# Standard Model Three-loop Beta-functions

## Gauge couplings.Yukawa couplings. Higgs self-interaction.


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in collaboration with: A.Bednyakov and V.Velizhanin

BLTP,JINR


Erice,2013

# Running couplings in the Standard Model

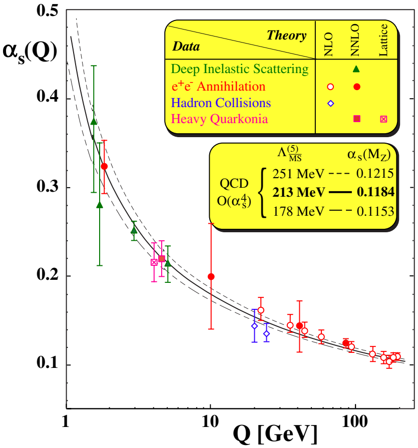


Figure : Famous running coupling

- Parameters from experiment at scale $\mu_0$
- Matching to $\overline{\mathrm{MS}}$
- Evolution from $\mu_0$ to $\mu$ using $\beta_{\alpha_s} = \frac{d\alpha_s(\mu)}{d\log(\mu)}$
- Obtain same at $\mu$
- Compare, tune parameters,repeat

**Scales available at experiment**

# Running couplings in the Standard Model

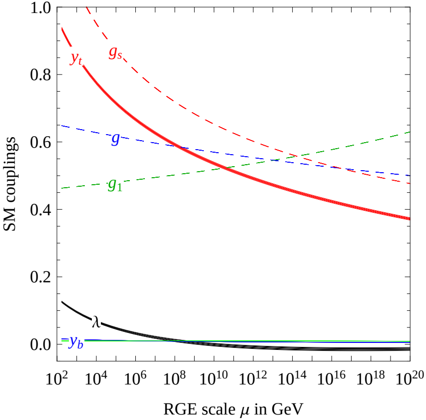


Figure : SM running couplings

- Parameters from experiment at scale $\mu_0$ still available
- Matching to $\overline{\mathrm{MS}}$, **precise** $m_t$ **and** $m_H$ **needed for** $y_t$ **and** $\lambda$
- Evolution from $\mu_0$ to $\mu$ using $\beta_i = \frac{dg_i(\mu)}{d\log(\mu)}$ $g_i = g_1, g_2, g_s, y_t, y_b, y_\tau, \lambda$
- Evolution up to scale $\mu$, but all equations are coupled $\beta = \beta(g_1, g_2, g_s, y_t, y_b, y_\tau, \lambda)$
- For extrapolation we need to extend all $\beta$ in full model from 2-loops to 3-loops

# Motivation

1. **Is the Standard Model still valid up to a Planck scale?**
   - At the scale $\Lambda \sim 10^{18} GeV, \quad \Lambda \gg \mu_0$ effective potential may be approximated:

$$V_{eff} \approx \lambda(\Lambda)\Phi^4 + O(\lambda^2(\Lambda), g_i^2(\Lambda))$$

   - Vacuum stable if $V_{eff} > 0$ from $\mu_0$ upto $\Lambda$
   - Equal to precise determination of $\lambda(\Lambda)$ sign at $\Lambda$-scale
   - Minimal stability bound $129 \pm 3$ GeV is very close to $m_H$

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2. **Calculations in full theory analyticaly w/o additional assumptions**
   - Computtional methods testing
   - $\overline{\mathrm{MS}}$-scheme is used
   - all fields are massless

# Available results for beta-functions

- **4-loop QCD**

  T. van Ritbergen, J.A.M. Vermaseren, and S.A. Larin. In: *Phys.Lett.* B400 (1997)

- **2-loop Standard Model**

  H. Arason, D.J. Castano, B. Keszthelyi, S. Mikaelian, E.J. Piard, et al. In: *Phys.Rev.* D46 (1992) Ming-xing Luo and Yong Xiao. In: *Phys.Rev.Lett.* 90 (2003) C. Ford, I. Jack, and D.R.T. Jones. In: *Nucl.Phys.* B387 (1992)

- **3-loop SM gauge couplings**

  Luminita N. Mihaila, Jens Salomon, and Matthias Steinhauser. In: *Phys.Rev.Lett.* 108 (2012)

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## Full SM results, this work

$\alpha_1, \alpha_2, \alpha_s$ — A.V. Bednyakov, A.F. Pikelner, and V.N. Velizhanin. In: *JHEP* 1301 (2013)

$y_t, y_b, y_\tau$ — A.V. Bednyakov, A.F. Pikelner, and V.N. Velizhanin. In: *Phys.Lett.* B722 (2013)

$\lambda, \mu$ — A.V. Bednyakov, A.F. Pikelner, and V.N. Velizhanin. In: *arXiv:1303.4364 [hep-ph]* (2013)

# Main ingridients

1. **Complicated Feynman rules**
   - Many fields and couplings
   - A lot of parameters in model

Solution: `LanHEP` + unbroken model

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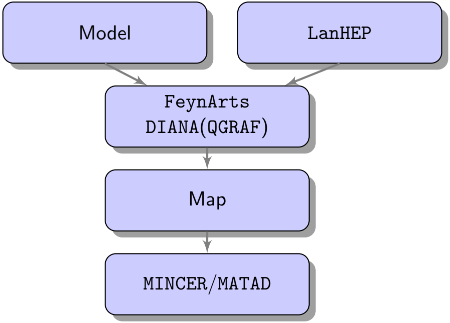
   - Permutations of legs

Solution: FeynArts/Diana(QGRAF)



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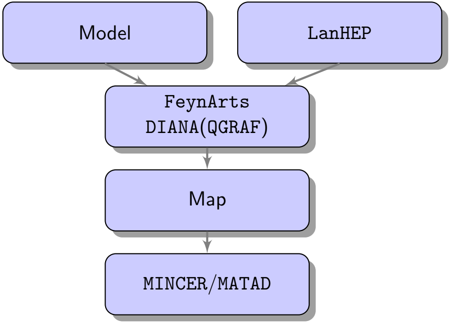
Solution: `FeynArts/Diana(QGRAF)`

3. **Complicated loop integrals**
   - 2,3,4-point diagrams
   - Spurious IR-poles

Solution: `MINCER`+IRR, Massive bubbles



# Gauge couplings.Background Field Method

## SM in Background Field gauge

$$\mathcal{L}_C(\hat{V}) \to \mathcal{L}_C(\hat{V} + V), \quad \sqrt{Z_{\hat{V}_i}} Z_{g_i} = 1$$

$$\hat{V}_i = (\hat{B}, \hat{W}, \hat{G}), \quad g_i = (g_1, g_2, g_s)$$

**Advantages**:

- Less gauge fixing parameters
- only 2-point functions needed: `MINCER`

**Disadvantages**:

- 2xN fields
- Complicated feynman rules

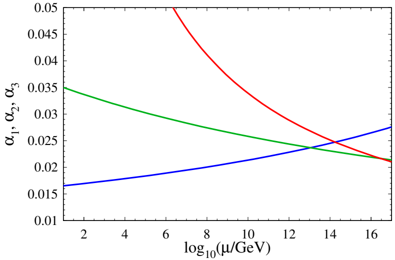
We implimmented SM in BGF formalism in `LanHEP` from: Ansgar Denner, Georg Weiglein, and Stefan Dittmaier. In: *Nucl.Phys.* B440 (1995)

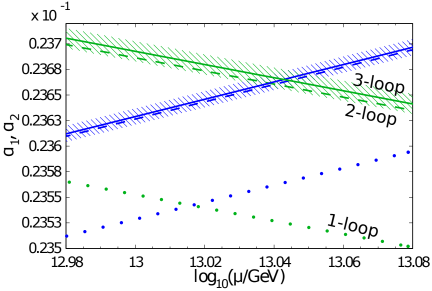
Our work is generalization for several gauge couplings of this:

A.G.M. Pickering, J.A. Gracey, and D.R.T. Jones. In: *Phys.Lett.* B510 (2001)

# Gauge couplings.Results

- Unification scale moves to higher energies
- Three-loop result is enough




$\alpha_1, \alpha_2, \alpha_s$

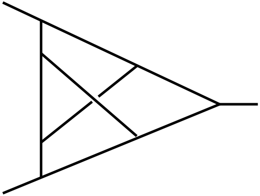
**Results for gauge couplings beta-functions, and fields anomalous dimensions in computer readable form:**

http://arxiv.org/src/1210.6873/anc

# Yukawa couplings.One more leg

In $\overline{\mathrm{MS}}$ scheme is no dependence on internal masses and distribution of external momentum

Sample Yukawa vertex 3-loop diagram. Higgs leg nullified



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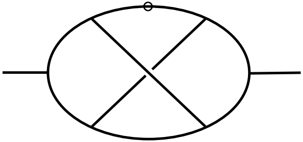


MINCER topology **NO**

Problem:

Problem: **Naive nullification is dangerous: spurious IR-poles**

- In general assymptotic expansion in external momentum needed

But: for $\bar{f}fH$-vertex it's not needed

# Yukawa couplings. Results

$$\beta_t^{(3)} \simeq \boxed{1.51 y_t^3 - 0.63 a_s y_t^2 + 0.22 a_s^2 y_t} \qquad \boxed{+0.07 y_t^2 \lambda - 0.06 a_s^3}$$

$\boxed{Already\ known\ result}$

## Yukawa couplings. Results

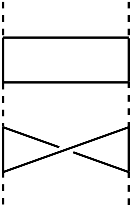
$$\beta_t^{(3)} \simeq \boxed{1.51 y_t^3 - 0.63 a_s y_t^2 + 0.22 a_s^2 y_t} - 0.11 a_2 y_t^2 \boxed{+0.07 y_t^2 \lambda - 0.06 a_s^3}$$

$$\beta_b^{(3)} \simeq 1.34 y_t^3 - 0.19 a_s^2 y_t - 0.09 a_s^3 - 0.06 a_2 y_t^2 - 0.04 a_2 a_s y_t + 0.03 a_s y_t^2$$

$$\beta_\tau^{(3)} \simeq 1.19 y_t^3 - 0.24 a_s y_t^2 + 0.09 a_s^2 y_t - 0.04 a_2 y_t^2 - 0.01 y_t^2 \lambda$$

**Full results for Yukawa couplings beta-functions, in computer readable form:** http://arxiv.org/src/1212.6829/anc

# Higgs self-coupling at three loops



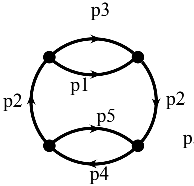
**Problem:**

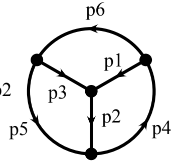
- Diagrams with four legs
- We cannot safely nullify two of them

**Possible solutions:**

1. Repeat assymptotic expansion in external momentum twice
2. Nullify one momentum, but reduce problem to calculation of three-loop IR-safe vertex integrals.
3. mass as IR-regulator

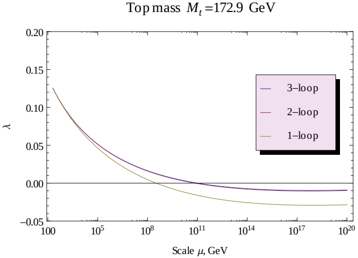
- ☐ Remove all legs
- ☐ Artificial mass for all propagators.
- ☐ Fully massive bubble integrals are known.

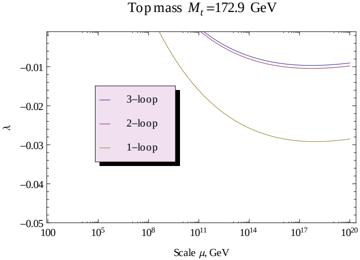


# Higgs self-coupling. Three-loop result

## Sign of $\lambda$ - as a test of vacuum stability



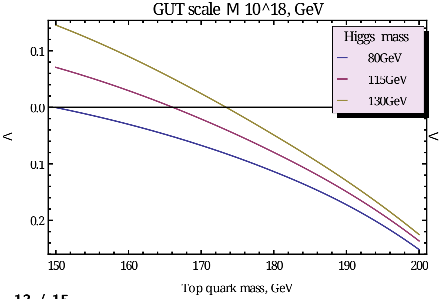

- Three-loop result is enough for precision
- Main uncertainty from $m_t$ and $m_H$

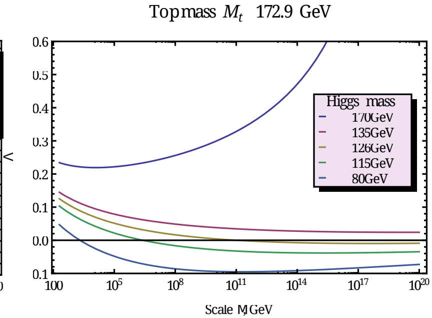
**Full results for Higgs self-coupling and mass parameter beta-functions, in computer readable form:**

http://arxiv.org/src/1303.4364/anc

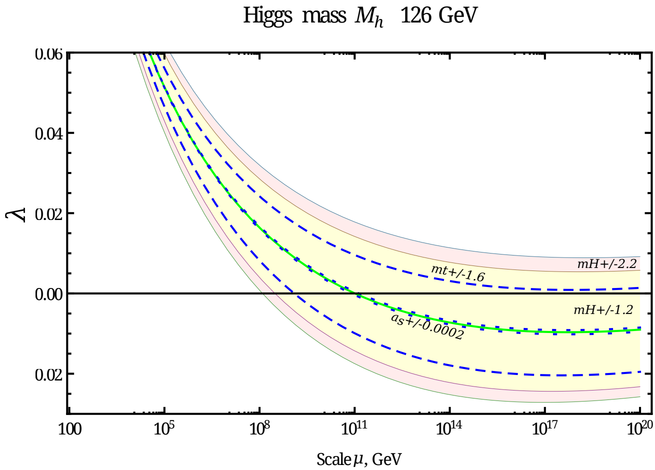
# Standard Model stability investigation with $\beta_\lambda$

- $\lambda < 0$ if $M_h < 111 GeV$ is a strong evidence for new physics, but excluded in direct measurements
- Metastable region coincides with measured $m_t$ and $m_H$, $\lambda 0$ SM asymptotycally free?
- Landau-pole is higher than Planck-scale, $\Lambda > M_{PL}$, SM is valid effective theory from Fermi to Planck scales



# Strong dependence on SM parameters deviations




dotted:$\alpha_s \pm 0.0002$ dashed:$m_t = 172.9 \pm 1.6 GeV$,

filled regions:$m_H = 126 \pm 1.2 GeV$,$m_H \pm 2.2 GeV$

# Conclusion

1. Gauge couplings beta-functions in SM are calculated - independent check
2. Yukawa couplings beta-functions - new result
3. Higgs self-coupling and mass parameter beta-functions
4. Set of programs for calculations in more complicated models.
5. **High precision available in calculations, but comparable precision in $m_t$ and $m_H$ determination needed from experiment**