"Universe In Problems" project

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Outline

Universe in Problems project

There are thousands of websites with names that contain the term **cosmology**. Many of them are devoted to discussion of fundamental questions: whether there is life on Mars, what was there when there was nothing and the like. Our aim is much more modest. We present here an online living version of our book of problems on cosmology.

The only way to rise above the **popular** level in any science is to master its alphabet, that is, to learn to solve problems, even if most simple at the beginning. To our best knowledge, there are no problem books on cosmology yet, that would include its spectacular recent achievements. Of course, most of excellent modern textbooks on cosmology include problems. However, a reader, exhausted by high theory, may often be thwarted by the lack of time and strength to solve them.

Finally, we hope that after getting to grip with the problems our reader not only preserves his interest to cosmology, but tries to make the next step: read original papers. If this transition is overcome seamlessly, we will have achieved our goal. **Welcome!**

www.universeinproblems.com

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The total number of the problems on this website:

1165

Cosmo warm-up

This is an introductory chapter, intended to remind the readers of the quantities and concepts to be used later in the actual problems. The scope is too large, so you will not learn a subject if you don't know it yet, but this may help you have some fun while recalling what you already know and provides a heads-up for some of the more interesting, sometimes controversial, questions.

If the Universe was infinitely old and infinitely extended, and stars could shine eternally, then in whatever direction you look the line of your sight should cross the surface of a star, and therefore all the sky should be as bright as the Sun's surface. This notion is known under the name of Olbers' paradox.

Let n be the mean density of stars in the Universe and L be the mean luminosity of stars. The observed luminosity (energy flow's surface density) on the Earth from a star with luminosity L at distance r is

$$F(r) = \frac{L}{4\pi r^2}.$$

Consider a spherical shell of radius r and thickness dr centered at Earth. The radiation intensity of stars inside this shell (the power which reaches a unit surface from one steradian) is

$$dP(r) = F(r) \cdot n \cdot r^2 dr = \frac{nL}{4\pi} dr.$$

It is important to note that total intensity of radiation from the shell is independent on the distance to it. Thus, the total intensity of radiation from all stars in the Universe

$$P = \int_0^\infty dP = \frac{nL}{4\pi} \int_0^\infty dr$$

diverges in the case of infinite stationary Universe.

Olbers' paradox is an example of the so-called "law of incorrect naming", which states that no law is called after a person who in fact discovered it. This paradox had been known 150 years before Olbers' formulated it (Diggers (1576)).

Dynamics of the Expanding Universe

Contemporary cosmology is based on understanding of the dynamics of the Universe, in terms of the General Theory of Relativity, under the strong simplifying assumptions of global homogeneity and isotropy. Here we deal with general questions, from the derivation of the FLRW metric to understanding of its dynamics with its many variants and (apparent) paradoxes, but without restricting ourselves to specific material content of the Universe. The second part provides a digest of General Relativity, which is necessary to know in order to deal with the majority of the whole collection of problems.

As interesting example of this section let us consider Cosmography paragraph.

In the following problems we use an approach to the description of the evolution of the Universe, which is called "cosmography"*. It is based entirely on the cosmological principle and on some consequences of the equivalence principle. The term "cosmography" is a synonym for " cosmo-kinematics". Let us recall that kinematics represents the part of mechanics which describes motion of bodies regardless of the forces responsible for it. In this sense cosmography represents nothing else than the kinematics of cosmological expansion."

In order to construct the key cosmological quantity $a(t)$ one needs the equations of motion (the Einstein's equation) and some assumptions on the material composition of the Universe, which enable one to obtain the energy-momentum tensor. The efficiency of cosmography lies in the ability to test cosmological models of any kind, that are compatible with the cosmological principle. Modifications of General Relativity or introduction of new components (such as dark matter or dark energy) certainly change the dependence $a(t)$, but they absolutely do not affect the kinematics of the expanding Universe.

The rate of Universe's expansion, determined by Hubble parameter $H(t)$, depends on time. The deceleration parameter $q(t)$ is used to quantify this dependence. Let us define it through the expansion of the scale factor $a(t)$ in a Taylor series in the vicinity of current time t_0:"

$$a(t) = a\left(t_0\right) + \dot{a}\left(t_0\right)\left[t - t_0\right] + \frac{1}{2}\ddot{a}(t_0)[t - t_0]^2 + \cdots$$

Let us present this in the form

$$\frac{a(t)}{a\left(t_0\right)} = 1 + H_0\left[t - t_0\right] - \frac{q_0}{2}H_0^2[t - t_0]^2 + \cdots$$

where the deceleration parameter is

$$q(t) \equiv -\frac{\ddot{a}(t)a(t)}{\dot{a}^2(t)} = -\frac{\ddot{a}(t)}{a(t)}\frac{1}{H^2(t)}.$$

Note that the accelerated growth of scale factor takes place for $q < 0$. When the sign of the deceleration parameter was originally defined, it seemed evident that gravity is the only force that governs the dynamics of Universe and it should slow down its expansion. The choice of the sign was determined then by natural wish to deal with positive quantities. This choice turned out to contradict the observable dynamics and became an example of historical curiosity.

In order to describe the kinematics of the cosmological expansion in more detail it is useful to consider the extended set of the parameters:

$$H(t) \equiv \frac{1}{a}\frac{da}{dt}$$

$$q(t) \equiv -\frac{1}{a}\frac{d^2a}{dt^2}\left[\frac{1}{a}\frac{da}{dt}\right]^{-2}$$

$$j(t) \equiv \frac{1}{a}\frac{d^3a}{dt^3}\left[\frac{1}{a}\frac{da}{dt}\right]^{-3}$$

$$s(t) \equiv \frac{1}{a}\frac{d^4a}{dt^4}\left[\frac{1}{a}\frac{da}{dt}\right]^{-4}$$

$$l(t) \equiv \frac{1}{a}\frac{d^5a}{dt^5}\left[\frac{1}{a}\frac{da}{dt}\right]^{-5}$$

Dynamics of the Universe in the Big Bang Model

There is a saying also that "cosmologists are often mistaken, but never in doubt". The model of the Hot Universe, however, though modified, still lies at the heart of modern cosmology.

Does the expansion of space mean that everything in it is stretched? Galaxies? Atoms? A shallow answer to this question is: "bounded" systems do not take part in the expansion. However, if space is stretched, then how can these systems not experience some, at least minimal, extension? Should bounded systems be stretched less intensively? In the **Influence of cosmological expansion on local systems** section are several problems attempt to clarify the question by the example of a simple model: a classical atom, which consists of a negatively charged electron with negligible mass, rotating around a positively charged massive nucleus.

Black Holes

In the last couple of decades black holes have turned from a very beautiful but equally abstract (mostly) theoretical concept to an object generally believed to constitute a very distinctive and important part of the observed Universe. On the other hand, study of black holes lies at the junction of General Relativity and quantum world, and led us to the holographic principle. This makes them a common area to a very wide field of research, including classical and quantum gravity, cosmology and astrophysics.

There are following paragraphs in this section:

Cosmic Microwave Background and Thermodynamics of Universe

In this section we consider such problems and questions as: **thermodynamics of Black-Body Radiation, time Evolution of CMB, statistical properties of CMB, primary anisotropy of CMB, CMB interaction with other components, thermodynamical properties of elementary particles, thermodynamics of non-relativistic gas, entropy of Expanding Universe, Connection between Temperature and Redshift, Peculiarities of Thermodynamics in Early Universe, The Saha equation, Primary Nucleosynthesis and etc.**

Dark Energy

The observed accelerated expansion of the Universe requires either modification of General Relativity or existence within the framework of the latter of a smooth energy component with negative pressure, called the "dark energy". This component is usually described with the help of the state equation $p = w\rho$. As it follows from Friedman equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p).$$

For cosmological acceleration it is needed that $w < -1/3$. The allowed range of values of w can be split into three intervals. The first interval $-1 < w < -1/3$ includes scalar fields named the quintessence. The substance with the state equation $p = -\rho$ $(w = -1)$ was named the cosmological constant, because $\rho = const$: energy density does not depend on time and is spacially homogeneous in this case. Finally, scalar fields with $w < -1$ were called the phantom fields. Presently there is no evidence for dynamical evolution of the dark energy. All available data agree with the simplest possibility - the cosmological constant. However the situation can change in future with improved accuracy of observations. That is why one should consider other cases of dark energy alternative to the cosmological constant.

Dark Matter

In the beginning of thirties of the last century a Swiss cosmologist F. Zwicky applied the virial theorem (in the gravitational field $2\langle E_{kin}\rangle + \langle E_{pot}\rangle = 0$) in order to estimate the mass of the Coma cluster (Berenice's Hair). He was surprised to discover that in order to support the finite motion of the galaxies belonging to the cluster, its mass must be at least two orders of magnitude greater than the observed mass (in the form of luminous galaxies). He was the first to introduce the term "**dark matter**" which strongly entered the vocabulary of modern cosmology. At present the term is understood as the non-baryon matter component which neither emits nor absorbs electromagnetic waves in any range.

We discuss such themes as Observational Evidence of the Dark matter Existence, Dark Matter Halo, Candidates for Dark Matter Particles, Dark Matter Detection, The Dark Matter in the Solar System, The Dark Stars and **Interactions in the Dark Sector.**

Holographic Universe

The traditional point of view assumed that dominating part of degrees of freedom in our World are attributed to physical fields. However it became clear soon that such concept complicates the construction of Quantum Gravity: it is necessary to introduce small distance cutoffs for all integrals in the theory in order to make it sensible. As a consequence, our world should be described on a three-dimensional discrete lattice with the period of the order of Planck length. Lately some physicists share an even more radical point of view: instead of the three-dimensional lattice, complete description of Nature requires only a two-dimensional one, situated on the space boundary of our World. This approach is based on the so-called "holographic principle". The name is related to the optical hologram, which is essentially a two-dimensional record of a three-dimensional object. The holographic principle consists of two main statements:

1. All information contained in some region of space can be "recorded" (represented) on the boundary of that region.
2. The theory, formulated on the boundaries of the considered region of space, must have no more than one degree of freedom per Planck area:

$$N \leq \frac{A}{A_{pl}}, \quad A_{pl} = \frac{G\hbar}{c^3}. \tag{1}$$

Thus, the key piece in the holographic principle is the assumption that all the information about the Universe can be encoded on some two-dimensional surface - the holographic screen. Such approach leads to a new interpretation of cosmological acceleration and to an absolutely unusual understanding of Gravity. The gravity is understood as an entropy force, caused by variation of information connected to positions of material bodies. More precisely, the quantity of information related to matter and its position is measured in terms of entropy. Relation between the entropy and the information states that the information change is exactly the negative entropy change $\Delta I = -\Delta S$. Entropy change due to matter displacement leads to the so-called entropy force, which, as will be proven below, has the form of gravity. Its origin therefore lies in the universal tendency of any macroscopic theory to maximize the entropy. The dynamics can be constructed in terms of entropy variation and it does not depend on the details of microscopic theory. In particular, there is no fundamental field associated with the entropy force. The entropy forces are typical for macroscopic systems like colloids and biophysical systems. Big colloid molecules, placed in thermal environment of smaller particles, feel the entropy forces. Osmose is another phenomenon governed by the entropy forces.

Probably the best known example of the entropy force is the elasticity of a polymer molecule. A single polymer molecule can be modeled as a composition of many monomers of fixed length. Each monomer can freely rotate around the fixation point and choose any spacial direction. Each of such configurations has the same energy. When the polymer molecule is placed into a thermal bath, it prefers to form a ring as the entropically most preferable configuration: there are many more such configurations when the polymer molecule is short, than those when it is stretched. The statistical tendency to transit into the maximum entropy state transforms into the macroscopic force, in the considered case - into the elastic force.

Let us consider a small piece of holographic screen and a particle of mass m approaching it. According to the holographic principle, the particle affects the amount of the information (and therefore of the entropy) stored on the screen. It is natural to assume that entropy variation near the screen is linear on the displacement Δx:

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x. \qquad (2)$$

The factor 2π is introduced for convenience, which the reader will appreciate solving the problems of this Chapter. In order to understand why this quantity should be proportional to mass, let us imagine that the particle has split into two or more particles of smaller mass. Each of those particles produces its own entropy change when displaced by Δx. As entropy and mass are both additive, then it is natural that the former is proportional to the latter. According to the first law of thermodynamics, the entropy force related to information variation satisfies the equation

$$F\Delta x = T\Delta S. \qquad (3)$$

If we know the entropy gradient, which can be found from (2), and the screen temperature, we can calculate the entropy force.

An observer moving with acceleration a, feels the temperature (the Unruh temperature)

$$k_B T_U = \frac{1}{2\pi}\frac{\hbar}{c}a. \qquad (4)$$

Let us assume that the total energy of the system equals E. Let us make a simple assumption that the energy is uniformly distributed over all N bits of information on the holographic screen. The temperature is then defined as the average energy per bit:

$$E = \frac{1}{2}Nk_B T. \qquad (5)$$

Equations (2)-(5) allow one to describe the holographic dynamics, and as a particular case - the dynamics of the Universe, and all that without the notion of Gravity.