Strange Particle Production from Nucleons

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Outline

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   - Motivation

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Total $\sigma$ for $\nu/\bar{\nu}$ CC processes,

$$\sigma^{Total} = \sigma^{QE} + \sigma^{Inelastic} + \sigma^{DIS}$$

[G. P. Zeller used NUANCE MC]
Introduction

Total \( \sigma \) for \( \nu / \bar{\nu} \) CC processes,

\[
\sigma^{Total} = \sigma^{QE} + \sigma^{Inelastic} + \sigma^{DIS}
\]

\[
\sigma^{Inelastic} = \sigma^{1\pi} + \sigma^{2\pi} + \cdots + \sigma^{YK} + \sigma^{1K} + \sigma^{1Y} + \cdots
\]

[G. P. Zeller used NUANCE MC]
MINERvA experiment is planning to study strange particle production with high statistics.

MiniBooNE, SciBooNE, K2K, T2K, NOvA are taking data in the few GeV region.

In performing the background studies in the analysis of neutrino oscillation experiments.
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Monte Carlo generators used in the analysis of the current experiments apply models that are not well suited to describe the strangeness production at low energies.

In estimation of atmospheric neutrino $\Delta S$ backgrounds for nucleon-decay searches.

LAGUNA plans to use detectors like GLACIER, MEMPHYS or LENA to test physics at the GUT scale.
R. E. Shrock (Phys. Rev. D 12, 2049, 1975) used a resonant Born model and estimated the charged and neutral \( \nu N \rightarrow \mu^- KY \) \((10^{-40} \text{ cm}^2)\) in the neutrino energy region up to 3 GeV.

A. A. Amer (Phys. Rev. D 18, 2290, 1978) used a harmonic oscillator quark model to estimate cross section for \( K^+ \Lambda \) production \(1.35 - 2.65 \times 10^{-41} \text{ cm}^2\).

H. K. Dewan (Phys. Rev. D 24, 2369, 1981) has studied Associated Production as well as S.C. C.C. reactions \( \nu_\mu N \rightarrow \mu^- KY \) using Born approximations with hyperon-nucleon transition form factors determined from the Cabibbo theory with SU(3) symmetry.

\[ Y \equiv \text{Hyperon (}\Delta S = 0\text{) and Nucleon (}\Delta S = 1\text{)} \]
Super Kamiokande set the experimental limit for proton decay. \( \tau > 5.4 \times 10^{33} \text{y} \) and \( \tau > 2.3 \times 10^{33} \text{y} \) for decay modes \( p \rightarrow e^+\pi^0 \) and \( p \rightarrow K^+\bar{\nu}^2 \)

- \( K^+ \) life time 12.8 ns and they decay into
  \( K^+ \rightarrow \mu^+\nu_\mu \) (63.43 %) and
  \( K^+ \rightarrow \pi^+\pi^0 \) (21.13 %)

- Main background sources are muon neutrinos produced by cosmic rays interactions

- These neutrinos interact with the detector producing muon in the energy range where search for proton decay is performed.
Weak processes accompanying strange particle

\[ \nu_{ln} \rightarrow l^- K^+ n \quad \bar{\nu}_{ln} \rightarrow l^+ K^- n \]
\[ \nu_{lp} \rightarrow l^- K^+ p \quad \bar{\nu}_{lp} \rightarrow l^+ K^+ p \]
\[ \nu_{ln} \rightarrow l^- K^0 p \quad \bar{\nu}_{lp} \rightarrow l^+ \bar{K}^0 n \]
\[ \nu_{ln} \rightarrow l^- K^0 \Sigma^+ \quad \bar{\nu}_{lp} \rightarrow l^+ K^+ \Sigma^- \]
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\[ \nu_{lp} \rightarrow l^- K^+ \Sigma^+ \quad \bar{\nu}_{ln} \rightarrow l^+ K^0 \Sigma^- \]
**Weak processes accompanying strange particle**

\[
\begin{align*}
\nu_l n & \rightarrow l^- K^+ n & \bar{\nu}_l n & \rightarrow l^+ K^- n \\
\nu_l p & \rightarrow l^- K^+ p & \bar{\nu}_l p & \rightarrow l^+ K^+ p \\
\nu_l n & \rightarrow l^- K^0 p & \bar{\nu}_l p & \rightarrow l^+ \bar{K}^0 n \\
\nu_l n & \rightarrow l^- K^0 \Sigma^+ & \bar{\nu}_l p & \rightarrow l^+ K^+ \Sigma^- \\
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\nu_l n & \rightarrow l^- K^+ \Sigma^0 & \bar{\nu}_l p & \rightarrow l^+ K^0 \Sigma^0 \\
\nu_l p & \rightarrow l^- K^+ \Sigma^+ & \bar{\nu}_l n & \rightarrow l^+ K^0 \Sigma^- \\
\end{align*}
\]

\(\bar{\nu}\) induced \((|\Delta S| = 1)\) charged-current reactions

\[
\begin{align*}
\bar{\nu}_\mu + p & \rightarrow l^+ + K^- + p \\
\bar{\nu}_l + p & \rightarrow l^+ + \bar{K}^0 + n \\
\bar{\nu}_l + n & \rightarrow l^+ + K^- + n.
\end{align*}
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Weak processes accompanying strange particle

\[ \nu_l n \rightarrow l^- K^+ n \quad \bar{\nu}_l n \rightarrow l^+ K^- n \]
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\( \bar{\nu} \) induced (\( |\Delta S| = 1 \)) charged-current reactions

\[ \bar{\nu}_\mu + p \rightarrow l^+ + K^- + p \]
\[ \bar{\nu}_l + p \rightarrow l^+ + \bar{K}^0 + n \]
\[ \bar{\nu}_l + n \rightarrow l^+ + K^- + n. \]

\( \Delta S = 1 \) processes have no NC contributions as they involve a change
**Formalism**

The basic reaction for the neutrino induced charged current kaon production is

\[ \nu(k) + N(p) \rightarrow l(k') + K(p_k) + X(p') \]

where \( X \) is

1. \( N (\Delta S = 1) \)
2. \( Y (\Delta S = 0) \)

\[ d^9 \sigma = \frac{1}{4ME(2\pi)^5} \frac{d\vec{k}'}{(2E_l)} \frac{d\vec{p}'}{(2E'_p)} \frac{d\vec{p}_k}{(2E_K)} \delta^4(k + p - k' - p' - p_k) \bar{\Sigma} \Sigma |\mathcal{M}|^2, \]
Formalism

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I. \( N (\Delta S = 1) \)

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\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} j_\mu^{(L)} J_\mu^{(H)} = \frac{g}{2\sqrt{2}} j_\mu^{(L)} \frac{1}{M_W^2} \frac{g}{2\sqrt{2}} J_\mu^{(H)},
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\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} j^{(L)}_\mu J^{(H)} = \frac{g}{2\sqrt{2}} j^{(L)}_\mu \frac{1}{M_W^2} \frac{g}{2\sqrt{2}} J^{(H)},
\]

\( j^{(L)}_\mu \) is Leptonic Current

\& \( J^{(H)} \) is Hadronic Current
Figure: Feynman diagrams contributing to the $J_{\mu}^{(H)}$
**Figure:** Feynman diagrams contributing to the $J^\mu(H)$

Obtained using **Chiral Perturbation Theory (\(\chi PT\))**
**Figure:** Feynman diagrams contributing to the $J_{\mu}^{(H)}$

Obtained using Chiral Perturbation Theory (χPT)

**WNN' vertex structure**

$$\langle Y'; \vec{p}' | J_{\mu}^{(H)} | N; \vec{p} \rangle \propto \bar{u}(\vec{p}') (V^\mu - A^\mu) u(\vec{p})$$
**GENIE (Generates Events for Neutrino Interaction Experiments) MC**

![Graph showing cross-sections for \( \nu_l + p \rightarrow l^- + K^- + p \)]

**Figure:** Comparison of cross-sections for \( \nu_l + p \rightarrow l^- + K^- + p \)
NUANCE Monte Carlo (Baker PRD’1981,83)
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![Graph showing data from Baker and Son (1981, 1983).](image)

DEVELOPMENT ??
**GENIE Cont.**

*Figure:* Comparison of cross-sections for $\nu_l + p \rightarrow l^- + K^- + p$
for \( \nu_\mu + p \rightarrow \mu^- + K^+ + p \)

\[\sigma \quad (10^{-41} \text{ cm}^2)\]

\[E_\nu \quad (\text{GeV})\]

\(v+p \rightarrow \mu+p+K^+\)
\( Q^2 \) Distribution

**Figure:** Differential Cross-section for the processes \( \bar{\nu}_\mu N \rightarrow \mu^+ N' \bar{K} \) (Solid lines) and \( \bar{\nu}_e N \rightarrow e^+ N' \bar{K} \) (Dashed lines)
\[ \sigma \quad \text{for} \quad \bar{\nu}_\mu + p \rightarrow \mu^+ + K^- + p \]

\[
\begin{array}{c|c|c|c|c|c}
E_{\bar{\nu}} (GeV) & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 \\
\sigma (cm^2) & 1e-41 & 2e-41 & 3e-41 & 4e-41 & & & \\
\end{array}
\]

- Full Model
- Contact
- \( \pi \)
- \( \Lambda \)
- \( \Sigma^* \)
Preliminary

\[ \frac{d\sigma}{dQ^2} \text{(cm}^2 \text{GeV}^{-2}) \]

\[ Q^2 \text{(GeV}^2) \]

\[ e^- K^+ n \]
\[ \mu^- K^+ n \]
\[ e^- K^0 p \]
\[ \mu^- K^0 p \]
\[ e^- K^- p \]
\[ \mu^- K^- p \]
Flux Convoluted with Cross Section

\[ \langle \sigma \rangle \left(10^{-41} \text{cm}^2\right) \text{ for } \bar{K} \text{ production with MiniBooNE } \bar{\nu}_\mu \text{ flux and neutral current } \pi^0 \text{ production (per nucleon) measured at MiniBooNE.} \]
Flux Convoluted with Cross Section

\(< \sigma > (10^{-41} cm^2)\) for \(\bar{K}\) production with MiniBooNE \(\bar{\nu}_\mu\) flux and neutral current \(\pi^0\) production (per nucleon) measured at MiniBooNE.

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<td>14.8 ± 0.5 ± 2.3</td>
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\( \langle \sigma \rangle \) for MinerVa is \( \sim 10^{-41} cm^2 \)
\( \langle \sigma \rangle \) for T2K is \( \sim 1.5 \times 10^{-42} cm^2 \)

\( E_l + E_k < 2. GeV \)
Conclusion

- We find the contribution of contact term to be significant in single kaon production as well as in the associated particle production.
- The threshold for associated antikaon production corresponds to the $K - \bar{K}$ channel and it is much higher than for the kaon case (kaon-hyperon).
- The study may be useful in the analysis of antineutrino experiments at MINERvA, NOvA, T2K and others with high statistics and/or higher antineutrino energies.
- Results of the associated particle production also require the contribution of resonant channels, which we are planning to include.
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Thanks

Grazie
backup
Meson-Meson Interaction

\[ \mathcal{L}_M^{(2)} = \frac{f_\pi^2}{4} \text{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{f_\pi^2}{4} \text{Tr}(\chi U^\dagger + U \chi^\dagger) , \]

\[ U(x) = \exp \left( i \frac{\phi(x)}{f_\pi} \right) \]

\[ D_\mu U \equiv \partial_\mu U - i r_\mu U + i U l_\mu \]

\[ u_\mu = i \left[ u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - il_\mu) u^\dagger \right] , \quad u^2 = U \]

\[ r_\mu = 0 , \quad l_\mu = - \frac{g}{\sqrt{2}} (W_\mu^+ T_+ + W_\mu^- T_-) , \text{ with } W^\pm \text{ the } W \text{ boson fields} \]

\[ T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \quad T_- = \begin{pmatrix} 0 & 0 & 0 \\ V_{ud} & 0 & 0 \\ V_{us} & 0 & 0 \end{pmatrix} . \]
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\[ D_\mu U \equiv \partial_\mu U - ir_\mu U + iUl_\mu \]

\[ u_\mu = i \left[ u'^\dagger (\partial_\mu - ir_\mu) u - u(\partial_\mu - il_\mu) u'^\dagger \right], \quad u^2 = U \]

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\[ L_{MB}^{(1)} = \text{Tr} \left[ \bar{B} (iD - M) B \right] - \frac{D}{2} \text{Tr} \left( \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \right) - \frac{F}{2} \text{Tr} \left( \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \right) \]
Meson-Baryon Interaction

\[ \mathcal{L}_{MB}^{(1)} = \text{Tr} \left[ \bar{B} (iD - M) B \right] - \frac{D}{2} \text{Tr} \left( \bar{B} \gamma^\mu \gamma_5 \{ u_\mu, B \} \right) \\
- \frac{F}{2} \text{Tr} \left( \bar{B} \gamma^\mu \gamma_5 [ u_\mu, B ] \right) \]

where \( M \) denotes the mass of the baryon octet, and the parameters \( D = 0.804 \) and \( F = 0.463 \) can be determined from the baryon semileptonic decays. The covariant derivative of \( B \) is given by
\[ \mathcal{L}_{MB}^{(1)} = \text{Tr} \left[ \bar{B} (iD - M) B \right] - \frac{D}{2} \text{Tr} \left( \bar{B} \gamma^\mu \gamma_5 \{ u_\mu, B \} \right) \]

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\[ D_\mu B = \partial_\mu B + [\Gamma_\mu, B], \]

\[ \Gamma_\mu = \frac{1}{2} \left[ u^\dagger (\partial_\mu - ir_\mu) u + u(\partial_\mu - il_\mu) u^\dagger \right], \]
**Meson-Baryon Interaction Cont.**

\[
\mathcal{L}_{\text{dec}} = C \left( \varepsilon^{abc} \tilde{T}^\mu_{ade} u_{\mu,b}^d B^e_c + h.c. \right)
\]

\(C \simeq 1\) has been fitted to the \(\Delta(1232)\) decay-width. The spin 3/2 propagator for \(\Sigma^*\) is given by

\[
G^{\mu\nu}(P) = \frac{P^{\mu\nu}_{RS}(P)}{P^2 - M_{\Sigma^*}^2 + iM\Sigma^*\Gamma\Sigma^*},
\]

where \(P = p + q\) is the momentum carried by the resonance, \(q = k - k'\) and \(P^{\mu\nu}_{RS}\) is the projection operator

\[
P^{\mu\nu}_{RS}(P) = \sum_{\text{spins}} \psi^\mu \bar{\psi}^\nu = -(P + M_{\Sigma^*}) \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{P^\mu P^\nu}{M_{\Sigma^*}^2} + \frac{1}{3} \frac{P^\mu \gamma^\nu - P^\nu \gamma^\mu}{M_{\Sigma^*}} \right],
\]
The $\Sigma^*$ width

$$\Gamma_{\Sigma^* \rightarrow Y, M} = \frac{C_Y}{192\pi} \left( \frac{C}{f_\pi} \right)^2 \frac{(W + M_Y)^2 - m^2}{W^5} \lambda^{3/2}(W^2, M_Y^2, m^2) \Theta(W - M_Y - m).$$

$$\Gamma_{\Sigma^*} = \Gamma_{\Sigma^* \rightarrow \Lambda \pi} + \Gamma_{\Sigma^* \rightarrow \Sigma \pi} + \Gamma_{\Sigma^* \rightarrow N\bar{K}},$$

Here, $m, M_Y$ are the masses of the emitted meson and baryon. $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and $\Theta$ is the step function.
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Here, $m, M_Y$ are the masses of the emitted meson and baryon. $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and $\Theta$ is the step function. The factor $C_Y$ is 1 for $\Lambda$ and $\frac{2}{3}$ for $N$ and $\Sigma$. 