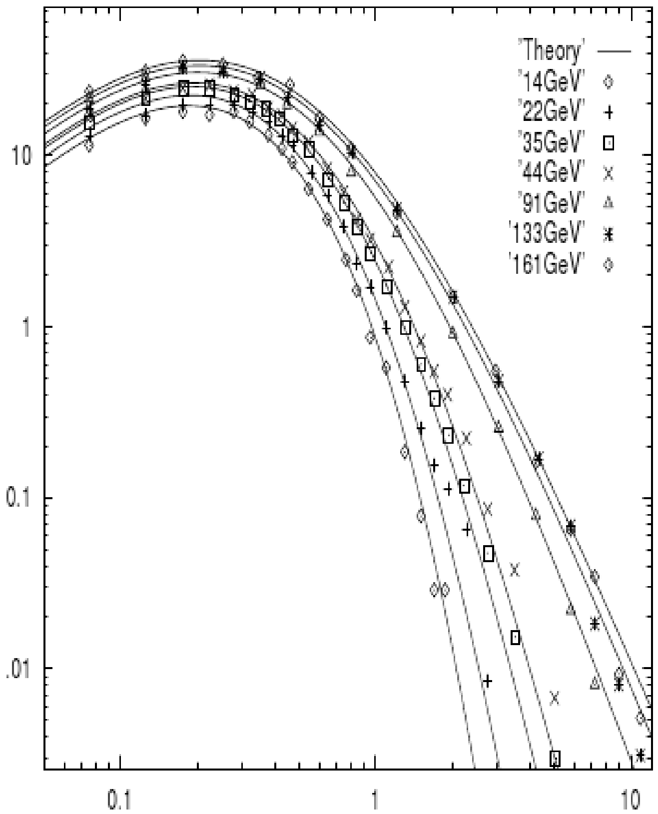
Multiplicity fluctuations hide event-by-event physics from our eyes.

We should measure the spectra separately in each multiplicity bin

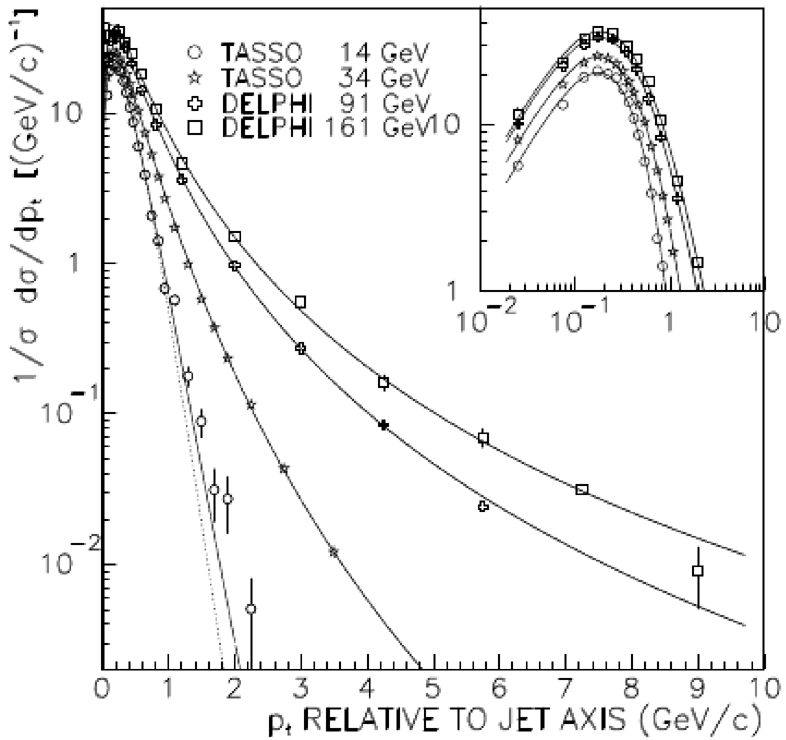
Back-up Slides……………………..

Hadronok jet-re merőleges impulzusának eloszlása elektron-pozitron ütközésekben

C. Beck, Physica A: 286, 164-180, 2000.



I. Bediaga, E. M. F. Curado, J. M. Miranda, Physica A: 286, 156-163, 2000.



$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} \propto p_T \int_0^{\infty} dp_L \underbrace{\left(1+(q-1)E/T\right)^{-q/(q-1)}}_{Tsallis \quad eloszlás}$$

$$E=\sqrt{p_L^2+p_T^2+m^2}$$

A kísérletiek azonban a multiplicitás fluktuációkra átlagolt spektrumokat mérik

A miltiplicitás eloszlás KNO-skálázást mutat (Koba-Nielsen-Olesen-féle)

$$p(N) = \frac{1}{\langle N(s) \rangle} \Psi \left(\frac{N - N_0}{\langle N(s) \rangle} \right)$$

- A. Rényi, Foundations of Probability, Holden-Day (1970).
- A. M. Polyakov, Zh. Eksp. Teor. Fiz. 59, 542 (1970).
- Z. Koba, H. B. Nielsen, P. Olesen, Nucl. Phys. B 40, 317 (1972).
- S. Hegyi, Phys. Lett. B: 467, 126-131, 1999.
- S. Hegyi, Proc. ISMD 2000, Tihany, Lake Balaton, Hungary, 2000
- Yu.L. Dokshitzer, Phys. Lett. B, 305, 295 (1993); LU-TP/93-3 (1993).

A kírérletekkel konzisztens konkrét függvényalak:

$$p(N) \propto (N - N_0)^{\alpha - 1} e^{-\beta (N - N_0)}$$

Amiből az átlag hadron eloszlás:

$$\frac{d\sigma}{d^D x} = \sum f_N(x) N p(N) \propto \frac{(1-x)^{D(N_0 - 1) - 1}}{\left(1 - a \ln(1-x)\right)^b}$$

Urmossy et. al., Phys. Lett. B, 701: 111-116 (2011), arXiv:1101.3023

Statisztikus fizikai eloszlásokat látunk, ha

- ***Az ütközésben keletkező anyag egyensúlyba kerül***
- ***vagy ha a heletkező $h_1, \ldots, h_N$ részecske keltési hatáskeresztmetszete***

$$d\sigma^{h_1,\ldots,h_N} = |M|^2 \, \delta^{(4)}\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\ldots,h_N}$$

olyan, hogy

$$d\sigma^{h_1,\ldots,h_N} \propto \delta\left(\sum_i \epsilon_{h_i} - E_{tot}\right) d\Omega_{h_1,\ldots,h_N}$$

Az entrópia mindkét esetben maximális, a keletkezett részecskék mikrokanonikus sokaságot alkotnak, így az 1-részecske eloszlás (m = 0)

$$f_N(x) \propto \frac{\Omega_{N-1}(E-\epsilon)}{\Omega_N(E)} \propto (1-x)^{D(N-1)-1}, \qquad x = \frac{\epsilon}{E} = \frac{p}{\sqrt{s}/2}$$

mivel az N-részecskés fázistérfogat

$$\Omega_N(E) = \int \prod d^D p_i \;\; \delta\left(E - \sum \epsilon_j\right) \propto E^{DN-1}$$

Hőtartállyal egyensúlyban lévő részrendszer eloszlása

Egyensúlyban egy ergodikus rendszer az azonos energiájú állapotaiban egyenlő valószínűséggel található meg. Az ilyen állapotok száma

$$\Omega(E) = \int d\mu\, \delta\left[H(\mu) - E\right]$$

Ha egy *r* részrendszer, *R* hőtartály és a teljes r+R rendszer állapotszámai és energiái között az

$$\Omega_{r+R}(E) = \tilde{h}\left[\Omega_r(\epsilon_r), \Omega_R(E_R)\right] \qquad E = h(\epsilon_r, E_R)$$

egyenletek írhatók fel, és ezek az egyenletek szétbonthatóak:

$$S\left[\Omega_{r+R}(E)\right] = S\left[\Omega_r(\epsilon_r)\right] + S\left[\Omega_R(E_R)\right]$$

$$L(E) = L(\epsilon_r) + L(E_R)$$

akkor az *r* részrendszer eloszlására, f(Є) = 1 / Ω_r(Є) -ra variációs feladat kapható:

$$\int d\mu_r f(\epsilon_r) S\left[1/f(\epsilon_r)\right] - \beta \int d\mu_r L(\epsilon_r) f(\epsilon_r) \;=\; max$$

***μ_r* az *r* részrendszer fázistere (ez az egyenlet a *Jaynes-elv*vel analog)**

A termodinamikai hőmérséklet

Ez a közelítés akkor jogos, ha $E_R \gg \epsilon_r$ **és így**

$$S\left[\Omega(E_R)\right] = S\left[L(E) - L(\epsilon_r)\right] \approx konst - \beta_R L(\epsilon_r)$$

Ez a hőmérséklet definíció konzisztens *r* és *R* termodinamikai egyensúlyának feltételével:

$$S_{r+R}(E) = S_r(\epsilon_r) + S_R(E_R) \;=\; max$$

$$L(E) = L(\epsilon_r) + L(E_R) \;=\; fix$$

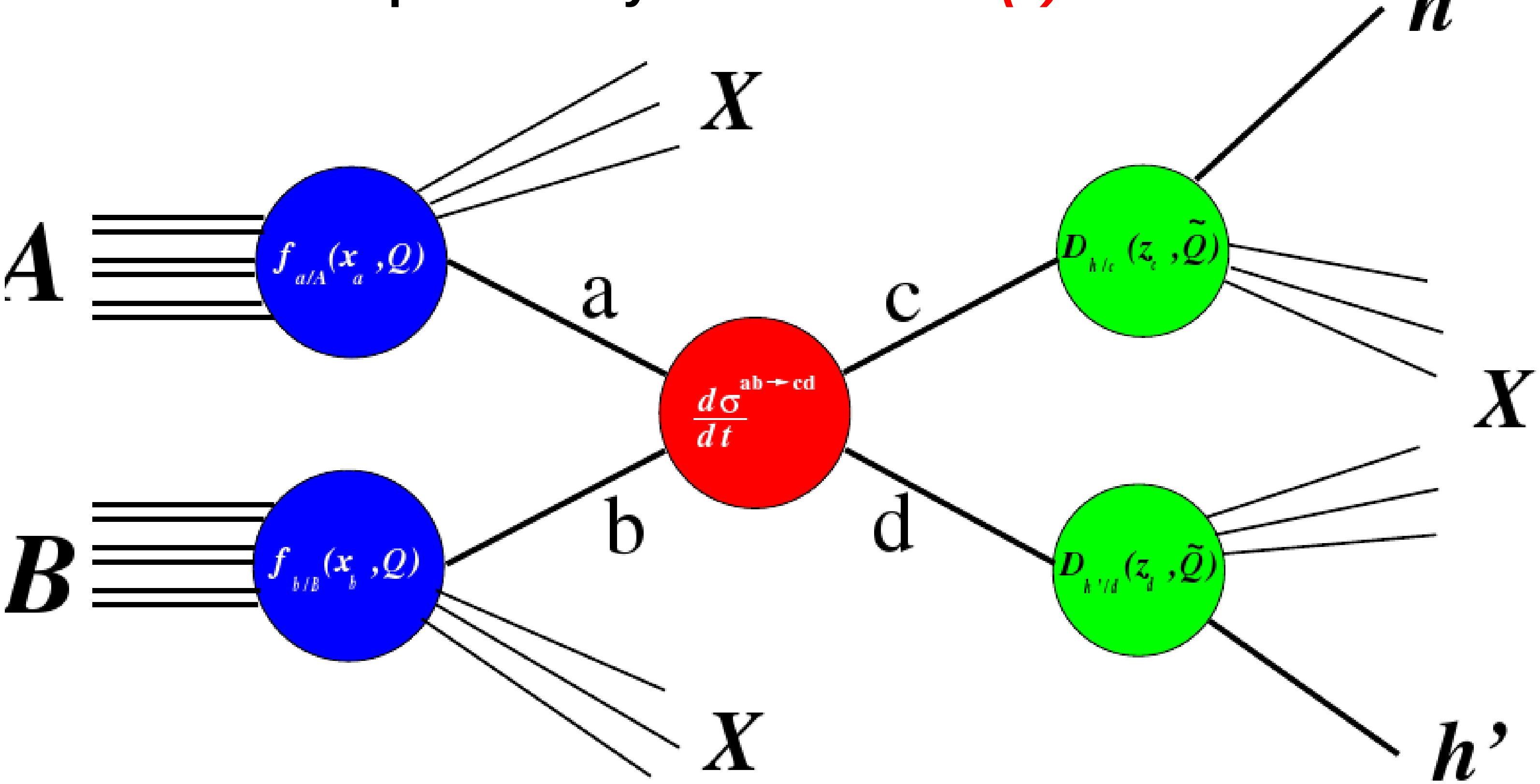
$$\frac{\partial S_R(E_R)}{\partial L(E_R)} \;=\; \beta_R \;=\; \frac{\partial S_r(\epsilon_r)}{\partial L(\epsilon_r)}$$

T.S. Biró, P. Ván, Physical Review E, 83, 061187, (2011)

Parton-model calculation in pp collisions

Idea: **partons *a, b* inside protons *A, B* scatter off of each other.**

- **A parton carries some momentum fraction *x* of the momentum of its proton with probability-distribution *f(x)*.**
  Throughout the scattering of *a* and *b*, partons *c* and *d* are produced.
- ***c* and *d* induce *jets* (showers of hadron, whose distribution is measured).**
  Hadrons inside the jets, carry momentum fraction *z* of the momentum of the leading parton *c* or *d* with probability distribution *D(z)*.



***Goal:* find a satisfactory model for** $D(z)$

Why to deal with fragmentation?

To calculate some *hadronic observables* in high-energy collisions (such as *transverse momentum spectra*, *angular distribution*, etc… in proton-proton --> hadrons reactions)
we need to *understand* the process of *hadronisation*.

Hadronisation is a *non-perturbative* phenomenon, and no models have yet been really successful in its description.

In *parton-model calculations*, people use *phenomenological functions* which, however, have no theoretical background.

My Goal is to propose a statictical physical model that describes these fragmentation functions.