TRANSITION RADIATION AND IONIZATION ENERGY LOSSES OF HIGH-ENERGY ‘HALF-BARE’ ELECTRON

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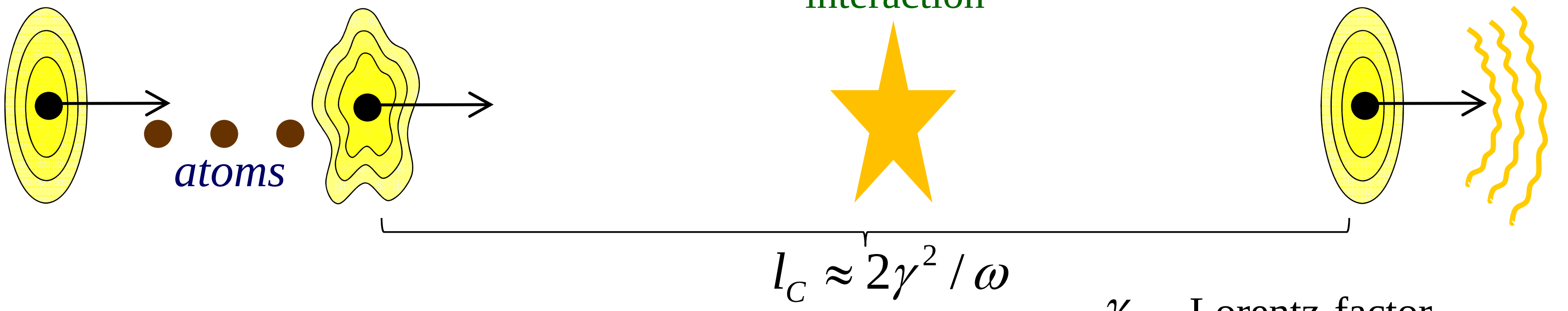
- *N.F. Shul’ga, S.V. Trofymenko, V.V. Syshchenko // JETP Lett. (2011)*

- *N.F. Shul’ga, S.V. Trofymenko // Chapter in the Book “Solutions and Applications of Scattering, Propagation, Radiation and Emission of Electromagnetic Waves”, InTech (2012)*

- *N.F. Shul’ga, S.V. Trofymenko // Phys. Lett. A (2012)*

COHERENCE LENGTH. ‘HALF-BARE’ PARTICLE

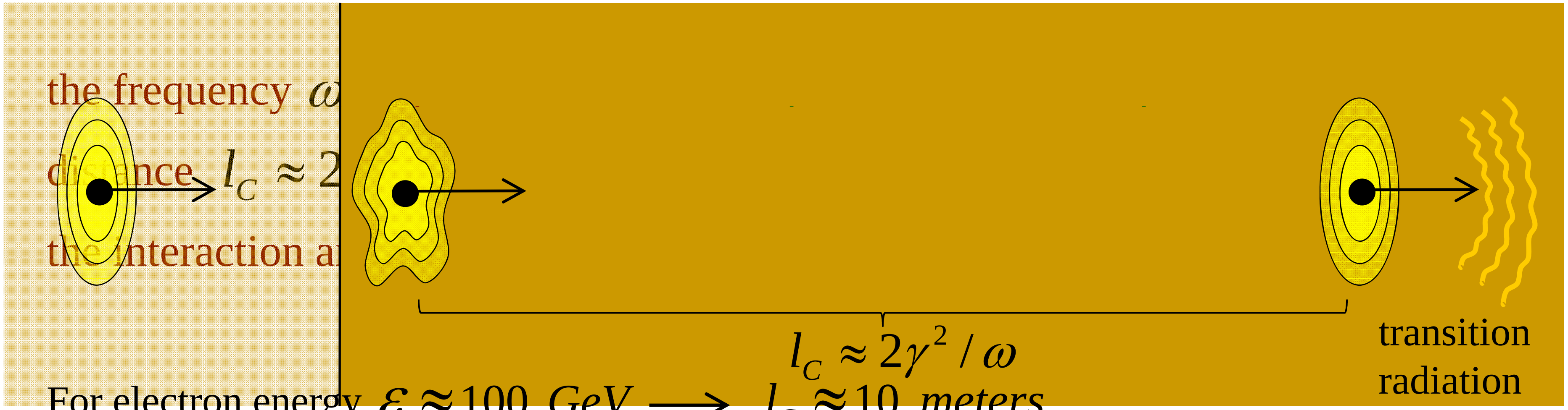
Ultra relativistic case ($\gamma >> 1$):



$$l_C \approx 2\gamma^2 / \omega$$

γ – Lorentz-factor

ω – radiated frequency



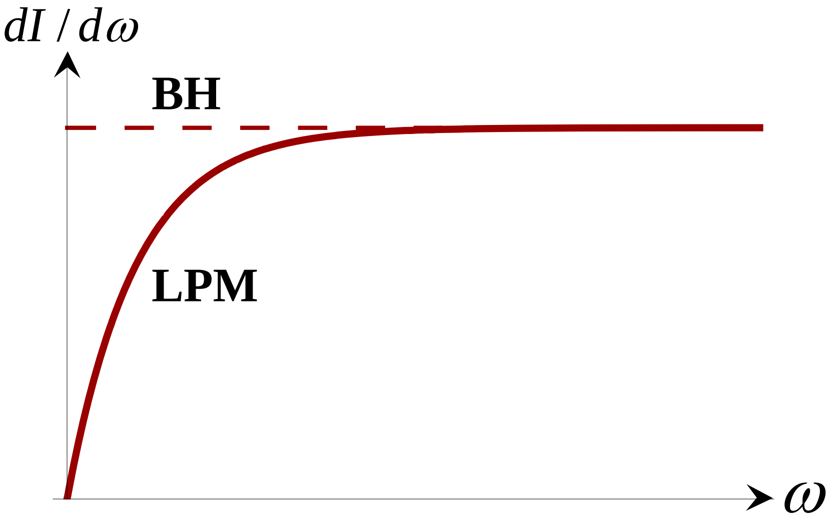
$$l_C \approx 2\gamma^2 / \omega$$

For electron energy $\varepsilon \approx 100 \;\; GeV \longrightarrow l_C \approx 10 \;\; meters$

boundary between
two substances

MANIFESTATION OF 'HALF-BARE' STATE OF ELECTRON IN ITS BREMSSTRAHLUNG

1) Coherent bremsstrahlung



2) Landau-Pomeranchuk-Migdal effect (suppression of Bethe-Heitler spectrum al low frequencies)

3) Ternovsky-Shul'ga-Fomin effect (modification of LPM-effect in thin layers of substance)

Observed in CERN NA63 experiment:

H.D. Thomsen, K.K. Andersen, J. Esberg, H. Knudsen, M. Lund, K.R. Hansen, U.I. Uggerhøj, et. al., (2009).

U. Uggerhøj : '… we have seen the 'half -bare' electron !'

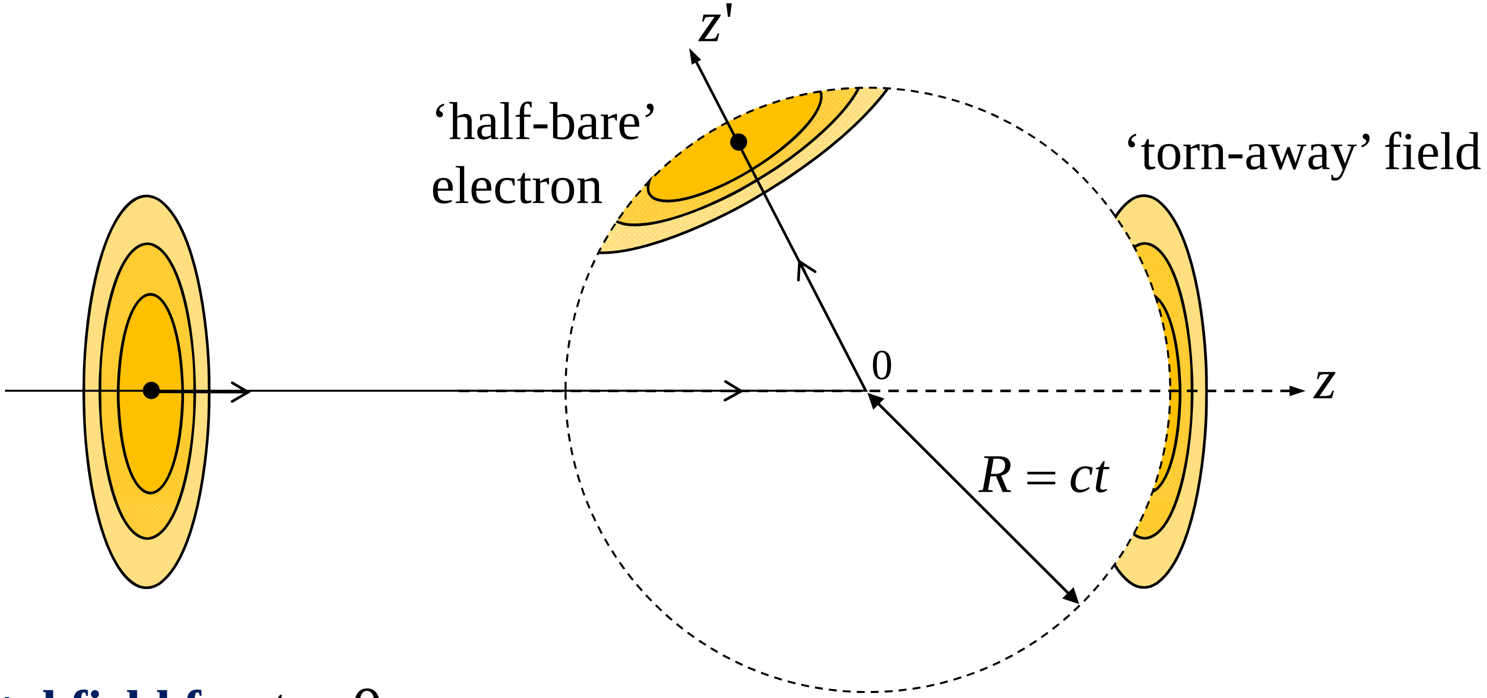
AIM OF THE PRESENT PAPER

Search for manifestation of ‘half-bare’ state of electron in its:

1) **Transition radiation**

2) **Ionization energy losses**

ELECTRON ‘UNDRESSING’ BY INSTANT SCATTERING

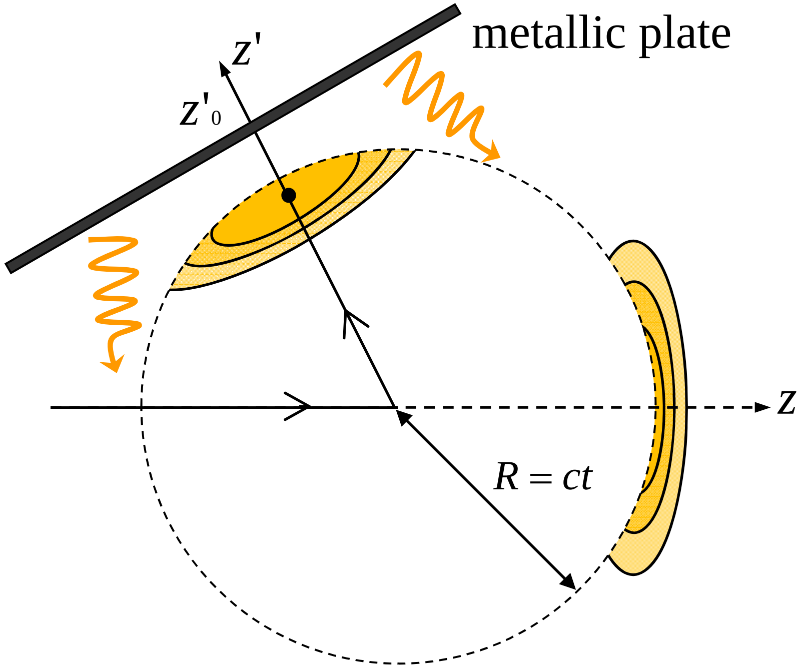


The total field for $t > 0$:

$$\varphi(\vec{r},t) = \theta(r-t)\varphi_{\vec{v}}(\vec{r},t) \ + \ \theta(t-r)\varphi_{\vec{v}'}(\vec{r},t)$$

A.I. Akhiezer, N.F. Shul’ga // High Energy Electrodynamics in Matter, 1996
N.F. Shul’ga, V.V. Syshchenko, S.N. Shul’ga // Phys. Lett. A, 2009

TRANSITION RADIATION BY ‘HALF-BARE’ ELECTRON



- suppression of radiation for $z'_0 << 2\gamma^2/\omega$
- period of oscillations $\Lambda = \dfrac{4\pi}{\omega(\vartheta^2+\gamma^{-2})}$
- the oscillations can be observed for $z'_0 < \dfrac{2\pi}{\Delta\omega\ (\vartheta^2+\gamma^{-2})}$

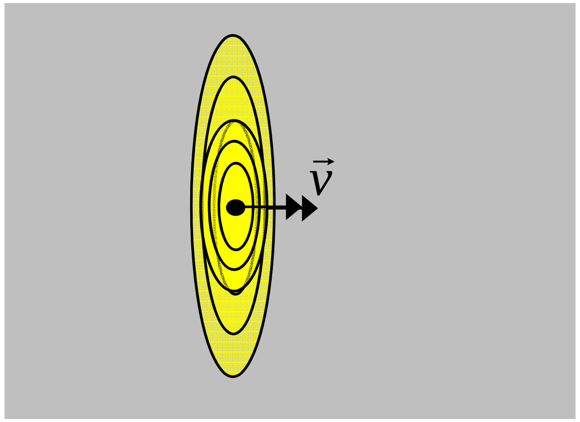
$\omega/\Delta\omega$ – detector resolution

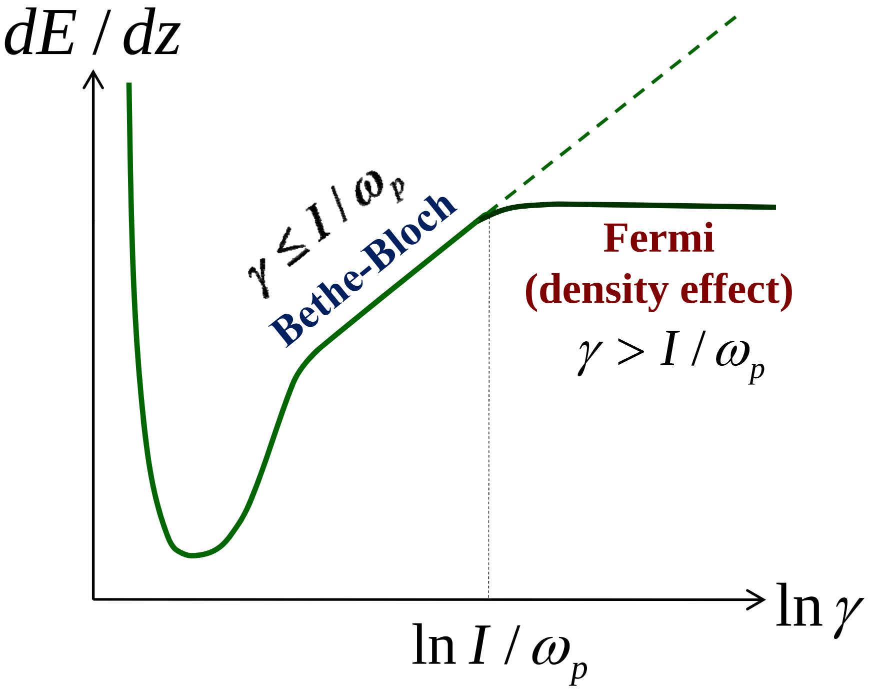
Transition radiation by ‘half-bare’ electron:

$$\frac{d\mathcal{E}}{d\omega\ do} = \frac{e^2}{\pi^2}\frac{\vartheta^2}{\left(\vartheta^2+\gamma^{-2}\right)^2}2\left\{1-\cos\left[\frac{\omega\ z'_0}{2}\left(\gamma^{-2}+\vartheta^2\right)\right]\right\}$$

FERMI AND BETHE-BLOCH FORMULAE

Infinite medium





Bethe-Bloch formula ($\gamma \leq I / \omega_p$):

$$\frac{d\mathcal{E}}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{\gamma}{bI}$$

Fermi formula ($\gamma > I / \omega_p$):

$$\frac{d\mathcal{E}}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{v}{b\omega_p}$$

γ – electron Lorentz-factor

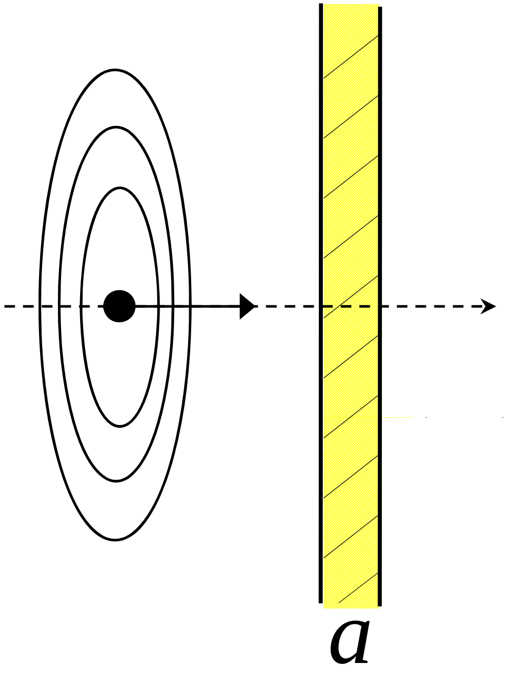
I – mean ionization potential

ω_p – plasma frequency

THIN LAYER OF SUBSTANCE

Bethe-Bloch and Fermi formulae are valid in boundless homogeneous substance

Garibian G.M.// JETP, 1959
Sørensen A. // Phys.Rev.A, 1987



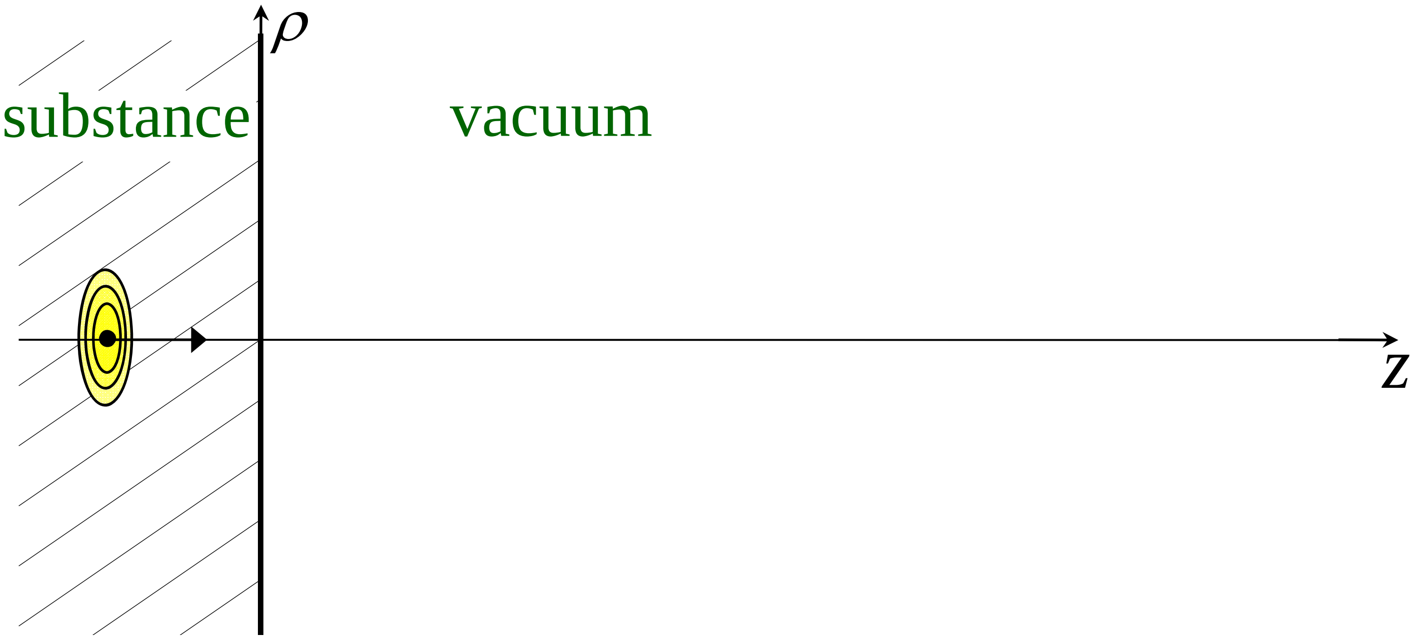
Total absence of the density effect in thin plates:

$$a \leq I / \omega_p^2$$

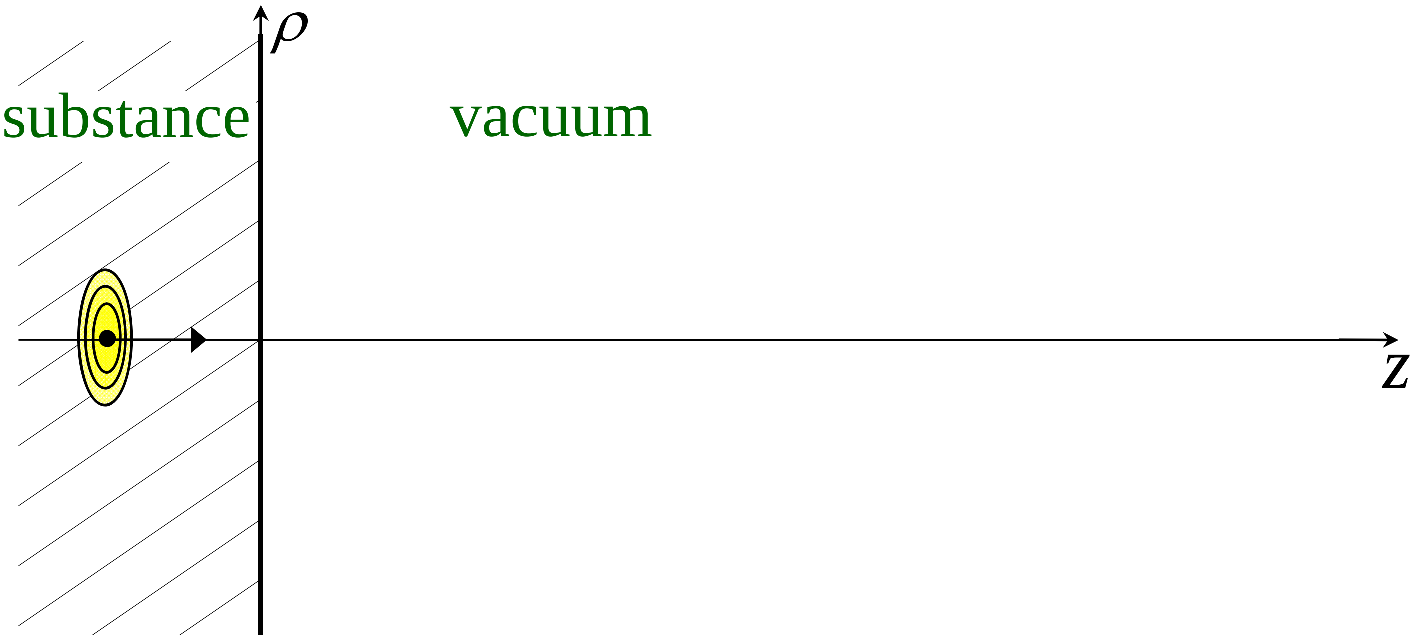
Particle energy loss:

$$\Delta E = \frac{\omega_p^2 e^2}{v^2} a \ln \frac{\gamma}{bI} \quad \textbf{for} \quad 1 \leq \gamma < \infty$$

EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM



EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM

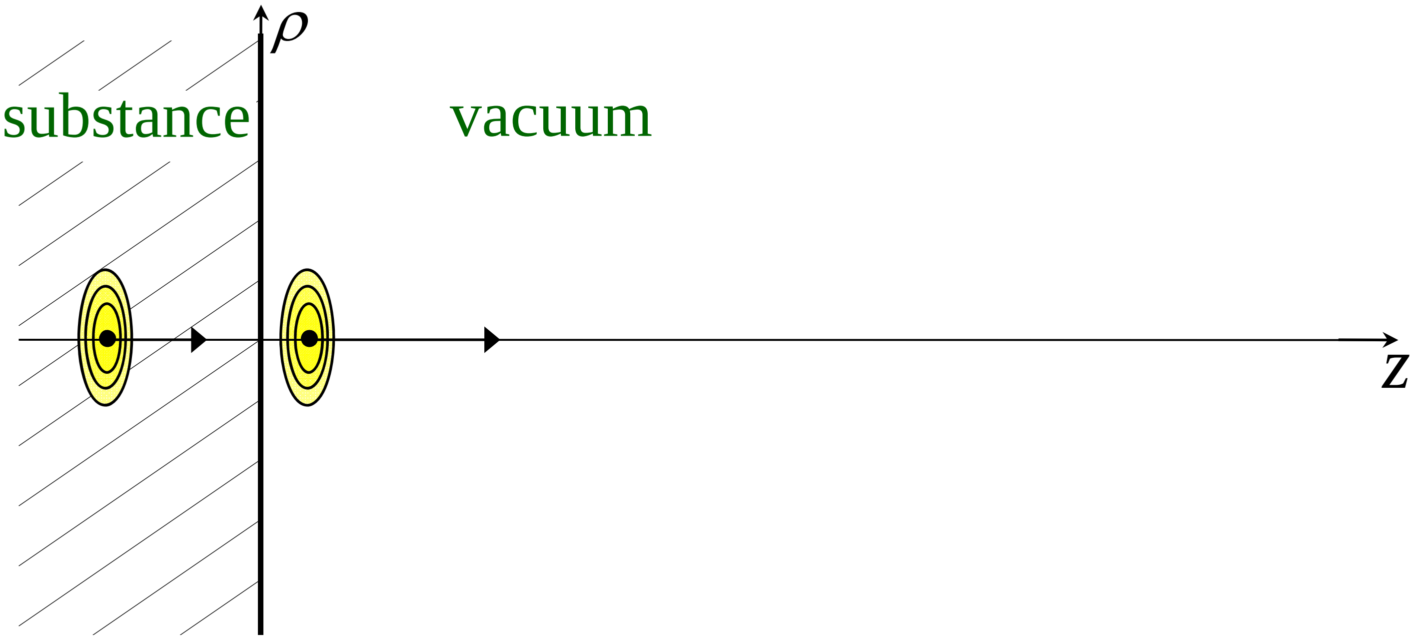


Fourier component of the total field ($\gamma >> 1$, $\omega >> \omega_p$):

$$E_\omega^\rho(\vec{r}) = 2\frac{e}{v}\int_0^\infty dq q^2 J_1(q\rho)\left\{ \underbrace{\left[\frac{1}{q^2+\omega_p^2+\frac{\omega^2}{\gamma^2}} - \frac{1}{q^2+\frac{\omega^2}{\gamma^2}}\right] e^{i\omega\, z\, -\, \frac{q^2 z}{2\omega}}}_{\text{transition radiation}} + \underbrace{\frac{e^{i\frac{\omega}{v}z}}{q^2+\frac{\omega^2}{\gamma^2}}}_{\text{Coulomb field}} \right\}$$

$J_1(x)$ – Bessel function

EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM



For $z \to 0$**:**

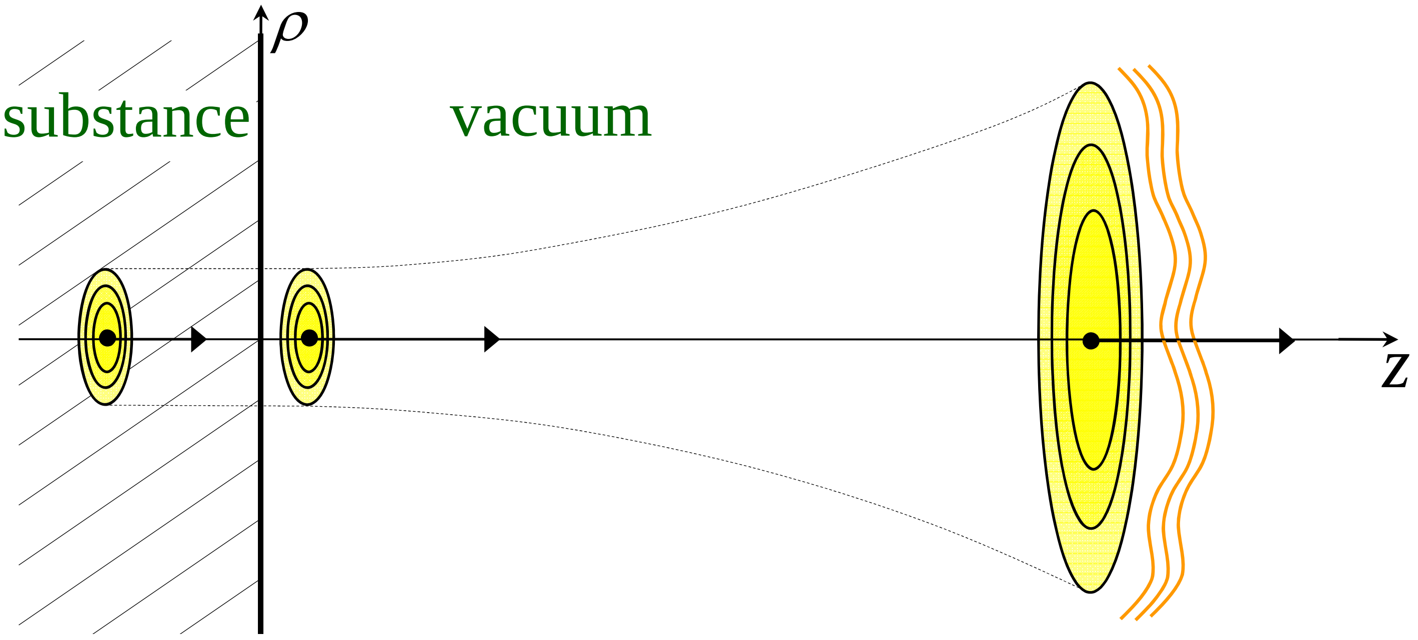
$$E_{\omega}^{\rho}(\rho) = \frac{2e}{v}\sqrt{\frac{\omega^2}{\gamma^2}+\omega_p^2}\,K_1\left(\rho\sqrt{\frac{\omega^2}{\gamma^2}+\omega_p^2}\right)e^{i\frac{\omega}{v}z}$$

suppressed frequencies $\omega \le \gamma\omega_p$

electron is ‘half-bare’

$K_1(x)$ – modified Bessel function

EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM



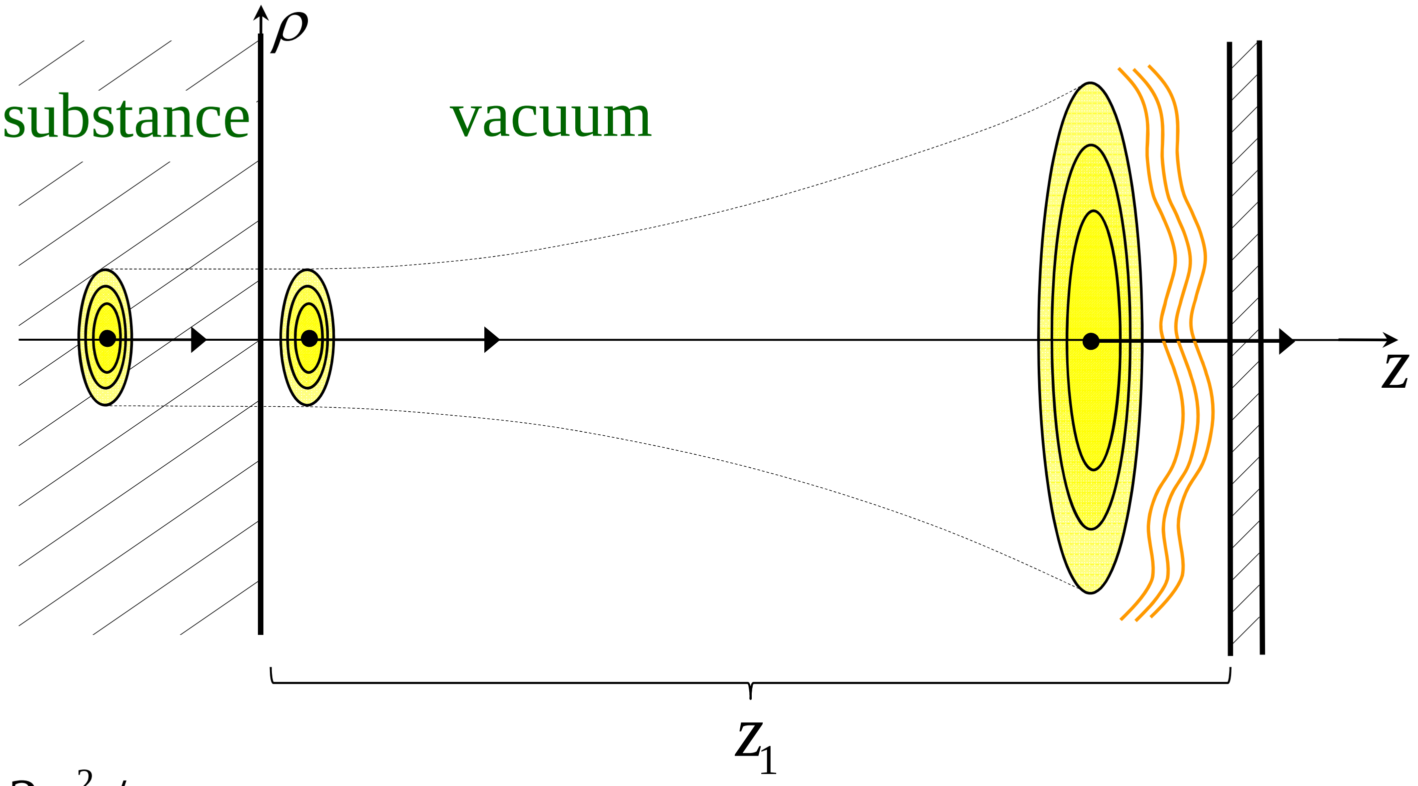
For $z > 2\gamma^2/\omega$ **:**

$$E_{\omega}^{\rho}(\rho,z) = \frac{2e}{v}\left\{ \frac{\omega}{\gamma}K_1\left(\frac{\omega}{\gamma}\rho\right)e^{i\frac{\omega}{v}z} + \frac{e^{i\omega r}}{r}F(\rho/z) \right\}$$

F – definite function of ρ/z

$$r = \sqrt{\rho^2 + z^2}$$

EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM



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IONIZATION ENERGY LOSSES OF ‘HALF-BARE’ ELECTRON

Total ionization per unit path ($q_0 >> \omega_p >> I/\gamma$ **):**

$$\frac{d\mathcal{E}}{dz} = \frac{\eta_p^2 e^2}{v^2}\left\{ \ln\frac{q_0 v\gamma}{I} + \ln\frac{\omega_p v\gamma}{I} + \right.$$

$$+ Ci(\lambda_\gamma) - \cos\lambda_p Ci(\lambda_p + \lambda_\gamma) - \sin\lambda_p Si(\lambda_p + \lambda_\gamma) +$$

$$\left. + \lambda_\gamma Si(\lambda_\gamma) + \cos\lambda_\gamma \right\}$$

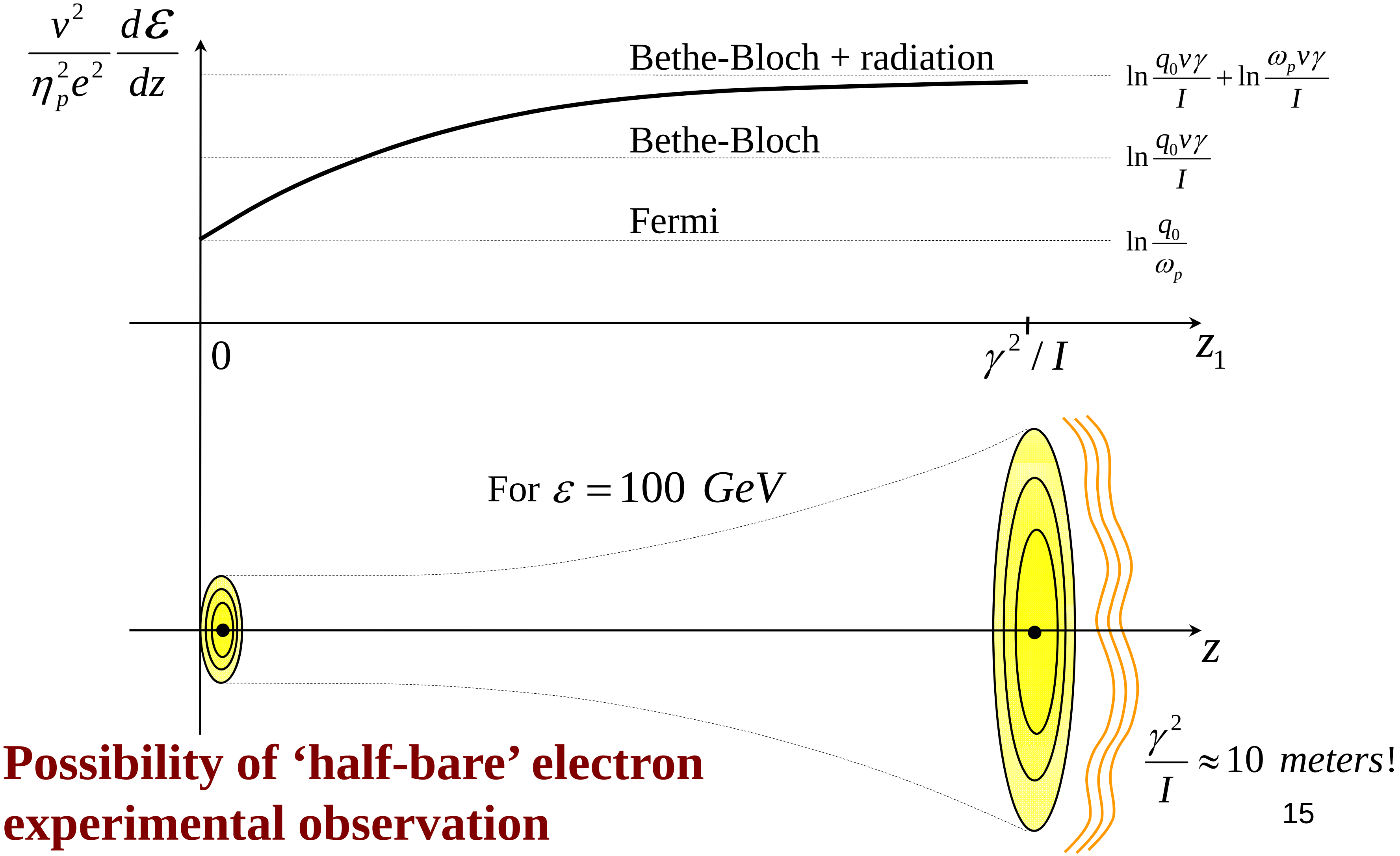
where:

$$\lambda_p = z_1\omega_p^2/2I \qquad \lambda_\gamma = Iz_1/2v^2\gamma^2$$

η_p – plasma frequency of the plate

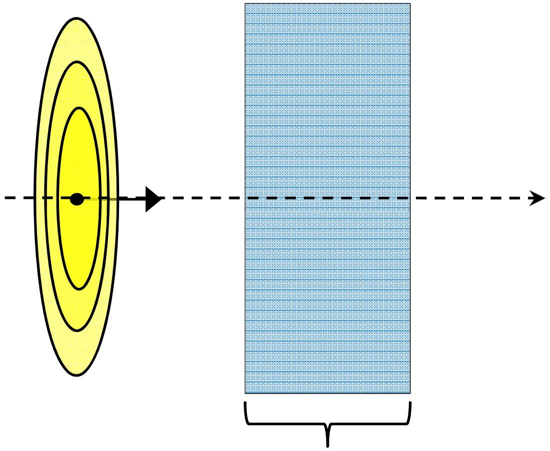
I – mean ionization potential

IONIZATION ENERGY LOSSES OF 'HALF-BARE' ELECTRON (from Fermi to Bethe-Bloch formula)

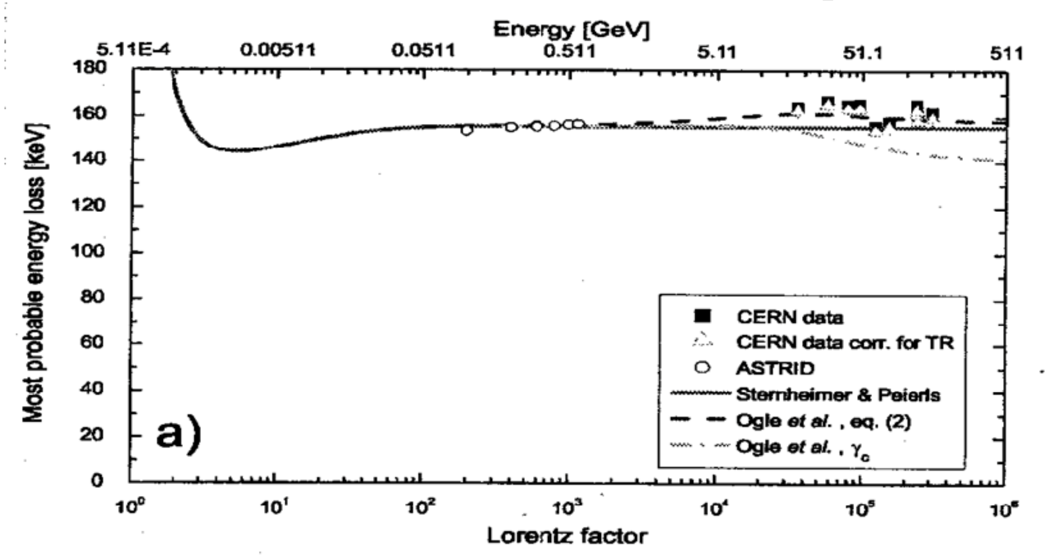


Possibility of 'half-bare' electron experimental observation

CERN NA63 EXPERIMENT (2010)



$$a >> I / \omega_p^2$$



K.K. Andersen, J. Esberg, K.R. Hansen, H. Knudsen, M. Lund, H.D. Thomsen, U.I. Uggerhøj, et. al., NIM B (2010).

CONCLUSIONS

‘Half-bare’ state of electron should be manifested in its transition radiation and ionization losses as well as in bremsstrahlung:

❖Dependence of electron transition radiation characteristics on distance between the plate and the scattering point

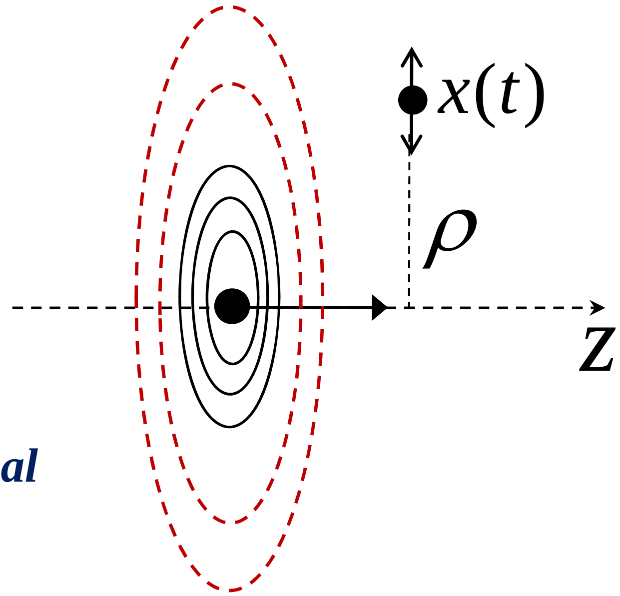
❖Existence of transition process in which ionization losses of the particle are defined by the mechanism of restoration of the field around it (not by absorption)

❖ Gradual increase of electron ionization losses in thin plate from the value with density effect (Fermi formula) to the value without it (Bethe-Bloch formula) supplemented by contribution to ionization from transition radiation

IONIZATION OF SUBSTANCE BY EXTERNAL FIELD

$$\omega_0 = I$$

$$\ddot{x} + \beta \dot{x} + \omega_0^2 x = \frac{e}{m}\left(E_\rho^C + E_\rho^F\right)$$



Energy transfer to a harmonic oscillator by external field:

$$\Delta \mathcal{E} = \frac{e^2}{2m}\left|E_{\omega_0}(\vec{r})\right|^2$$

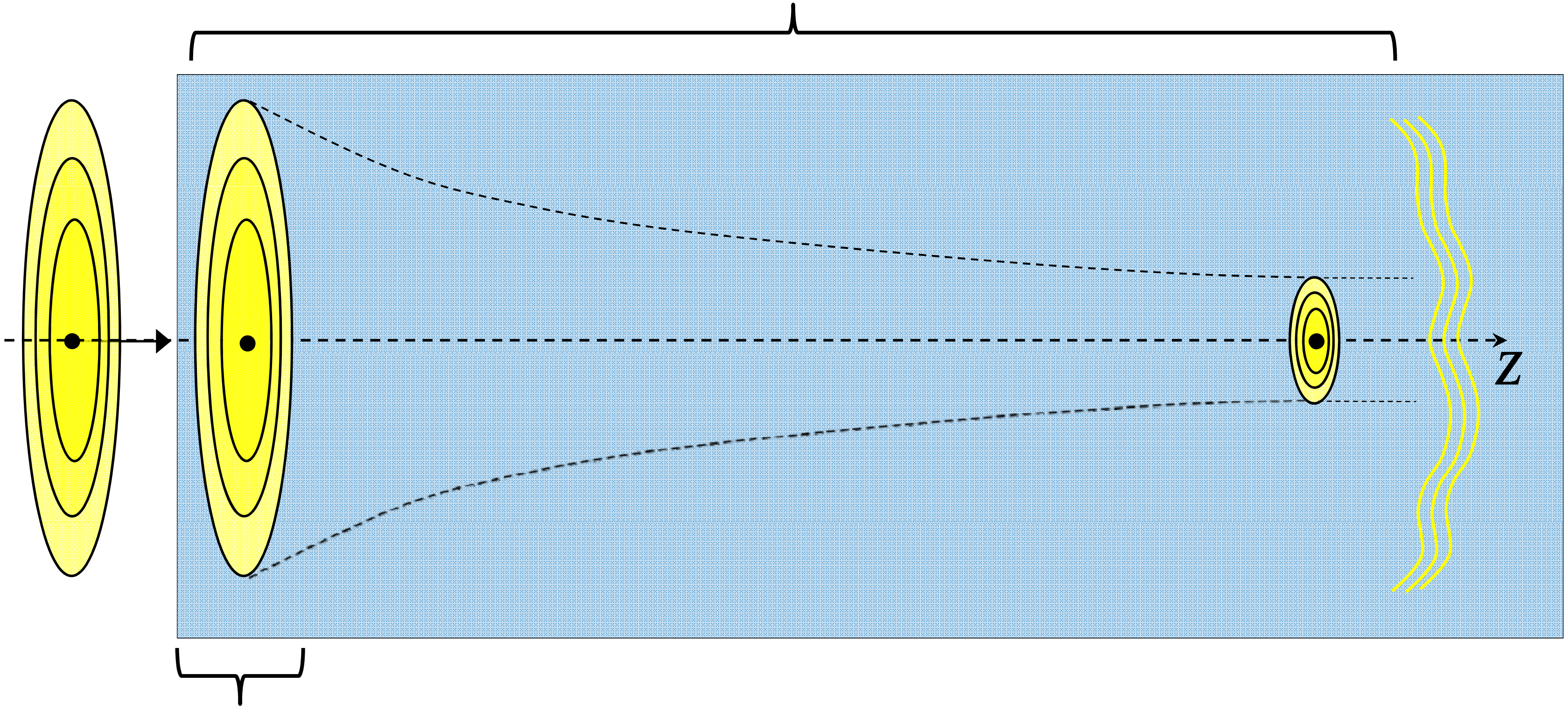
J.D. Jackson // Classical electrodynamics, 1999

ω_0 – oscillator’s own frequency

Total ionization per unit path:

$$\frac{d\mathcal{E}}{dz} = n\frac{e^2}{2m}\int_0^\infty d\rho 2\pi\rho \left|E_{\omega_0}^\rho(\vec{r})\right|^2$$

REBUILDING OF THE FIELD AND IONIZATION LOSSES



rebuilding of the field $L \approx \gamma^2 / I$

change of ionization losses

$L \approx I / \omega_p^2$ ≈ **absorption length**

I – mean ionization potential

IONIZATION ENERGY LOSSES OF ‘HALF-BARE’ ELECTRON

Assumption: $q_0 >> \omega_p >> I/\gamma$

For $z \to 0$

$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2}{v^2} \ln \frac{q_0}{\omega_p}$$

For $I/\omega_p^2 << z << 2\gamma^2/I$

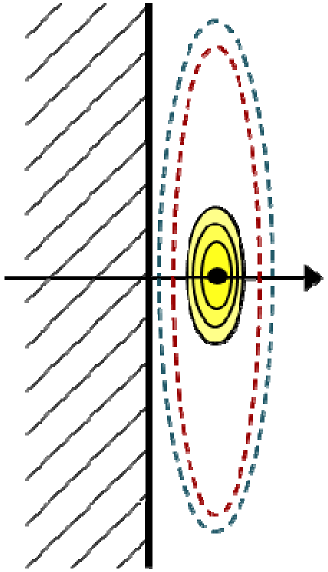
$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2}{v^2} \ln \frac{q_0 z \omega_p}{2I}$$

For $z \geq 2\gamma^2/I$

$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2}{v^2} \left\{ \ln \frac{q_0 v \gamma}{I} + \ln \frac{\omega_p v \gamma}{I} \right\}$$

$$F(\vartheta) = \frac{\omega_p^2}{\omega^2} \frac{\vartheta}{(\vartheta^2 + \omega_p^2/\omega^2 + 1/v^2\gamma^2)(\vartheta^2 + 1/v^2\gamma^2)}$$

EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM AT ULTRA HIGH ENERGIES



Total field:

$$\vec{E} = \vec{E}^{C} + \vec{E}^{F}$$

Boundary conditions:

$$\left.\begin{aligned} \left(E_{\rho}^{C} + E_{\rho}^{F}\right)^{diel}\Big|_{z=0} &= \left(E_{\rho}^{C} + E_{\rho}^{F}\right)^{vac}\Big|_{z=0} \\ \left(E_{z}^{C} + E_{z}^{F}\right)^{diel}\Big|_{z=0} &= \widehat{\varepsilon}\left(E_{z}^{C} + E_{z}^{F}\right)^{vac}\Big|_{z=0} \end{aligned}\right\} \quad + \quad div\vec{E}^{F} = 0$$

Fourier expansion of the free field:

$$E_{\rho}(\vec{r},t) = \frac{e}{\pi\ v}\int_{0}^{\infty} dq q^{2} J_{1}(q\rho)\times$$

$$\times\int_{-\infty}^{+\infty} d\omega \frac{\sqrt{\omega^{2}-q^{2}}}{\sqrt{\omega^{2}\varepsilon-q^{2}}+\varepsilon\sqrt{\omega^{2}-q^{2}}}\left[\frac{1+\frac{v}{\omega}\sqrt{\omega^{2}\varepsilon-q^{2}}}{q^{2}+\frac{\omega^{2}}{v^{2}}-\varepsilon\omega^{2}}-\frac{\varepsilon+\frac{v}{\omega}\sqrt{\omega^{2}\varepsilon-q^{2}}}{q^{2}+\frac{\omega^{2}}{v^{2}}-\omega^{2}}\right]e^{i\sqrt{\omega^{2}-q^{2}}z-i\omega\ t}$$

$\vec{q}$ – component of vector $\vec{k}$ orthogonal to z axis

EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM AT ULTRA HIGH ENERGIES

Own Coulomb field of the particle:

$$E_\rho^C(\vec{r},t) = -\frac{\partial}{\partial\rho}\frac{e}{\sqrt{\rho^2\gamma^{-2}+(z-vt)^2}} = \frac{e}{\pi\ v}\int_{-\infty}^{\infty}d\omega\int_0^{+\infty}dq\ \frac{q^2J_1(q\rho)}{q^2+\frac{\omega^2}{v^2\gamma^2}}e^{i\frac{\omega}{v}z}$$

Total field:

$$E_\rho = E_\rho^C + E_\rho^F$$

NIM B, Volume 268, Issue 9, 1 May 2010, Pages 1412–1415