

# Heavy-meson physics and flavour violation with a single generation<sup>1</sup>

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<sup>1</sup>based on [JHEP 1208 \(2012\) 136](#)

# Motivation

Quarks	$u$ up	$c$ charm	$t$ top
	$d$ down	$s$ strange	$b$ bottom
	$\nu_e$ e- Neutrino	$\nu_\mu$ $\mu$ - Neutrino	$\nu_\tau$ $\tau$ - Neutrino
Leptons	$e$ electron	$\mu$ muon	$\tau$ tau
	I	II	III
The Generations of Matter			

Problems:

- Gauge hierarchy
- Family replication
- Mass hierarchy

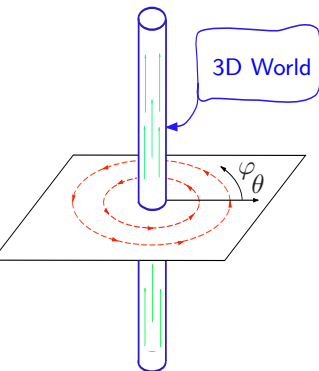
# Model with LED and a single generation

- Model with large extra dimensions and a single generation [Libanov, Troitsky, Frere '00, Libanov, Troitsky, Frere, E. Nugaev '03]
- Extra dimensions are compactified into the two-dimensional sphere

$$M^4 \otimes S^2 \quad (R - \text{radius of the sphere})$$

- Processes with FCNC and flavour violation appear in the model due to KK modes of gauge bosons
- One could bound  $R$  via studying rare processes: lepton number violation and *flavour* violation

# Model with LED and a single generation: vortex



- Observable world is a core of Abrikosov-Nielsen-Olesen vortex.

$$\mathcal{L}_V = \sqrt{|g|} \left( -\frac{1}{4} F_{AB} F^{AB} + (D^A \Phi)^\dagger D_A \Phi - \frac{\lambda}{2} (|\Phi|^2 - v^2)^2 \right),$$

where  $A$  is  $U_g(1)$  gauge field and  $\Phi$  is a scalar

- There is only one generation of six-dimensional fermions

# Model with LED and a single generation: fermions

- Zero modes of fermions localized in a core of vortex due to interaction with vortex:

$$\mathcal{L}_{int} = g \phi \bar{\Psi} \frac{1 - \Gamma_7}{2} \Psi$$

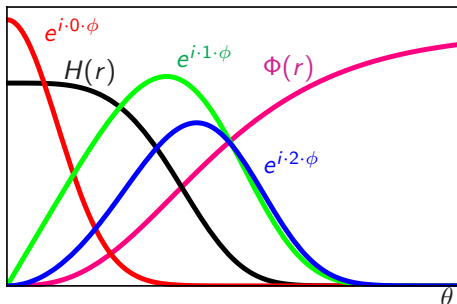
- Generalized momentum

$$J = -i\partial_\phi - k \frac{1 + \Gamma_7}{2} \quad (1)$$

is conserved in a vortex background.

- 6D zero modes  $\leftrightarrow$  4D chiral fermionic families
- $J \leftrightarrow$  generation number in 4D effective theory

# Origin of mass hierarchy



- Zero modes with different momentum ( $n = 0, 1, 2$ ) have different shape in extra dimensions:

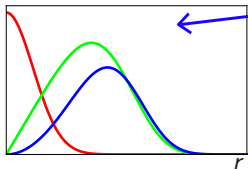
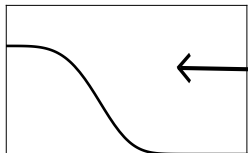
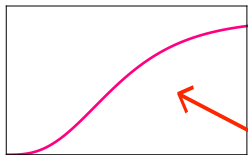
$$\hat{J} \Psi_n \equiv - \left( i \partial_\varphi + 3 \frac{1 + \Gamma_7}{2} \right) \Psi_n = n \Psi_n$$

$$\Psi_n(\theta \rightarrow 0) \sim (\theta)^{3-n} \cdot e^{i(3-n)\phi}$$

- Four dimensional masses generated by Higgs field  $H$ :

$$m_{nm} \propto \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta \bar{\Psi}_n \Psi_m H \Phi$$

# Field content



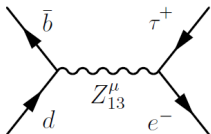
Fields	Profiles	Charges		Representations	
		$U_g(1)$	$U_Y(1)$	$SU_W(2)$	$SU_C(3)$
scalar $\Phi$	$F(r)e^{i\varphi}$ $F(0) = 0, F(\infty) = v$	+1	0	1	1
vector $A_\varphi$	$A(r)/e$ $A(0) = 0, A(\infty) = 1$	0	0	0	0
scalar $X$	$X(r)$ $X(0) = v_X, X(\infty) = 0$	+1	0	1	1
scalar $H$	$H(r)$ $H_i(0) = \delta_{2i}v_H, H_i(\infty) = 0$	-1	+1/2	2	1
fermion $Q$	3 L zero modes	axial (3, 0)	+1/6	2	3
fermion $U$	3 R zero modes	axial (0, 3)	+2/3	1	3
fermion $D$	3 R zero modes	axial (0, 3)	-1/3	1	3
fermion $L$	3 L zero modes	axial (3, 0)	-1/2	2	1
fermion $E$	3 R zero modes	axial (0, 3)	-1	1	1
SM gauge fields	$Z_\mu, \dots$ Kaluza-Klein spectrum	0	SM	SM	SM

# Rare processes

KK vector modes carry **angular momentum = family number**. In the absence of fermion mixings, family number is an exactly conserved quantity  $\Rightarrow$  processes with  $\Delta G = \Delta J \neq 0$  are suppressed by orders of small mixing parameter  $\epsilon^{\Delta G}$ ,  $\epsilon \sim 0.1$ .

$$B^0 \rightarrow \tau e$$

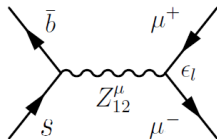
$$\Delta G = \Delta J = 0$$



$$\sim \frac{\#}{M_{Z'}^2}$$

$$B_s^0 \rightarrow \mu\mu$$

$$\Delta G = \Delta J = 1$$



$$\sim \epsilon \frac{\#}{M_{Z'}^2}$$

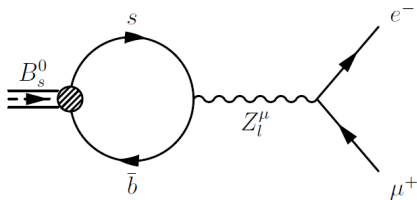


# Calculation of $B_s^0 \rightarrow \mu e$ decay width

- $B_s^0 \rightarrow \mu e$  decay  $BR(B_s^0 \rightarrow \mu e) < 2.0 \cdot 10^{-7}$   
[Beringer *et al.* '12]

- Coupling in the effective four-dimensional Lagrangian:

$$\frac{g}{2 \cos \theta_W} \sum_{l=1}^{\infty} Z_{l,1}^{\mu} \left\{ E_{23}^{l,1} \bar{s} \gamma_{\mu} \left( -\frac{1}{2} \gamma_5 \right) b + E_{12}^{l,1} \bar{e} \gamma_{\mu} \left( 2 \sin^2 \theta_W - \frac{1}{2} - \frac{1}{2} \gamma_5 \right) \mu \right\}$$



# Calculation of $B_s^0 \rightarrow \mu e$ decay width

- Decay width:

$$\Gamma(B_s^0 \rightarrow \mu^+ e^-) = \frac{G_F^2 m_Z^4 \zeta^2 R^4 f_{B_s}^2 m_{B_s} m_\mu^2 (1 + (1 - 4 \sin^2 \theta_W)^2)}{128\pi},$$

where  $\sum_{l=1}^{\infty} \frac{E_{23}^{l,1} E_{12}^{l,1} R^2}{l(l+1)} = \zeta R^2$ ,  $\zeta \simeq 0.47$

- Using

$$BR(B_s^0 \rightarrow \mu e) = \Gamma(B_s^0 \rightarrow \mu e) \tau_{B_s^0} < 2.0 \cdot 10^{-7}$$

- We obtain restriction on  $R$ :

$$\frac{1}{R} > 0.7 \text{ TeV.}$$

# $B^0 \rightarrow K^0 \mu e$ decay

- Three-particle decay  $B^0 \rightarrow K^0 \mu e$

- Width:

$$\Gamma = m_W^4 \frac{G_F^2 \zeta^2 R^4 F_1^2(0) m_{B_d}^5 (C_V^2 + C_A^2)}{6\pi^3},$$

where  $C_V$  and  $C_A$  - numerical coefficients.

- We get restriction:

$$\frac{1}{R} > 3.3 \text{ TeV.}$$

# Processes with $\Delta G \neq 0$

- Total change of generation number  $\Delta G = 0$  due to the conservation of momentum  $J$ .
- Processes with  $\Delta G \neq 0$  are allowed due to the mixings.
- For  $B^0 \rightarrow \mu e$  decay:

$$\frac{g}{2 \cos \theta_W} \sum_{l=2}^{\infty} Z_{l,2}^{\mu} E_{13}^{l,2} \left\{ \bar{b} \gamma_{\mu} \left( -\frac{1}{2} \gamma_5 \right) d + \epsilon_L \alpha_L \bar{e} \gamma_{\mu} \left( 2 \sin^2 \theta_W - \frac{1}{2} - \frac{1}{2} \gamma_5 \right) \mu \right\}$$

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$$\frac{1}{R} > m_Z \left( \frac{G_{F\xi}^2 (\epsilon_L \alpha_L)^2 f_{B_d}^2 m_{B_d} m_{\mu}^2 \tau(B_d) (1 + (1 - 4 \sin^2 \theta_W)^2)}{64 \pi B_{B^0 \rightarrow \mu e}} \right)^{\frac{1}{4}} = 0.15 \text{ TeV.}$$

# Experimental data and results

	BR	$\Delta G$	$1/R >, \text{TeV}$
$B_s^0 \rightarrow \mu e$	$< 2.0 \cdot 10^{-7}$	0	0.7
$B^0 \rightarrow \tau e$	$< 2.8 \cdot 10^{-5}$	0	0.65
$B^0 \rightarrow K^0 \mu e$	$< 2.7 \cdot 10^{-7}$	0	3.3
$D^0 \rightarrow \mu e$	$< 8.0 \cdot 10^{-7}$	0	0.3
$K_L^0 \rightarrow \mu e^2$	$< 2.4 \cdot 10^{-12}$	0	60
$B^0 \rightarrow \mu e$	$< 6.4 \cdot 10^{-8}$	1	0.15
$B_s^0 \rightarrow \mu^+ \mu^-$	$3.2 \cdot 10^{-9}$	1	0.46
$D^0 \rightarrow \mu^+ \mu^-$	$< 1,3 \cdot 10^{-6}$	1	0.11
$B_s^0 \leftrightarrow \bar{B}_s^0$	$\Delta m_{B_s^0} \approx 1.17 \cdot 10^{-8} \text{ MeV}$	2	0.09

<sup>2</sup>The stringent bound from kaon physics, [[hep-ph/0309014](https://arxiv.org/abs/hep-ph/0309014)]

# Conclusions

- We found that the best limit  $1/R > 3.3 \text{ TeV}$  arises from the three-particle decay  $B \rightarrow K\mu e$ . This bound is much less stringent than the constraint arising from the two-particle decay  $K \rightarrow \mu e$  in kaons. The reason is the still too poor statistics: the experimental bound on the branching ratio of  $K \rightarrow \mu e$  is  $2.4 \cdot 10^{-12}$  while for the  $B$ -meson decay is  $2.7 \cdot 10^{-7}$ .
- The distinctive feature of the model would be an observation of  $K \rightarrow \mu e$  and  $B \rightarrow K\mu e$  decays without observations of other flavour-changing processes at the same precision level.

**THANK YOU!**

- Effective 4-dimensional interaction of the fermionic zero modes with KK tower of gauge boson is given by (all fields depend on 4-dimensional coordinates only)

$$\mathcal{L}_4 = e \cdot \text{Tr}(\mathbf{A}^\mu \mathbf{j}_\mu^*),$$

with

$$\mathbf{A}^\mu = (\mathbf{A}^\mu)^\dagger = \sum_{l=0}^{\infty} \begin{pmatrix} E_{11}^{l,0} A_{l,0}^\mu & E_{12}^{l,1} A_{l,1}^\mu & E_{13}^{l,2} A_{l,2}^\mu \\ E_{21}^{l,1} A_{l,1}^{\mu*} & E_{22}^{l,0} A_{l,0}^\mu & E_{23}^{l,1} A_{l,1}^\mu \\ E_{31}^{l,2} A_{l,2}^{\mu*} & E_{32}^{l,1} A_{l,1}^{\mu*} & E_{33}^{l,0} A_{l,0}^\mu \end{pmatrix},$$

and

$$\mathbf{j}_{mn}^\mu = a_m^\dagger \bar{\sigma}^\mu a_n,$$

where  $a_n$  are two-component Weyl spinors.



- To incorporate quark mixings in a model, additional field  $X$  is required.
- If one rewrites the current  $\mathbf{j}_\mu$  in terms of the mass eigenstates, then the matrix  $\mathbf{A}^\mu$ , should be replaced by

$$\tilde{\mathbf{A}}^\mu = S_d^\dagger \mathbf{A}^\mu S_d.$$

- Explicit form of  $\mathbf{A}^\mu$ :

$$\left( \begin{array}{ccc} \mathbf{A}_{11} - 2\text{Re}(\epsilon^* \alpha \mathbf{A}_{12}) & \mathbf{A}_{12} + \epsilon \alpha (\mathbf{A}_{11} - \mathbf{A}_{22}) - \gamma \epsilon^* \mathbf{A}_{13} & \mathbf{A}_{13} + \epsilon (\gamma \mathbf{A}_{12} - \alpha \mathbf{A}_{23}) \\ \mathbf{A}_{12}^* + \epsilon^* \alpha (\mathbf{A}_{11} - \mathbf{A}_{22}) - \epsilon \gamma \mathbf{A}_{13}^* & \mathbf{A}_{22} + 2\text{Re}(\epsilon^* (\alpha \mathbf{A}_{12} - \gamma \mathbf{A}_{23})) & \mathbf{A}_{23} + \epsilon^* \alpha \mathbf{A}_{13} + \epsilon \gamma (\mathbf{A}_{22} - \mathbf{A}_{33}) \\ \mathbf{A}_{13}^* + \epsilon^* (\gamma \mathbf{A}_{12}^* - \alpha \mathbf{A}_{23}^*) & \mathbf{A}_{23}^* + \epsilon \alpha \mathbf{A}_{13}^* + \epsilon^* \gamma (\mathbf{A}_{22} - \mathbf{A}_{33}) & \mathbf{A}_{33} + 2\text{Re}(\epsilon^* \gamma \mathbf{A}_{23}) \end{array} \right)^\mu$$