Form factors of charmonium radiative dacays from lattice QCD

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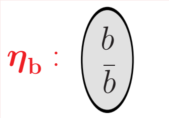
Based on "*Lattice QCD study of the radiative decays $J/\psi \to \eta_c \gamma$ and $h_c \to \eta_c \gamma$*"
D.Becirevic and F.Sanfilippo, arXiv:1206.1445

June 27, 2012

The η_b mass puzzle

Facts on η_b meson



- η_b is the Pseudo-scalar ($J^{PC} = 0^{-+}$) lighter $b\bar{b}$ meson,
- only recently seen at BaBar (2009),
- still omitted from summary table in PDG.

Hyperfine splitting

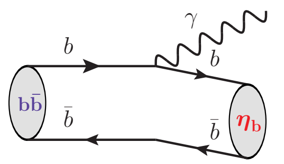
- Mass of $\Upsilon(1S)$ well established, $\boldsymbol{m_{\Upsilon(1S)} = 9460.30 \pm 0.26\,\mathrm{MeV}}$.
- We therefore look at $\Delta_b \equiv m_{\Upsilon(1S)} - m_{\eta_b}$.

Theoretical prediction of η_b mass from perturbative QCD:

- $\boldsymbol{\Delta_b = 44 \pm 11}\,\mathbf{MeV}$, S. Recksiegel et al. (Phys. Lett. B 578, 2004)
- $\boldsymbol{\Delta_b = 39 \pm 14}\,\mathbf{MeV}$, B. A. Kniehl et al. (Phys. Rev. Lett. 92, 2004)

Experimental determination of η_b meson mass

Mass only reconstructed from phase space of $\Gamma\left(b\bar{b} \to \eta_b\gamma\right)$ parametrization



$$\Gamma\left(b\bar{b} \to \eta_b\gamma\right) = \ldots$$

- From $\Upsilon\,(2,3S) \to \eta_b\gamma$: **(69.3 ± 2.8)** $\mathbf{MeV}$ (CLEO)
- From $h_b\,(1P) \to \eta_b\gamma$: **(59.6 ± 2.7)** $\mathbf{MeV}$ (Belle)

- Form factors used to determine m_{η_b} computed in quark models.
- Using Non Relativ. QCD would lead to smaller value.

The η_b mass puzzle

Can be tracked to New Physics?

- Extension of SM with > 1 Higgs doublet: light parity odd Higgs boson A^0.
- This could mix with η_b and modify QCD prediction on m_{η_b}.

Lattice post-diction

- Fermilab action: **(54 ± 12)** $\mathbf{MeV}$, T. Burch et al. (PRD81, 2010).
- Non relativ. QCD: **(70 ± 10)** $\mathbf{MeV}$, HPQCD coll. (PRD85, 2012).
- Non relativ. QCD: **(60 ± 8)** $\mathbf{MeV}$, S. Meinel, (PRD82, 2010).

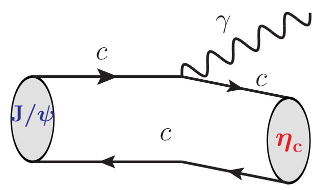
Much closer to the $\sim 70\,\mathrm{MeV}$ experimental measure.

Can we say something more?

- Nowadays lattice tools allows to compute these quantities in full QCD:
  - measure m_{η_b} in full relativistic QCD,
  - compute form factors used for experimental determination.
- As a starting point we will focus on charmonium system.

Charmonium radiative decays

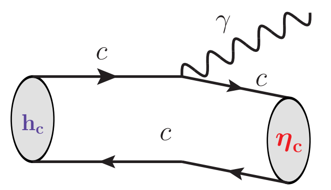
$J/\psi \to \eta_c \gamma$



$\Gamma\,(J/\psi \to \eta_c\gamma)$ puzzle:

- Potential Models: $\mathbf{(2.85 \pm ??)}$ **KeV**, [E. Eichten et al., RMP80 (2008)],
- Non Relativistic QCD: $\mathbf{(1.5 \pm 1.0)}$ **KeV** [N.Brambilla et al, PRD73 (2006)].
- Experimentally: **1.58(37) KeV** [PDG].

$h_c \to \eta_c\gamma$ radiative decay



Open questions on initial state:

- h_c only recently seen at CLEO (2005).
- $\mathrm{Br}\,(h_c \to \eta_c\gamma) = 53\,(7)\,\%$ at BESIII 2010
- Lifetime of h_c not measured yet.
- Possibility to make a prediction!

No modern lattice complete computation (quenched, no continuum limit).

Lattice in a nutshell

Discretize of the theory

- Change space-time with $N_x \times N_y \times N_z \times N_t$ points with spacing a.
- Write a discretized action having QCD as limit when $a \to 0$.

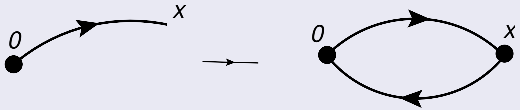
Observables computed as normal multi-dimensional integrals

$$\langle O \rangle = Z^{-1} \int D\left[A, \psi\right] O e^{-S(\psi, U)}$$

- sample the fields configuration space $[A, \psi]$ with weight $Z^{-1} \exp\left(-S\right)$,
- measure observable of interests: $\langle O \rangle = N^{-1} \sum_{i=1}^{N} O_{[A,\psi]_i}$.

Correlation functions

- Compute propagators by numerically solving Dirac equation: $D_{x,y} S_{y,0} = \delta_{x,0}$,
- combine them building correlation functions:



At the end take the **continuum limit** ($a \to 0$).

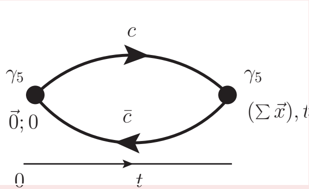
Our lattice setup

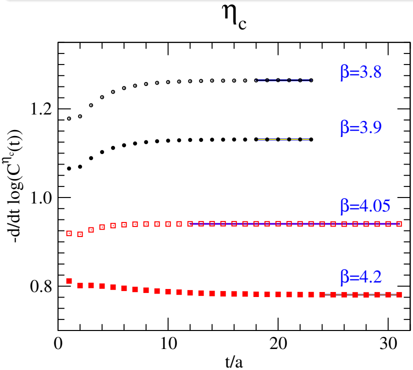
- Simulation with 2 light quarks
- Twisted mass QCD:
  - action designed to have $\mathcal{O}\left(a^2\right)$ effects (at maximal twist)
  - easier to take continuum limit w.r.t naive Wilson regularization
- **4 different lattice spacing**
- discarding disconnected diagrams contributions.

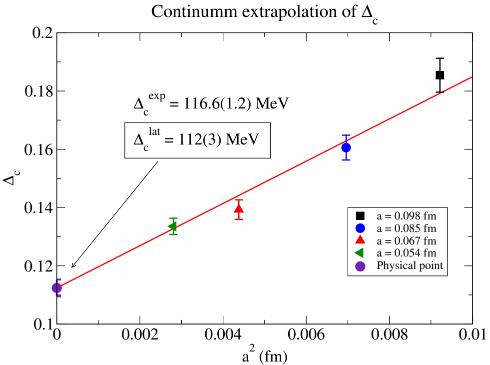
Hyperfine splitting determination

Two points functions

$$C^{\eta_c}(t) = \langle \mathrm{Tr}\left[S_c(0,0;\vec{x},t)\gamma_5 S_c(\vec{x},t;\vec{0},0)\gamma_5\right]\rangle =$$
$$\underset{t\to\infty}{\simeq} \langle \eta_b \,|\gamma_5|\, 0\rangle^2 \exp\left(-M_{\eta_c} t\right)$$





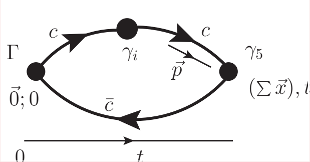


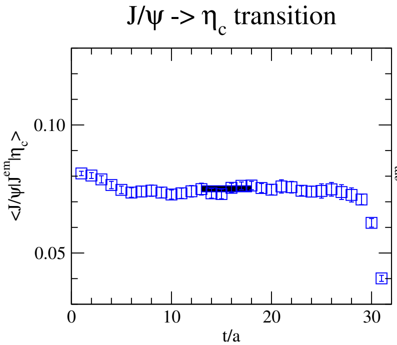
Form factors determination

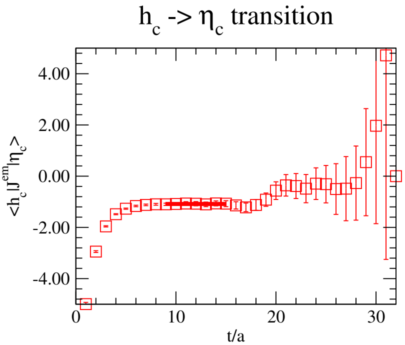
Three points functions

$$C^{\eta_c}_{i\Gamma}(t) = \langle \mathrm{Tr}\left[S_c(y;0)\gamma_i S_c(0,x)\Gamma S_c^{\vec{p}}(x,y)\gamma_5\right]\rangle =$$

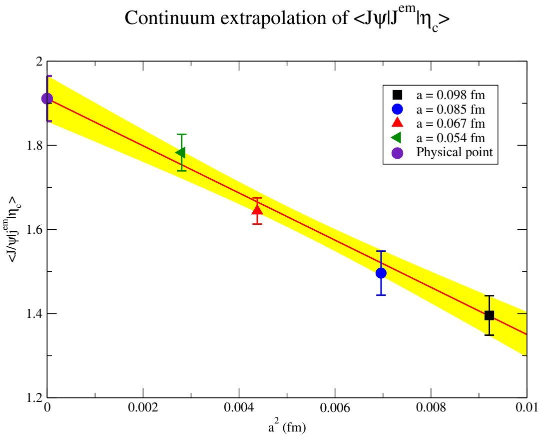
$$\underset{0\ll t\ll T}{\simeq} C \exp[(E_{\eta_c} - M_\Gamma)t]\langle\Gamma|J_i^{em}|\eta_c\rangle$$







$J/\Psi \rightarrow \eta_c\gamma$ decay



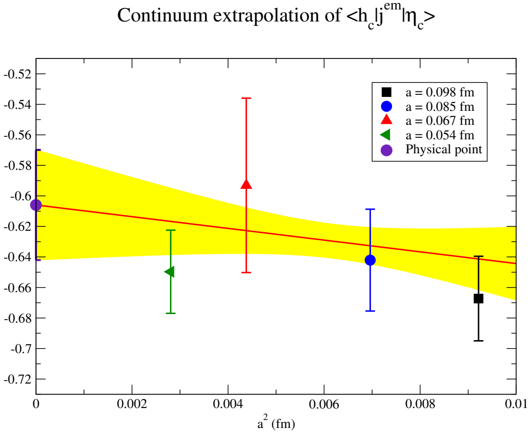
$$\Gamma(J/\psi \rightarrow \eta_c\gamma) = \frac{8}{27}\ \alpha_{\rm em}\ \times$$

$$\times (m_{J/\psi} + m_{\eta_c}) \left(\frac{\Delta}{m_{J/\psi}}\right)^3 \times$$

$$\times \left\langle J/\Psi \middle| J_i^{em} \middle| \eta_c \right\rangle^2$$

- Putting together everything: $\Gamma(J/\psi \rightarrow \eta_c\gamma) = 2.58\,(13)\ \mathrm{keV}$
- Compatible with QCD sum rules, quark models, dispersive models, but with smaller error and controllable uncertainties.
- Historically experiments found $\Gamma(J/\psi \rightarrow \eta_c\gamma) = 1.58\,(7)\ \mathrm{keV}$.
- Quite recently (2010), KEDR found $\Gamma(J/\psi \rightarrow \eta_c\gamma) = 2.2\,(6)\ \mathrm{keV}$.
- Theoretical computations agree, experimental situation is quite unclear.

$h_c \to \eta_c \gamma$ decay



$$\Gamma(J/\psi \to \eta_c \gamma) = \frac{8}{27} \ \alpha_{\rm em} \ \times$$

$$\times \frac{m_{h_c}^2 - m_{\eta_c}^2}{m_{h_c}} \times$$

$$\times \left\langle h_c \middle| J_i^{em} \middle| \eta_c \right\rangle^2$$

- Putting together everything: $\Gamma(h_c \to \eta_c \gamma) = 0.71(6)\,\mathrm{keV}$
- Only $\mathrm{Br}(h_c \to \eta_c \gamma) = (53 \pm 7)\,\%$ measured (BESIII).
- We have the occasion to predict h_c lifetime:
  $\Gamma_{h_c} = \frac{\Gamma(h_c \to \eta_c \gamma)}{\mathrm{Br}(h_c \to \eta_c \gamma)} = 1.36(23)\,\mathrm{MeV}$.
- Would be very interesting to compare with measure when available.

Conclusions

What have we done

First full determination of J/Ψ and $h_c \to \eta_c \gamma$ form factors and of Δ_c

- unquenching of light quarks
- continuum extrapolation under control

Results

- **Full agreement** of hyperfine splitting Δ_c with experimental results
- Hope to have **clarified** theoretical prediction for $\Gamma\left(J/\psi \to \eta_c \gamma\right)$
- Given a **prediction for h_c lifetime**

Future perspective

- Recheck with dynamical strange and charm
- Check irrelevance of disconnected diagrams

Promising for the bottomonium system.