The Next Step for the LHC
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Classical and Quantum Theories

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The Conformal Black Hole

minitalk:

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Quantum black hole physics today is in the same state of confusion as elementary particle physics before the advent of the Standard Model.
horizon
singularity
imploding matter

geodesic
lightcone
local

I

II

S

imploding matter

time
space
Hawking’s result: what is perceived as a vacuum to an observer who goes through the horizon, is a state of entangled particles for an outside observer.
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Hawking’s result: what is perceived as a vacuum to an observer who goes through the horizon, is a state of entangled particles for an outside observer. Since the outside observer does not see the particles that disappeared into the black hole, the states he does observe form a *quantum mixed state*, to be described by a *density matrix*.

A text book procedure should be found to replace that by a *quantum mechanical pure state*. But whatever you do, you then modify the state described by the observer who went in.

*Now, the observer who went in sees particles: a firewall.*
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In a given coordinate frame, the metric tensor $g_{\mu\nu}(\vec{x}, t)$ has 10 independent components. One of these is the overall factor ("conformal factor"): $g_{\mu\nu} = \omega^2 \hat{g}_{\mu\nu}$. The light cones only depend on $\hat{g}_{\mu\nu}$, not on $\omega$. *Our two observers disagree about $\omega$!!*
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Indeed they do! The outside observer sees that the hole shrinks to zero; the inside observer sees a whole whose mass did not change as he passed the $t = \infty$ line (the horizon). One can write the Schw. metric as

$$ds^2 = M^2(t)\left(-dt^2(1 - 2/r) + \frac{dr^2}{1 - 2/r} + r^2d\Omega^2\right)$$
We promote ω to a “local gauge field”, without modifying the physics! The field ω always was a dynamical variable in Einstein Hilbert gravity.

Now, splitting off the field ω, it appears to behave just as a scalar “Higgs” particle, except that it has negative energy (It sits at the wrong end in Einstein’s gravity equation: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$). Here, $G_{\mu\nu}$ is the ω contribution to the energy momentum tensor.
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New observation: demand laws of nature to stay regular as $\omega \to 0$. This helps. This is great. But:

... it gives too strong constraints on the physics (all anomalies have to cancel, which gives too many equations for Nature’s constants, and they appear to be in conflict.
The above considerations lead to a theory that has more equations than unknowns (all anomalies must cancel!) All parameters are fixed, including all masses, all couplings, Newton’s constant, and the cosmological constant. Unfortunately, they all seem to become of order 1 at the Planck scale; no physically reasonable solutions were found.
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End of minitalk on black holes
Dirac’s quantum mechanics

Axioms (with some subtle modifications)

• **States:** We have a vector space of bras, $\langle \psi |$, and a vector space of kets, $| \psi \rangle$ (usually infinite-dimensional). They represent physical states. $\psi$ stands short for a description of a state.

• We have the usual operations on these states: an anti-linear mapping $| \psi \rangle \leftrightarrow \langle \psi |$, an inner product $\langle \psi_1 | \psi_2 \rangle$, and $\langle \psi | \psi \rangle$ is a positive norm.

• **Dynamics:**
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- **Dynamics:** There is a linear Schrödinger equation:

  $$\frac{d}{dt} | \psi \rangle = -iH | \psi \rangle$$

  Here, $H$ can be any hermitean operator.
Axioms that will *not* be imposed:

- The probability that any one state $|\psi_1\rangle$ is *equal* to another state $|\psi_2\rangle$ is given by $|\langle \psi_1 | \psi_2 \rangle|^2$.
- The expectation value for an operator $X$, when measured in any state $|\psi\rangle$, is given by $\langle X \rangle = \langle \psi | X | \psi \rangle$

Instead, we will use the *weaker* conditions:
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Instead, we will use the weaker conditions:

- There is an initial condition: The universe begins in one, given state $|\psi(0)\rangle$.

- The probability that that state coincides with a “template state” $|\psi\rangle$ is given by $|\langle \psi | \psi(0) \rangle|^2$, and the expectation value of an operator $X$ in the state $|\psi(0)\rangle$ is $\langle \psi(0) | X | \psi(0) \rangle$.
  - In an other state $|\psi\rangle$, the expectation value $\langle X \rangle$ is $\langle \psi | \psi(0) \rangle \langle \psi(0) | X | \psi(0) \rangle \langle \psi(0) | \psi \rangle$. 
As states $|\psi\rangle$, we may use template states, such as states describing single particles (photons, nucleons, etc.). In terms of these template states, $|\psi(0)\rangle$ may be entangled:

$$|\psi(0)\rangle = \alpha|\psi_1\rangle|\psi_2\rangle + \beta|\psi_3\rangle|\psi_4\rangle + \cdots$$

Our weakened axioms should not imply observable changes or limitations in the general concept of Quantum Mechanics.

But they open the door to “underlying theories”.
Classical theories that can be mapped onto Dirac quantum systems:

<table>
<thead>
<tr>
<th>Classical theory</th>
<th>Quantum theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Discrete periodic system</td>
<td>Finite number of equally spaced energy levels</td>
</tr>
<tr>
<td>• Continuous periodic system</td>
<td>Harmonic oscillator</td>
</tr>
<tr>
<td>• flat classical sheet</td>
<td>First quantized massless non-interacting fermion</td>
</tr>
<tr>
<td>• many classical sheets</td>
<td>Second-quantized massless fermions</td>
</tr>
<tr>
<td>• Classical string theory on a lattice (26 or 10 dimensions)</td>
<td>Quantum (super-) string theory on the continuum (26 or 10 dimensions)</td>
</tr>
</tbody>
</table>
The scale anomaly.

Take a conformally flat background spacetime. Consider a complete EH + matter Lagrangian:

\[ \mathcal{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} D_\mu \omega^2 - \frac{1}{6} \tilde{\kappa}^2 \Lambda \omega^4 - \frac{1}{2} D_\mu \varphi^2 - \frac{1}{8} \lambda \varphi^4 - \frac{1}{2} (\tilde{\kappa}^2 m_s^2) \omega^2 \varphi^2 - \frac{1}{6} (\tilde{\kappa} g_3) \omega \varphi^3 - \bar{\psi} (\gamma D + \tilde{\kappa} m_d \omega + y \varphi) \psi \]

All parameters here are dimensionless, and, taking \( \omega \) and \( \varphi \) to scale the same way, this lagrangian is entirely conformally invariant ….

\[ \tilde{\kappa}^2 = \frac{1}{6} \kappa^2 = \frac{4}{3} \pi G_N \]

… classically! Quantum mechanically only if the beta function(s) vanish!
The equations
\[
\frac{\mu d}{d\mu} (\tilde{\Lambda}, g, \lambda, y, y^5, \tilde{\kappa} g_3, \tilde{\kappa} m_s, \tilde{\kappa} m_d, \ldots) = 0
\]
\[
\bar{\beta}(\tilde{\Lambda}, g, \lambda, y, y^5, \tilde{\kappa} g_3, \tilde{\kappa} m_s, \tilde{\kappa} m_d, \ldots) = 0
\]

Have only isolated solutions! All coupling parameters, including \( \Lambda \), are completely fixed by these equations, which can be worked out.

The only choices we have are discrete parameters: the group structures and symmetry patterns.

a LANDSCAPE of “Standard Models”
The only difference with conventional quantum gravity:

Demanding that the $\omega$ field behaves regularly at the origin, just as the other scalars $\phi$.

This is a statement about the small distance limit.

but such a postulate may well be permissible after having made the theory conformally invariant.