Correlation femtoscopy of small systems

Volodymyr SHAPOVAL, Yuri SINYUKOV

Bogolyubov Institute for Theoretical Physics, NAS of Ukraine

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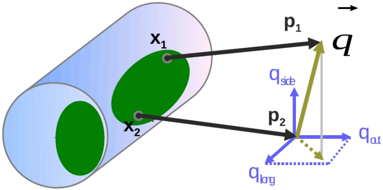
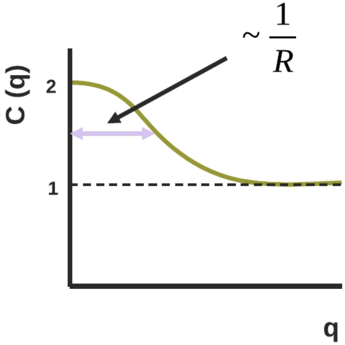
Intensity interferometry

- The correlation femtoscopy, or intensity interferometry method, is the direct tool to measure the spatial and temporal scales of extremely small and short-lived systems created in particle and nuclear collisions with accuracy of 10^{-15} m and 10^{-23} sec correspondingly.
- The femtoscopic space-time structure of the systems is typically represented in terms of the interferometry radii. They are result of a Gaussian fit of the correlation function defined as a ratio of the two-particle spectra to the product of the single-particle ones.





- The method is based on the Bose-Einstein (BE) or Fermi-Dirac (FD) symmetric properties of the quantum states and exploits quantum correlations between identical particles stemming from multiparticle wavefunction (anti)symmetrization (*G.Goldhaber, S.Goldhaber, W. Lee, A. Pais Phys. Rev.* ***120*** *(1960) 325*).
- It has a deep analogy with the intensity interferometry telescope that was proposed by Hanbury Brown and Twiss for measurements of angular sizes of remote stars (*R. Hanbury Brown, R.Q. Twiss, Nature,* ***177*** *(1956) 27*).
  Unlike the standard telescopic and microscopic techniques based on the registration of light or particles intensities coming from the object, this method deals with the correlations between intensities of the source radiation registered by two (many) spatially separated parts of detecting devices.
- In pioneer papers (*G.I. Kopylov, M.I. Podgoretsky, Sov. J. Nucl. Phys.* ***15*** *(1972) 219; G. Cocconi, Phys. Lett. B* ***49*** *(1974) 459*) the measured interferometry radii were interpreted as the geometrical sizes of the systems.

- Later on it was found that for typical systems formed in experiments with heavy ions, the above geometrical interpretation needs to be generalized.
  The treatment of the interferometry radii as the homogeneity lengths (*Yu.M. Sinyukov, Nucl.Phys. A 566 (1994) 589*) in the systems and crucial suggestion about femtoscopy scanning of the source radiation in different momentum bins brought the possibility to analyze different parts of the source and explain the behavior of the interferometry radii.
- The practical method of use the final state interactions (FSI) and effects of long-lived resonances to extract the BE correlations in relatively large systems created in heavy ion collisions has been proposed (*Yu.M.Sinyukov, R.Lednický, J.Pluta, B.Erazmus, S.V.Akkelin Phys. Lett B432 (1998) 248*).
- Another challenge, which is still actual, concerns the femtoscopy analysis of relatively small systems created in particle interactions such as pp and e^+e^- where the observed femtoscopic scales are approximately 1 fm or smaller.
- This talk deals with the femtoscopy of such small systems accounting for the uncertainty principle and coherence of the radiation from spatially very closely set emitters.

Standard approach to correlation femtoscopy

Correlation function can be defined as:

$$C(p_1, p_2) = \frac{W(p_1, p_2)}{W(p_1)W(p_2)}$$

For one particle with momentum $p = (E, \mathbf{p})$ the registration probability is

$$W(p) = |\psi(p, t)|^2,$$

where

$$\psi(p, t) = e^{-iEt} e^{-i\mathbf{x}\mathbf{p}}$$

is particle's wavefunction in momentum representation, $|\psi(p, t)|^2 = 1$.
The probability of registration of the pair with momenta p_1 i p_2

$$W(p_1, p_2) = |\psi(p_1, p_2; t)|^2,$$

where $\psi(p_1, p_2; t) = \frac{1}{\sqrt{2}} \left[e^{i\mathbf{p_1}\mathbf{x_1}} e^{i\mathbf{p_2}\mathbf{x_2}} + e^{i\mathbf{p_2}\mathbf{x_1}} e^{i\mathbf{p_1}\mathbf{x_2}} \right] e^{-i(E_1+E_2)t}$ is (anti)symmetrized pair wavefunction in momentum representation.
The probability of registration of a pair in case of two-point emission

$$W_{x_1, x_2}(p_1, p_2) = |\psi_{x_1, x_2}(p_1, p_2; t)|^2 = 1 + \cos\left[(p_1 - p_2) \cdot (x_1 - x_2)\right]$$
$$= 1 + \cos\left(q \cdot \Delta x\right) = C(p_1, p_2)$$

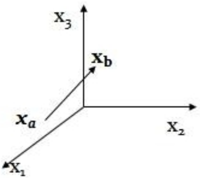
The previous formula is typically generalized for the case of emission from some spatio-temporal region D characterized by emission centers distribution function $\rho(x)$ just by averaging the expression for two-point correlator over this region

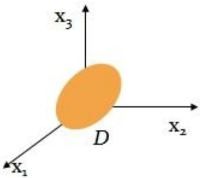
$$W_D(p_1,p_2) = \int d^4x_a d^4x_b \rho(x_a)\rho(x_b) W_{x_a,x_b}(p_1,p_2)$$

$$C_D(p_1,p_2) = 1 + \left| \int d^4x \rho(x) e^{iq\cdot x} \right|^2 \qquad (1)$$

If we suppose $\rho(x) \propto \exp\left[-\sum_{i=1}^{3} \frac{x_i^2}{2R_i^2}\right] \delta(t-t_0)$ then

$$C_D(p_1,p_2) = 1 + \exp\left[-\sum q_i^2 R_i^2\right]$$





However the standard approach stated above gives incorrect results in the case of system with small number of emitters.
Let us consider the case of the system consisting only of two emitting points, $\rho(x) = \frac{1}{2}(\delta^4(x - x_1) + \delta^4(x - x_2))$. Then if we apply the formula (1), we arrive at the following expression

$$W_{x_1,x_2}(p_1, p_2) = \sum_{i,j=1,2} \rho_i \rho_j (1 + \cos\left[(p_i - p_j)(x_i - x_j)\right]) = 1 + \cos^2\left[\frac{1}{2}(p_1 - p_2)(x_1 - x_2)\right] \quad (2)$$

This result is incorrect!

Let us consider the one- and two-boson emission probabilities from the source consisting only of two points x_1 and x_2, from where particles are emitted **independently**, in **distinguishable** quantum states. For the one boson case we have to add two probabilities corresponding to two different emission variants:

$$\rho_1: \quad A_1(p) = e^{i\mathbf{p}\mathbf{x_1}} e^{-iEt} \quad \text{and} \quad \rho_2: \quad A_2(p) = e^{i\mathbf{p}\mathbf{x_2}} e^{-iEt}; \quad \rho_1 + \rho_2 = 1.$$

$$W(p) = \rho_1 |A_1(p)|^2 + \rho_2 |A_2(p)|^2 = 1$$

For the two bosons case we have three different final states

$$\rho_{11}: \; A_{11}(p_1, p_2) = e^{i\mathbf{p_1}\mathbf{x_1}} e^{i\mathbf{p_2}\mathbf{x_1}} e^{-i(E_1+E_2)t}; \; \rho_{22}: \; A_{22}(p_1, p_2) = e^{i\mathbf{p_1}\mathbf{x_2}} e^{i\mathbf{p_2}\mathbf{x_2}} e^{-i(E_1+E_2)t};$$

$$\rho_{12}: \quad A_{12}(p_1, p_2) = \frac{1}{\sqrt{2}} \left(e^{i\mathbf{p_1}\mathbf{x_1}} e^{i\mathbf{p_2}\mathbf{x_2}} + e^{i\mathbf{p_1}\mathbf{x_2}} e^{i\mathbf{p_2}\mathbf{x_1}} \right) e^{-i(E_1+E_2)t} \; ; \; \rho_{11} + \rho_{22} + \rho_{12} = 1.$$

If we put $\rho_{ii} = \rho_i^2$ and $\rho_1 = \rho_2 = 1/2$, then $\rho_{11} = \rho_{22} = 1/4$, $\rho_{12} = 1/2$.
Thus we obtain

$$W_{x_1,x_2}(p_1, p_2) = \underbrace{\sum}_{i \leq j = 1,2} \rho_{ij} \left| A_{ij} \right|^2 = 1 + \frac{1}{2} \cos\left(q \cdot \Delta x\right) = C(p_1, p_2)$$

And if the states of particles emitted from the points x_1 and x_2 are **indistiguishable**, and not independent at all (full coherence) then to calculate the resulting probability one should add the amplitudes of particular ways of final state realization, not probabilities:

$$A_{x_1,x_2}(p) = A_1(p) + A_2(p)$$

$$A_{x_1,x_2}(p_1, p_2) = A_{x_1,x_2}(p_1)A_{x_1,x_2}(p_2)$$

$$C(p_1, p_2) = \frac{W_{x_1,x_2}(p_1, p_2)}{W_{x_1,x_2}(p_1)W_{x_1,x_2}(p_2)} = 1.$$

The both cases (of distinguishable and indistinguishable states) can be described within the partially correlated phases formalism:

$$A_{x_1,x_2}(p) = (e^{i\mathbf{p}\mathbf{x_1}} e^{i\phi(x_1)} + e^{i\mathbf{p}\mathbf{x_2}} e^{i\phi(x_2)})e^{-iEt}$$

$$A_{x_1,x_2}(p_1,p_2) = A_{x_1,x_2}(p_1)A_{x_1,x_2}(p_2)$$

After averaging over the ensemble of emission events with random phases $\phi(x_i)$ we obtain

$$C(p_1,p_2) = \frac{\left\langle |A(p_1,p_2)|^2 \right\rangle}{\left\langle |A(p_1)|^2 \right\rangle \left\langle |A(p_2)|^2 \right\rangle} = \begin{cases} 1 + \frac{1}{2}\cos(q \cdot \Delta x), & \text{if } \left\langle e^{i(\phi(x_1)-\phi(x_2))} \right\rangle = \delta^4(x_1 - x_2) \\ 1, & \text{if } \left\langle e^{i(\phi(x_1)-\phi(x_2))} \right\rangle = 1 \end{cases}$$

$$\left\langle e^{i(\phi(x_2)-\phi(x_1))} \right\rangle = \delta^4(x_1 - x_2) = I_{12}\delta(t_1 - t_2) \quad (3)$$

where I_{12} is the overlap integral

$$I_{ij} = \int d^3\mathbf{x} \left| \psi_{x_i}(t,\mathbf{x})\psi_{x_j}(t,\mathbf{x}) \right|$$

To be able to distinguish the quantum states of the particles emitted from the points x_1 and x_2 one needs the distance between these points to be much greater than the widths of the wave packets corresponding to the particles, so corresponding overlap integral should be small

$$I_{ij} << 1,$$

and

$$(x_i - x_j)^2 \geq 1/\Delta p^2, \quad (t_i - t_j)^2 \geq 1/\Delta p^2$$

where the last two inequalities express the uncertainty principle.
We see that due to the uncertainty principle, all emission points can be fully distinguished only in the case of flat momentum spectrum, $\Delta p = \infty$.

$$f(\mathbf{p}) = \tilde{f}(\mathbf{p})^2 = const, \quad I_{ij} = \delta(\mathbf{x}_i - \mathbf{x}_j)$$

For more realistic spectra the points set closely in the space-time cannot be distinguished, and delta-function in the phase correlator (3) should be smeared, e.g. for the gaussian momentum spectrum

$$f(\mathbf{p}) = \tilde{f}(\mathbf{p})^2 = \frac{1}{(2\pi p_0^2)^{3/2}} e^{-\frac{\mathbf{p}^2}{2p_0^2}}, \quad I_{ij} = e^{-\frac{p_0^2 (x_i - x_j)^2}{2}}$$

$$\delta(t_i - t_j) \longrightarrow e^{-\frac{\Delta p^2 (t_1 - t_2)^2}{2}}$$

$$\psi_x(p,t) = e^{i(\mathbf{p}\mathrm{x}-p^0 t)} e^{i\varphi(x)} \tilde{f}(\mathbf{p})$$

$$\rho(x) = \frac{1}{2\pi R_T \sqrt{R_L T}} e^{-\frac{x_T^2}{4R_T^2} - \frac{x_L^2}{4R_L^2} - \frac{t^2}{4T^2}}$$

$$f(\mathbf{p}) = \tilde{f}^2(\mathbf{p}) = \frac{1}{(2\pi k^2)^{3/2}} e^{-\frac{\mathbf{p}^2}{2k^2}}$$

The emission amplitude

$$A(p,t) = c \int d^4 x \psi_x(p,t) \rho(x)$$

The emission probability averaged over random phases $\varphi(x)$ is

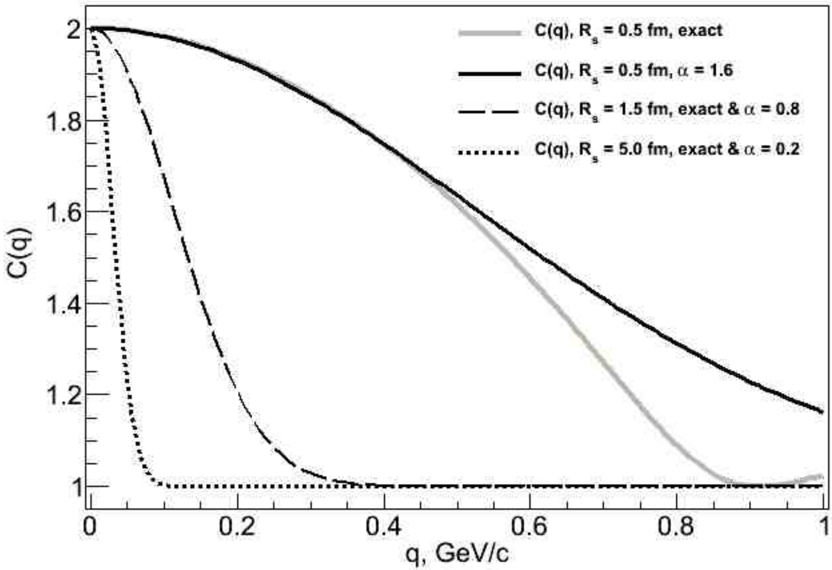
$$\overline{W(p)} = c^2 \int d^4 x d^4 x' e^{ip(x-x')} \rho(x) \rho(x') \langle e^{i(\varphi(x)-\varphi(x'))} \rangle f(\mathbf{p}).$$

Overlap integral

$$I_{x_i x_j} = \left| \int d^3 \mathbf{r} \psi_{x_i}(t,\mathbf{r}) \psi^*_{x_j}(t,\mathbf{r}) \right| = \frac{e^{-\frac{k^2(\mathrm{x}_i - \mathrm{x}_j)^2}{2(1+k^4(t_i-t_j)^2/m^2)}}}{\sqrt[4]{1+k^4(t_i-t_j)^2/m^2}}$$

$$(t_i - t_j)^2 \to a\langle (t_i-t_j)^2 \rangle = a \int d^4 x_i d^4 x_j \rho(x_i)\rho(x_j)(t_i-t_j)^2 = 4aT^2 \equiv \alpha T^2$$

$$G_{xx'} = \langle e^{i(\varphi(x)-\varphi(x'))} \rangle = \frac{e^{-\frac{k^2(\mathrm{x}-\mathrm{x}')^2}{2(1+\alpha k^4 T^2/m^2)}}}{\sqrt[4]{1+\alpha k^4 T^2/m^2}} e^{-k^2(t-t')^2/2}$$



C(q), R_s = 0.5 fm, exact
C(q), R_s = 0.5 fm, α = 1.6
C(q), R_s = 1.5 fm, exact & α = 0.8
C(q), R_s = 5.0 fm, exact & α = 0.2
C(q)
q, GeV/c
2
1.8
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1

One particle spectrum ($R_T = R_L = R$)

$$\overline{W(p)} = N e^{-\frac{p^2}{2k^2} - \frac{2p^2R^2}{1+4k_0^2R^2} - \frac{p^4T^2}{2m^2(1+4k^2T^2)}}$$

where $k_0^2 = k^2/(1+\alpha k^4 T^2/m^2)$.

The two-particle spectrum averaged over events with partially coherent phases is

$$\overline{W(p_1,p_2)} = c^4 \int d^4x_1 d^4x_2 d^4x_1' d^4x_2' e^{i(p_1x_1+p_2x_2-p_1x_1'-p_2x_2')} f(\mathbf{p}_1) f(\mathbf{p}_2) \cdot \\ \cdot \rho(x_1)\rho(x_2)\rho(x_1')\rho(x_2')\langle e^{i(\varphi(x_1)+\varphi(x_2)-\varphi(x_1')-\varphi(x_2'))}\rangle .$$

$$\langle e^{i(\varphi(x_1)+\varphi(x_2)-\varphi(x_1')-\varphi(x_2'))}\rangle = G_{x_1x_1'}G_{x_2x_2'} + G_{x_1x_2'}G_{x_2x_1'} - G_{x_1x_2'}G_{x_2x_1'}G_{x_1x_2}$$

$$C(\mathbf{p},\mathbf{q}) = \frac{\overline{W(p_1,p_2)}}{\overline{W(p_1)}\,\overline{W(p_2)}} = 1 + e^{-q_T^2R_T^2\frac{4k_0^2R_T^2}{1+4k_0^2R_T^2} - q_L^2R_L^2\frac{4k_0^2R_L^2}{1+4k_0^2R_L^2} - \frac{(\mathbf{q}\cdot\mathbf{p})^2T^2}{m^2}\frac{4k^2T^2}{1+4k^2T^2}} - C_d(\mathbf{p},\mathbf{q})$$

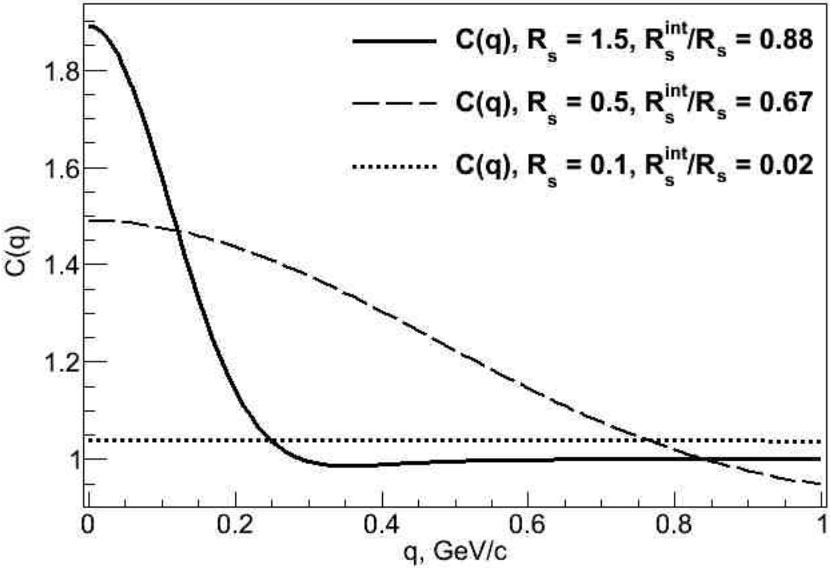
$$C_d(\mathbf{p},\mathbf{q}) = F(k_0^2R^2, k^2T^2) e^{-\frac{2\mathbf{q}^2k_0^2R^4(1+8k_0^2R^2)}{(1+4k_0^2R^2)(1+8k_0^2R^2+8k_0^4R^4)} - \frac{2k^2T^4(\mathbf{p}\cdot\mathbf{q})^2(1+8k^2T^2)}{m^2(1+4p^2T^2)(1+8k^2T^2+8k^4T^4)}}$$

$$F(k_0^2R^2, k^2T^2) = \sqrt{\frac{k_0}{k}} \frac{(1+4k^2T^2)^{1/2}\left(1+4k_0^2R^2\right)^{3/2}}{(1+8k^2T^2+8k^4T^4)^{1/2}\left(1+8k_0^2R^2+8k_0^4R^4\right)^{3/2}}$$

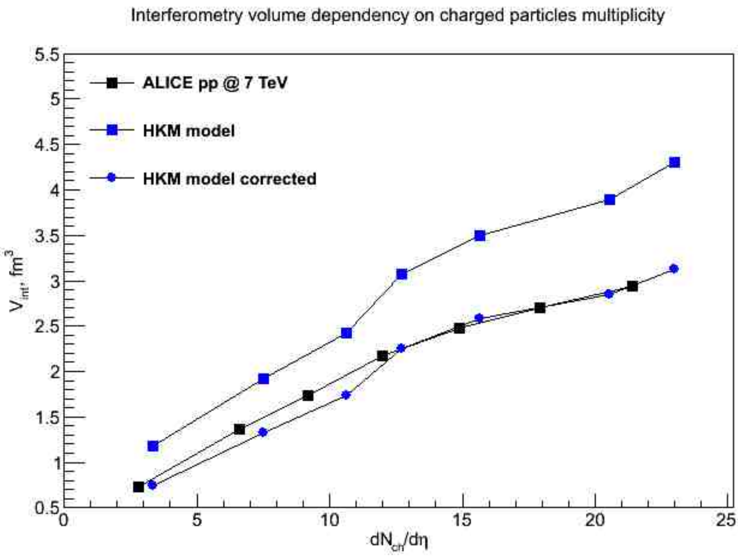
Such consideration accounting for the small system size and impossibility of complete distinguishing the emission points due to the uncertainty principle leads to the reduction of apparent interferometry radii.
Neglecting the correction for the double account, we get the following analytical formulas for radii reduction:

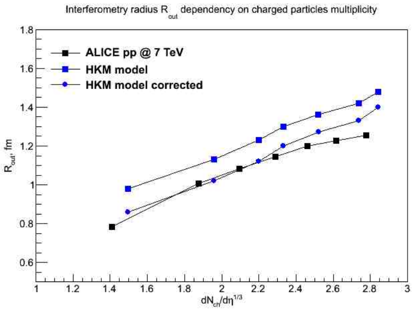
$$\begin{aligned}\frac{R_S^2}{R_{S,st}^2} &= \frac{4k_0^2R_T^2}{1+4k_0^2R_T^2}\\ \frac{R_O^2}{R_{O,st}^2} &= \left(R_T^2\frac{4k_0^2R_T^2}{1+4k_0^2R_T^2}+T^2v_{out}^2\frac{4k^2T^2}{1+4k^2T^2}\right)/\left(R_T^2+T^2v_{out}^2\right) \qquad (4)\\ \frac{R_L^2}{R_{L,st}^2} &= \frac{4k_0^2R_L^2}{1+4k_0^2R_L^2}\end{aligned}$$

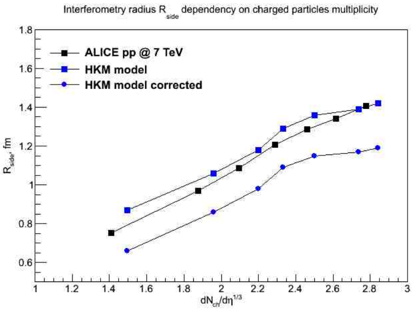
where $v_{out}=p_{out}/m\ll 1$.

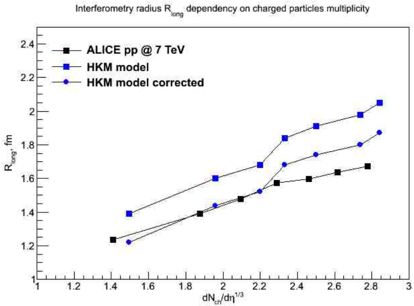

C(q), R_s = 1.5, R_s^int/R_s = 0.88
C(q), R_s = 0.5, R_s^int/R_s = 0.67
C(q), R_s = 0.1, R_s^int/R_s = 0.02
C(q)
1
1.2
1.4
1.6
1.8
0
0.2
0.4
0.6
0.8
1
q, GeV/c

Application to pp collisions femtoscopy within HKM model (S.V. Akkelin, Y. Hama, Iu.A. Karpenko, Yu.M. Sinyukov, Phys.Rev. C **78** 034906 (2008))




Interferometry radius R out dependency on charged particles multiplicity
ALICE pp @ 7 TeV
HKM model
HKM model corrected
R out, fm
dN ch/dη 1/3


Interferometry radius R side dependency on charged particles multiplicity
ALICE pp @ 7 TeV
HKM model
HKM model corrected
R side, fm
dN ch/dη 1/3


Interferometry radius R long dependency on charged particles multiplicity
ALICE pp @ 7 TeV
HKM model
HKM model corrected
R long, fm
dN ch/dη 1/3

Conclusions

- For small systems with size R $\sim$ 1 fm and the slopes of spectrum typical for pp and e^+e^- collisions the particles emission points cannot be completely distinguished due to uncertainty principle. Thus, they should be considered as a collection of sources with equal or partially correlated random phases and sum the amplitudes of emission from different points instead of summing the probabilities. As a result, in this case the specific reduction of the apparent interferometry radii is observed.
- For obtaining the correct expression for the two-particle correlation function the decomposition of the 4-point phase averages into the products of the irreducible 2-point ones must be accompanied by the double account elimination. Such approach leads to the suppression of the Bose-Einstein correlations for small sources.

Thank you for your attention!

Simple analytic model of non-femtoscopic two-pion correlations in the small systems

Three particles emission amplitude

$$A(\vec{p}_1, \vec{p}_2, \vec{p}_3) = A(\vec{p}_1)A(\vec{p}_2)A(\vec{p}_3) \to A(\vec{p}_1)A(\vec{p}_2)A(\vec{p}_3)Q(\vec{p}_2, \vec{p}_3)$$

The probabilities of single- and two-particle registration

$$W(\vec{p}_1) = \sum_i \frac{1}{3} \int dp_1^* dp_2^* dp_3^* \delta(\vec{p}_1 - \vec{p}_i^*) |A(\vec{p}_1^*, \vec{p}_2^*, \vec{p}_3^*) Q(\vec{p}_2^*, \vec{p}_3^*)|^2$$

$$W(\vec{p}_1, \vec{p}_2) = \sum_{i \neq j} \frac{1}{3} \int dp_1^* dp_2^* dp_3^* \delta(\vec{p}_1 - \vec{p}_i^*) \delta(\vec{p}_2 - \vec{p}_j^*) |A(\vec{p}_1^*, \vec{p}_2^*, \vec{p}_3^*) Q(\vec{p}_2^*, \vec{p}_3^*)|^2$$

The structure of correlation function in the case of clusters/jets

$$C(\vec{p}, \vec{q}) = \Lambda(\vec{p}) (\frac{2}{3} C^{mix}(\vec{p}, \vec{q}) + \frac{1}{3} C^{jet}(\vec{p}, \vec{q}))$$

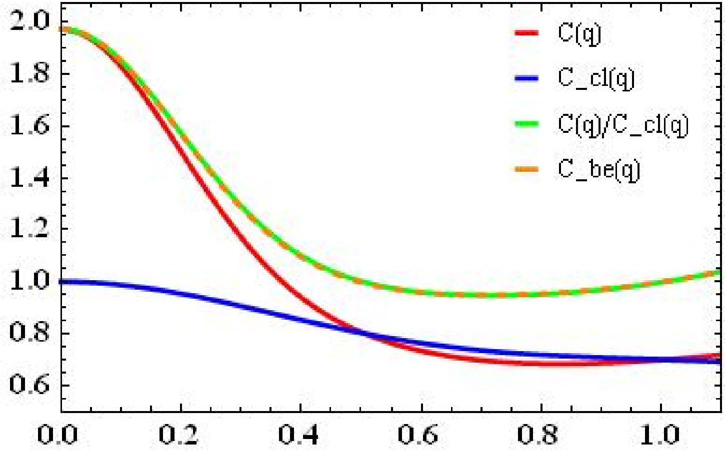
$$Q(\vec{p}_i, \vec{p}_j) = \exp\left(-\frac{(\vec{p}_i - \vec{p}_j)^2}{\alpha^2} \right)$$

"Soft"momentum conservation law

$$\delta(\vec{p}_1^* + \vec{p}_2^* + \vec{p}_3^*) \to \Delta(\vec{p}_1^*, \vec{p}_2^*) = Ce^{(\vec{p}_1^* + \vec{p}_2^* + \vec{p}_3^*)^2 / d^2}$$

$$C(\vec{p}, \vec{q}) = C_{BE} C_{NF} ?$$

Correlation function in the case of clusters/jets formation



$p_0 = 0.1 GeV/c, \alpha = 0.5 GeV/c, p = 0.35 GeV/c, d >> 1$

Non-femtoscopic correlations caused by emission function (initial conditions for hydro) fluctuations

The distribution function is modified:

$$f(\vec{p}) \propto e^{-\frac{\vec{p}^2}{2p_0^2}} \to e^{-\frac{(\vec{p}-\vec{u})^2}{2p_0^2}}$$

The weight:

$$\rho(u) = \frac{k^3}{\pi^{3/2}} e^{-u^2 k^2}$$

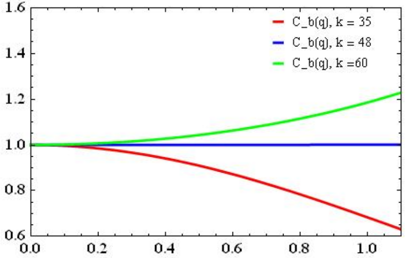
Momentum conservation law in the soft form:

$$\delta(p_1 + p_2 + \ldots) \to \Delta(p_1, p_2) = Ce^{(p_1+p_2+\ldots)^2/d^2}$$

Non-femtoscopic correlation function part

$$C_{NF}(p_1, p_2) = \frac{\sum_i \rho(u_i) W(p_1; u_i) W(p_2; u_i) \Delta(p_1, p_2)}{\sum_i \rho(u_i) W(p_1; u_i) \sum_j \rho(u_j) W(p_2; u_j)}$$

Non-femtoscopic correlations due to emission function fluctuations



$p = 0.35 GeV/c, d = 1 GeV/c, k = 35/GeV$