Embedding of Cosmic Inflation

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with Bhaskar Dutta and Kuver Sinha
Inflation **Works**.
(Prof. Riazuelo)

Inflation brings **novel concepts**.

Inflation **raises the bar** on model building.
Inflation **Works.**

**Inflation brings novel concepts.**

(Prof. Mukhanov)

Inflation **raises the bar** on model building.
Inflation Works.

Inflation brings novel concepts.

Inflation raises the bar on model building.
(Prof. Nanopoulos)
Embedding Inflation into larger models is difficult.

There is no distinguished candidate model.

The landscape of possible vacua is the dark sector makes this a pressing issue.
Embedding inflation is a Complex Problem.

A Problem of Scales: The Kallosh-Linde Problem

A Problem of History: The Measure Problem
Embedding inflation is a **Complex** Problem.

A Problem of **Scales:** The Kallosh-Linde Problem

*structural classification of potentials*

A Problem of **History:** The Measure Problem

*“fixed point” in the inflaton dynamics*
Slow-Roll Inflation

Inflaton Trajectory versus $N$

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Embedding of Cosmic Inflation
Attractor Dynamics and the Scalar Potential

\[ t \to N = \log[a(t)] \]

\[ \phi'' = \frac{1}{2}(\phi' + \sqrt{6})(\phi' - \sqrt{6})(\phi' + \frac{\partial \log V}{\partial \phi}) \]

\[ \phi' \approx -\frac{V_\phi}{V} \]

\[ \frac{V_{\phi\phi}}{V} - \left(\frac{V_\phi}{V}\right)^2 \approx 0 \]
Attractor Dynamics and the Scalar Potential

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“Inflection Point” Inflation

The field can start in some configuration.
“Inflection Point” Inflation

Small-field models inflate during a small field excursion
“Inflection Point” Inflation

It hits a minimum and reheating occurs.
The Kallosh-Linde Problem
The Kallosh-Linde Problem

Tension between high scale inflation and weak scale supersymmetry in string compactifications
Horrendously Complicated Potentials

\[ V = \frac{A^2 e^{2f - \frac{4\pi \Phi}{m}} \pi}{m \phi^2} + \frac{AB e^{2f - \frac{2\pi \phi}{m} - \frac{2\pi \phi}{n}} \pi}{m \phi^2} + \frac{B^2 e^{2f - \frac{4\pi \phi}{n}} \pi}{n \phi^2} + \frac{AB e^{2f - \frac{2\pi \phi}{m}} \pi}{m \phi^2} + \frac{2A^2 e^{2f - \frac{4\pi \phi}{m}} \pi^2}{3m^2 \phi} + \frac{2B^2 e^{2f - \frac{4\pi \phi}{n}} \pi^2}{3n^2 \phi} + \frac{4AB e^{2f - \frac{2\pi \phi}{m}} \pi^2}{3mn \phi} + \frac{A e^{f - \frac{2\pi \phi}{m}} \pi W_0}{m \phi^2} + \frac{B e^{f - \frac{2\pi \phi}{n}} \pi W_0}{n \phi^2} + \frac{4C}{\phi^3} \]
Racetrack Scalar Potential

Pre and Post Uplifting Scalar Potential

(Conlon, Quevedo), (Blanco-Pillado, et al.), (Cicoli, et al.),
(Kallosh, Linde), (Olechowski, et al.), (Allahverdi, Dutta, Sinha)...

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Embedding of Cosmic Inflation
The Kallosh-Linde Problem

Start at the quartic point...

Scalar Potential
The Kallosh-Linde Problem

... watch a cubic point appear ...

Scalar Potential
The Kallosh-Linde Problem

... Inflation scale rises as the barrier falls ...

Scalar Potential
The Kallosh-Linde Problem

... Inflation scale rises as the barrier falls ...

Scalar Potential
The Kallosh-Linde Problem

... Inflation scale rises as the barrier falls ...

Scalar Potential
The Kallosh-Linde Problem

... until the barrier disappears.

Scalar Potential
Structurally equivalent to something far simpler

\[ V = \frac{1}{5} \phi^5 + \frac{1}{3} a\phi^3 + \frac{1}{2} b\phi^2 + c\phi + \text{constant} \]
The $A_4$ (Swallowtail) Germ

Various Domains of Swallowtail Parameter Space ($a=-1$)

(Thom), (du Val), (Arnold), ...
The Kallosh-Linde Problem Revisited

Start at the (quartic) $A_3$ Point...

$A_4$ Parameter Space

Scalar Potential
The Kallosh-Linde Problem Revisited

... watch the (cubic) $A_2$ point appear ...
The Kallosh-Linde Problem *Revisited*

... Inflation scale rises as the barrier falls ...

![Diagram](image.png)
The Kallosh-Linde Problem Revisited

... Inflation scale rises as the barrier falls ...
The Kallosh-Linde Problem Revisited

... Inflation scale rises as the barrier falls ...
The Kallosh-Linde Problem *Revisited*

... until the barrier disappears.
ADE Classification of Inflection Point Models

Structural classification based on V.I. Arnold’s Theory of Singularities

**MSSM-inflation:** improved tuning by twelve orders of magnitude

(Allahverdi,SD,Dutta ’11)

**IIB Racetrack-inflation:** sketched a measurement scheme for extracting potential data from the CMB/LHC

(SD,Dutta,Sinha ’11).
The Measure Problem
Initial Conditions? Exponential Supression? $\mathcal{P} = e^{-3N}$

(Gibbons,Hawking,Stewart),(Gibbons,Turok)

**Dynamical fixed point** property of inflection point inflationary trajectories (Itzhaki,Kovetz)

Allowing couplings to vary leads to $\mathcal{P} = 1/N^3$.

(SD,Dutta,Sinha)

Good **agreement** with Monte Carlo analysis!

(Liddle,et. al.),(McAllister et. al. ’11)
\[ \mathcal{A}_3 : V = \frac{1}{4} \chi^4 + \alpha \chi^3 + \frac{27}{4} \alpha^4 \]

(SD, Dutta, Sinha)
Field Trajectory in a $D_5$ Potential
Conclusion

Kallosh-Linde Problem:
structural classes for inflection point inflation

Measure Problem:
important contribution of inflection point models

Prospects:
Curious but nontrivial similarity with Starbobinsky-like models. Are there “Universal” properties beyond inflection points? Will this aid in vacuum selection? Something new to look for?
Structural Class Representative Potentials

\( A_k : x^{k+1} + \ldots \)
\( D_k : x^{k-1} + y^2 x + \ldots \)
\( E_6 : x^4 + y^3 + \ldots \)
\( E_7 : y^3 + yx^4 + \ldots \)
\( E_8 : x^5 + y^3 + \ldots \)
Racetrack Inflation in IIB Flux Compactifications

(Complicated Hidden sector model)

Effective 4D $\mathcal{N} = 1$ Supergravity

Model Data

\[
K = -3 \log(T + \bar{T})
\]
\[
W = W_0 + Ae^{-\frac{2\pi}{m} (T-f)} + Be^{-\frac{2\pi}{n} (T-f)}
\]
\[
V = e^K \left( |DW|^2 - 3|W|^2 \right)
\]
The Kallosh-Linde Problem Revisited

Separating the scales of Inflation and SUSY Breaking REQUIRES

Large critical point separations

\[ H_{\text{inf}}^2 \propto \zeta_1^4 \zeta_2 \quad m_{3/2}^2 \propto \zeta_1 \zeta_2 \left( \zeta_1^3 - \zeta_2^3 \right) \]

A\textsubscript{4} Scalar Potential
Trigger?

Mirage Mediation... (Choi et al., '04, '05, '06) (Dutta, Kamon, et al. '11)

...Gaugino masses unify early

\[ a_{\text{mir}} \propto \zeta_1 \]

Ratio of the effects of moduli and anomaly mediation...

\[
\left( a_{\text{mir}} = \frac{m_{3/2}}{M_{\text{mod}} \log(M_P/m_{3/2})} = \frac{(\zeta_1 + \alpha - f)}{32} \right)
\]
Connection with SUSY Breaking Soft Terms

Density Perturbations

\[ \Delta_R^2 \propto \alpha^6 \zeta_1^6 \zeta_2^3 \]

Gaugino Soft Mass

\[ M = \frac{8\pi |\Delta_R| \alpha^3}{3N_e^2 \zeta_1 \zeta_2 (\zeta_1 + \alpha - f)} \left( 1 + a_{\text{mir}} \frac{bg^2}{16\pi^2} \log\left( \frac{M_P}{m_{3/2}} \right) \right) \]
Any overshoot problem? No.
The Itzhaki-Kovetz Fixed Point

(Itzhaki-Kovetz ’08)

\[ V = \beta \phi^3 + 1 \]

\[ \beta_c \approx 0.774 \]
The Slow Roll Trajectory:

\[ \dot{\phi} \approx -\frac{V_{\phi}}{V} \]

\[ V = \beta (\phi^3 + 1/\beta) \]

Quartic Potential:

\[ V = \frac{1}{4} \phi^4 + \alpha \phi^3 + \frac{27}{4} \alpha^4 \]

\[ \alpha_c \approx 0.661. \]

Even More Gory Details: S.D., Dutta, Sinha:

“Attractors, Universality and Inflation” (1203.6892)
\( \alpha_c \) is just the "tip of the iceberg"
This fact informs **likelihood** of inflation

Enlarge the **Gibbons-Hawking-Stewart** ('87) ensemble to include **all possible couplings**.

**Gibbons-Turok** ('08) likelihood of $e^{-3N_e}$ is enhanced to...

$$1/N_e^3$$

A power law. In agreement with a recent Monte Carlo analysis of D-brane inflation.

(Agarwal et.al 2011)
Low Power at Large Scales

(SD, Dutta ’12)

\[ \Xi = \frac{\phi''}{\phi'} \]

Evolution of the \( \Xi \) parameter
Low Power at Large Scales

Fibre Inflation–like Potentials

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Embedding of Cosmic Inflation
Low Power at Large Scales

\[ 2l(l+1)C_l \]

Low modes of Fibre Inflation

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Embedding of Cosmic Inflation
Low Power at Large Scales

Low modes of Fibre Inflation, mk2

$2l(l+1)C_l$
Inflation along a **D-Flat** Direction in the MSSM

(Gherghetta, Kolda, Martin’96)

\[
e_{i}L_{j}L_{k} \quad \text{or} \quad u_{d_{m}}d_{n}
\]

no fundamental gauge singlets required.
Lifted by higher dimensional operators.

\[
W \sim \frac{\Phi^{3n}}{M_{P}^{3(n-1)}}
\]
F-Terms from $W = \lambda_1 \phi^3 + \lambda_2 \frac{\phi^6}{M_p^3} + \lambda_3 \frac{\phi^9}{M_p^6}$

... Give a scalar potential.

$V = |\phi|^4 \left| \lambda_1 + \lambda_2 \phi^3 + \lambda_3 \phi^6 \right|^2$
The tuning here is

\[ \lambda_2^2 = \frac{64}{25} \lambda_1 \lambda_3, \]

\[ \frac{\delta \rho}{\rho} \sim 10^{-5} \Rightarrow \lambda_1 \left( \frac{16 \lambda_3}{5 \lambda_2} \right)^{1/3} \sim 10^{-8} \]

so

\[ \lambda_1 < 10^{-8} \]

Smallness of \( \lambda_1 \) technically **natural**, tied to neutrino sector?

Here, \( H \gg \mathcal{O}(\text{TeV}). \)