Perturbation Theory in Flat FLRW Cosmology and Beyond

Alex Leithes

Why use flat FLRW (Friedmann-Lemaître-Robertson-Walker) Cosmology?

- Problems of Hot Big Bang Model
  - Flatness: Latest Planck data - flat within 0.2%
  - Early deviation from flatness rapidly grows. Why so flat now?
  - Horizon: Visible horizon/Hubble radius early times << area visible today
    e.g. CMB ≈ 1deg << whole sky
  - Flatness: Accelerated expansion rapidly drives universe towards flat.
  - Horizon: Universe inflates - superluminal expansion
    Small region in thermal equilibrium inflated beyond horizon.
  - Inflation: Flattens, Causally connects.

- Homogeneous versus Inhomogeneous Cosmologies and Inflation
  - Large Scale Structure: Beyond ≈ 100 Mpc, homogeneous.
  - CMB: No temperature variation > 10^{-5} K, homogeneous.
  - Clusters and voids: small scale inhomogeneities.
  - Temperature Fluctuations (∼ 10^{-5}): small scale inhomogeneities.
  - Inflation driven by: simplest - scalar field, called Inflaton.
  - Quantum fluctuations seed perturbations.

Figure: Universe timeline, including inflationary period - ESA

Figure: Left: ESA Planck CMB δT map - Right: SDSS(III) Cluster and Void survey

Perturbation Theory in flat FLRW Cosmology.

Scalar Field Inflation and Perturbations in Brief

- Power Spectrum of scalar field perturbations,
  \[ P_S(k) = \left( \frac{H}{2\pi} \right)^2 \left( \frac{H}{2\pi} \right)^2 \bigg|_{k=\alpha H} \]

- Related to Power Spectrum Scalar (density) perturbations,
  \[ P_S(k) = \left( \frac{H}{2\pi} \right)^2 \left( \frac{H}{2\pi} \right)^2 \bigg|_{k=\alpha H} \]

- Metric:
  \[ ds^2 = -(1 + 2\phi) dt^2 + 2aB_i dx^i dt + a^2 (\delta_{ij} + 2C_{ij}) dx^i dx^j \]
  Note: \( C_{ij} \) may be further decomposed: \( C_{ij} = E_{ij} \delta_{ij} \)

- 4-velocity: \( u^\mu = \left[ 1 - \phi \right] \frac{\nu^\mu}{a} \]

- Energy Density and Pressure:
  \[ \rho = \rho_{(0)} + \delta \rho, \quad P = P_{(0)} + \delta P \]

- Perturbed portion of Energy Conservation \( \rightarrow \)
  \[ \partial_t \delta \rho + 3H (\delta \rho + \delta P) + (\delta v^i \nabla^2 + \delta \nabla^i) \left( \rho_{(0)} + P_{(0)} \right) = 0 \]

- Curvature perturbation \( \zeta = -\psi - H \frac{\nu^i}{\rho_{(0)}} \)
  \[ \therefore \zeta = -\phi \bigg|\bigg|_{\delta p=0} \]
  and is defined on uniform density hypersurfaces.

- Uniform density curvature perturbation, \( \zeta \), conserved for barotropic fluid, on large scales.

Why Perturb Cosmologies beyond flat FLRW (my goal)?

- LTB as an alternative cosmology
  - Spherically symmetric spacetime.
  - FLRW is a subclass of LTB.
  - Unperturbed Metric:
    \[ ds^2 = -dt^2 + X^2(r,t)dr^2 + Y^2(r,t)ds^2 + \left( \frac{d\theta^2}{\sin^2\theta} \right) + \left( d\phi^2 + d\psi^2 + d\omega^2 \right) \]

- Two scale factors, not independent.
- Seeking to apply perturbation theory at linear order to LTB.
- To recover Perturbed Conservation Equations for LTB - is there a \( \zeta \) in LTB?
- More general than flat FLRW.
- Only perturbations around flat FLRW background well understood.
- Planck 2013 results: North-south CMB power asymmetry - quadrupole/octopole alignment fuelled interest in inhomogeneous cosmologies
- Void models, structure formation - e.g. Sussman 2013 intro reviews field.
- Deeper understanding of FLRW.