Form factors of charmonium radiative decays from lattice QCD

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D. Becirevic and F. Sanfilippo, arXiv:1206.1445

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The $\eta_b$ mass puzzle

Facts on $\eta_b$ meson

- $\eta_b$ is the Pseudo-scalar ($J^{PC} = 0^{-+}$) lighter $b\bar{b}$ meson,
- only recently seen at BaBar (2009),
- still omitted from summary table in PDG.

Hyperfine splitting

- Mass of $\Upsilon(1S)$ well established, $m_{\Upsilon(1S)} = 9460.30 \pm 0.26$ MeV.
- We therefore look at $\Delta_b \equiv m_{\Upsilon(1S)} - m_{\eta_b}$. 
Theoretical prediction of $\eta_b$ mass from perturbative QCD:


Experimental determination of $\eta_b$ meson mass

Mass only reconstructed from phase space of $\Gamma (b\bar{b} \rightarrow \eta_b\gamma)$ parametrization

$$\Gamma (b\bar{b} \rightarrow \eta_b\gamma) = \ldots$$

- From $\Upsilon (2, 3S) \rightarrow \eta_b\gamma$: $(69.3 \pm 2.8)$ MeV (CLEO)
- From $h_b (1P) \rightarrow \eta_b\gamma$: $(59.6 \pm 2.7)$ MeV (Belle)

- Form factors used to determine $m_{\eta_b}$ computed in quark models.
- Using Non Relativ. QCD would lead to smaller value.
The $\eta_b$ mass puzzle

Can be tracked to New Physics?
- Extension of SM with $> 1$ Higgs doublet: light parity odd Higgs boson $A^0$.
- This could mix with $\eta_b$ and modify QCD prediction on $m_{\eta_b}$.

Lattice post-diction
- Fermilab action: $(54 \pm 12)$ MeV, T. Burch et al. (PRD81, 2010).
- Non relativ. QCD: $(70 \pm 10)$ MeV, HPQCD coll. (PRD85, 2012).
- Non relativ. QCD: $(60 \pm 8)$ MeV, S. Meinel, (PRD82, 2010).

Much closer to the $\sim 70$ MeV experimental measure.

Can we say something more?
- Nowadays lattice tools allows to compute these quantities in full QCD:
  - measure $m_{\eta_b}$ in full relativistic QCD,
  - compute form factors used for experimental determination.
- As a starting point we will focus on charmonium system.
Charmonium radiative decays

### $J/\psi \to \eta_c \gamma$

#### $\Gamma (J/\psi \to \eta_c \gamma)$ puzzle:
- Potential Models: $(2.85 \pm 2.2) \text{ KeV}$, [E. Eichten et al., RMP80 (2008)],
- Non Relativistic QCD: $(1.5 \pm 1.0) \text{ KeV}$ [N. Brambilla et al., PRD73 (2006)].
- Experimentally: $1.58(37) \text{ KeV}$ [PDG].

### $h_c \to \eta_c \gamma$ radiative decay

#### Open questions on initial state:
- $h_c$ only recently seen at CLEO (2005).
- $\text{Br} (h_c \to \eta_c \gamma) = 53 (7) \%$ at BESIII 2010
- Lifetime of $h_c$ not measured yet.
- Possibility to make a prediction!

No modern lattice complete computation (quenched, no continuum limit).
Lattice in a nutshell

Discretize of the theory
- Change space-time with $N_x \times N_y \times N_z \times N_t$ points with spacing $a$.
- Write a discretized action having QCD as limit when $a \to 0$.

Observables computed as normal multi-dimensional integrals
\[
\langle O \rangle = Z \int D [A, \psi] O e^{-S(\psi, U)}
\]
- sample the fields configuration space $[A, \psi]$ with weight $Z^{-1} \exp (-S)$,
- measure observable of interests: $\langle O \rangle = N^{-1} \sum_{i=1}^{N} O_{[A, \psi],i}$.

Correlation functions
- Compute propagators by numerically solving Dirac equation: $D_{x,y} S_{y,0} = \delta_{x,0}$,
- combine them building correlation functions:

At the end take the continuum limit ($a \to 0$).
Our lattice setup

- Simulation with 2 light quarks
- Twisted mass QCD:
  - action designed to have $\mathcal{O}(a^2)$ effects (at maximal twist)
  - easier to take continuum limit w.r.t naive Wilson regularization
- 4 different lattice spacing
- discarding disconnected diagrams contributions.
Hyperfine splitting determination

Two points functions

\[ C^{\eta_c}(t) = \langle \text{Tr} \left[ S_c(0,0;\vec{x},t) \gamma_5 S_c(\vec{x},t;\vec{0},0) \gamma_5 \right] \rangle = \sim_{t \to \infty} \langle \eta_b \mid \gamma_5 \mid 0 \rangle^2 \exp(-M_{\eta_c} t) \]

\[ \Delta_c^{\text{exp}} = 116.6(1.2) \text{ MeV} \]
\[ \Delta_c^{\text{lat}} = 112(3) \text{ MeV} \]
Form factors determination

Three points functions

\[ C_{Tc}^{\eta_c}(t) = \langle \text{Tr} \left[ S_c(y;0) \gamma_i S_c(0,x) \Gamma S_c^\dagger(x,y) \gamma_5 \right] \rangle = \]

\[ \sim 0 \ll t \ll T \exp\left[ (E_{\eta_c} - M_{\Gamma}) t \right] \langle \Gamma | J_i^{em} | \eta_c \rangle \]

\[ \text{J/\psi} \rightarrow \eta_c \text{ transition} \]

\[ \text{h}_c \rightarrow \eta_c \text{ transition} \]
**$J/\psi \rightarrow \eta_c \gamma$ decay**

Continuum extrapolation of $\langle J_\psi| J_{\text{em}}^\text{em} | \eta_c \rangle$

Putting together everything: \( \Gamma(J/\psi \rightarrow \eta_c \gamma) = 2.58 (13) \text{ keV} \)

Compatible with QCD sum rules, quark models, dispersive models, but with smaller error and controllable uncertainties.

Historically experiments found \( \Gamma(J/\psi \rightarrow \eta_c \gamma) = 1.58 (7) \text{ keV} \).

Quite recently (2010), KEDR found \( \Gamma(J/\psi \rightarrow \eta_c \gamma) = 2.2 (6) \text{ keV} \).

Theoretical computations agree, experimental situation is quite unclear.

\[
\Gamma(J/\psi \rightarrow \eta_c \gamma) = \frac{8}{27} \alpha_{\text{em}} \times \\
	imes (m_{J/\psi} + m_{\eta_c}) \left( \frac{\Delta}{m_{J/\psi}} \right)^3 \times \\
	imes \langle J/\psi | J_{i\text{em}}^\text{em} | \eta_c \rangle^2
\]
$h_c \to \eta_c \gamma$ decay

Continuum extrapolation of $\langle h_c | j^e m | \eta_c \rangle$

- Putting together everything: $\Gamma(h_c \to \eta_c \gamma) = 0.71(6)$ keV
- Only $\text{Br}(h_c \to \eta_c \gamma) = (53 \pm 7)\%$ measured (BESIII).
- We have the occasion to predict $h_c$ lifetime:
  $\Gamma_{h_c} = \frac{\Gamma(h_c \to \eta_c \gamma)}{\text{Br}(h_c \to \eta_c \gamma)} = 1.36(23)$ MeV.
- Would be very interesting to compare with measure when available.
Conclusions

What have we done

First full determination of $J/\Psi$ and $h_c \rightarrow \eta_c \gamma$ form factors and of $\Delta_c$
- unquenching of light quarks
- continuum extrapolation under control

Results

- **Full agreement** of hyperfine splitting $\Delta_c$ with experimental results
- Hope to have **clarified** theoretical prediction for $\Gamma(J/\Psi \rightarrow \eta_c \gamma)$
- Given a **prediction for** $h_c$ **lifetime**

Future perspective

- Recheck with dynamical strange and charm
- Check irrelevance of disconnected diagrams

Promising for the bottomonium system.