Real-time Yang-Mills Simulation and the Thermalization Problem at RHIC and LHC

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Relativistic Heavy-Ion Collision

RHIC

LHC

Heavy-ions collide $\rightarrow$ A new state of matter
(Au, Pb, …)
(Quark-gluon plasma)
Complexity $\rightarrow$ Simplicity

A system of heavy ions is very much complicated.

Complicated dynamics of strong interactions leads to
→ Thermalization
→ Characterized with macroscopic variables
  (Equation of state, $T$, $\mu$, …)

Success of the hydrodynamical models to understand experimental observables (i.e. elliptic flow, etc).

**Thermalization achieved at $\tau \sim 0.5\text{fm/c}$**
How?

Perturbative QCD – No chance
Instabilities? – Maybe

Turbulence (Turbulent Flow)

\[ \text{Re} = \frac{(\text{velocity}) \times (\text{size})}{(\text{kinetic viscosity})} \]
The central problem that people need to solve is not the problem of elementary particles or unified field; that is the problem of turbulence... the last great unsolved problem of classical physics.

Feynman (1969)
**Schematic View of Four Regimes**

- **Soft and coherent gluons**
  - Color Glass Condensate (CGC)
  - Initial (quantum) fluctuations

- **Instabilities → Isotropization**
  - Color Glass + Plasma = Glasma
  - Quantum fluctuations
  - Particle (entropy) production

- **→ Thermalization**

- **Hydrodynamic evolution + cascade**
  - Relativistic Hydrodynamics

- **Hadronization → Observation**
  - Particle yields, distributions

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Important Scales in QCD

\[ \Lambda_{\text{QCD}} \quad - \quad \text{perturbative or non-perturbative} \]

\[ Q_s(x) \quad - \quad \text{linear or non-linear} \quad (x \sim t / s) \]

\[ Q_s \sim 2 \text{GeV at RHC} \]
\[ \sim 5 \text{GeV at LHC} \]

Transverse size of a parton \( \sim 1/Q \)
Non-linearity → Simplicity

Saturation or non-linearity is realized when

\[ A \sim \frac{1}{g} \]

then we cannot treat \( gA \) as perturbation, and need to resum an infinite number of diagrams.

→ Solving the “classical” equations of motion

\[ D_\mu F^{\mu\nu} = j^\nu \]

→ Physical quantities scale with \( Q_s(x) \)

Geometric Scaling
Initial Condition

Fields made by two colliding sources

Initial condition is known on the light-cone

\[ A_i = \alpha_i^{(1)} + \alpha_i^{(2)} \]
\[ A_z = 0 \]
\[ \mathcal{E}^i = 0 \]
\[ \mathcal{E}^z = ig \left[ \alpha_i^{(1)}, \alpha_i^{(2)} \right] \]

Initial Condition ~ Coherent Fields

Lappi-McLerran (2006)

* Boost Invariant
* Coherent Fields
  (amp. ~ 1/g)
* Flux Tube
  (size ~ 1/Q_s)

* Expanding

Force from the tube should be overcome

McLerran-Lappi (2006)
Topological Effects

Parallel $E$ and $B$ $\rightarrow$ Topological Charge Density $\rightarrow$ Effective $\theta(x)$ (strong $\theta$ angle)

Local Parity Violation  (detectable through topological effects)
Chromo-Electric and Magnetic Fields

Longitudinal and Transverse Fields

\[
\frac{g^2 E_{L,T}^2}{(g^2 \mu)^4}, \quad \frac{g^2 B_{L,T}^2}{(g^2 \mu)^4} < \frac{1}{Q_s}
\]

\[
\sim 0.1 \text{ fm/c}
\]

free-streaming

\[
g = 2
\]

\[
g^2 \mu a = 120 / L
\]

Lappi-McLerran (2006)
Fukushima-Gelis (2011)

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**Longitudinal and Transverse Pressure**

\[
P_T = \frac{1}{2} \left\langle T^{xx} + T^{yy} \right\rangle = \left\langle \text{tr} \left[ E_L^2 + B_L^2 \right] \right\rangle,
\]

\[
P_L = \left\langle \tau^2 T^{mn} \right\rangle = \left\langle \text{tr} \left[ E_T^2 + B_T^2 - E_L^2 - B_L^2 \right] \right\rangle.
\]

(A)lmost free-streaming

Isotropization

\[
P_T = P_L
\]

Fukushima-Gelis (2011)
Negative Longitudinal Pressure

Flux tubes have a positive energy

Attractive Force

Flux tubes have a positive energy
Classical Statistical Simulation

Boost Invariant $E$ and $B$

Classical Dynamics + (Quantum) Fluctuations

Schwinger mechanism Hawking radiation

Same theoretical formulation as the dynamics in the Early Universe
Effects of (Minimal) Fluctuations

$$\delta E^i{}^a = \overline{E}^i{}^a \cos \left( 2 \pi \frac{z}{La} \right)$$

$$\delta E^z{}^a$$ from the Gauss law

64×64×128
**Spatial Structure with $k_z$**

$$\epsilon(x, k_z) = \int dy \left\langle E(x, y, -k_z) E(x, y, k_z) + B(x, y, -k_z) B(x, y, k_z) \right\rangle$$
Dynamical Pattern Formation

Large amplitude at non-zero $k_z$ spatially localized

$g^2 \mu t = 0.1$

$g^2 \mu t = 10$

$g^2 \mu t = 20$

$g^2 \mu t = 30$

$g^2 \mu t = 40$
Similarity to Magnetization

Spontaneous pattern formation from “uniform” to “non-uniform” distribution of coherent fields

Movie of dynamical pattern formation in YM

Spontaneous pattern formation from “ordered” to “disordered” state in spin systems (decoherence)

Movie of dynamical pattern formation in spin system
Still, the numerical simulation cannot fully explain the time scale of the thermalization... but...
Summary

Early-time evolution of the relativistic heavy-ion collision is to be investigated in the classical statistical simulation of the Yang-Mills theory.

Spontaneous pattern formation in the transverse plane has been observed in an analogous way to the magnetization decoherence.

Not fast enough to explain the thermalization – a first step toward turbulence in the YM

Full quantum fluctuations? Renormalization?