Tractor beams, Cosmic strings, and related phenomena

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1. Introduction

Forces acting on a **scatterer**

- multichannel problem
- energy and momentum conservation

Tractor beam or **Negative radiation pressure (NRP)**: force on the scatterer acts in the opposite direction as the wave travels

Other approaches

- **gain media** Mizraki, Fainman, [1]
- **structured beam** scattered dominantly into the direction behind the scatterer, Sukhov, Dogariu [2]
- **Negative index materials** force opposite to wave number, but not to Poynting vector [3]
- **Higher harmonics** due to **nonlinearities**, Romańczukiewicz; Forgács, Lukács and Romańczukiewicz [4]

2. One dimensional examples

Two channels, u and d, for channel u:

$$F_u/A_u^2 = k_u(1+|R_{uu}|^2-|T_{uu}|^2)+k_d(|R_{du}|^2-|T_{du}|^2)\,, \qquad (1)$$

- R_{ij} reflection coefficient, incoming wave j into outgoing mode i
- T_{ij} transmission coefficient similarly
- k_u, k_d: wave numbers

Eq. (1) follows immediately by taking into account that a plane wave $A\exp(ikx)/\sqrt{k}$, carries momentum $k|A|^2$. We note that the energy flux of such a wave is $\omega|A|^2$.
Energy conservation: $\sum_i(|R_{ij}|^2+|T_{ij}|^2)=1$.

A macroscopic example

- Also applicable to electomagnetic wave propagation
- Material with $n_x > n_y$
- Propagation in z direction, modes: E_x, E_y **polarizations**
- Scatterer: same material rotated by $45°$ around z axis
- Incoming wave of energy flow 10^3 W/cm^2 = 10^7 W/m^2 in x polarization radiation pressure: -1.05 Pa
- For total reflection : 2.45 Pa

A birefringent platelet

- The same system for light scattering
- outer medium: ordered liquid crystal, $n_x > n_y$
- Scatterer: birefringent, axes $45°$ rotated wrt. axes of outer medium
- using data of 5CB, wavelenght 5893 Å, 0.1 mm thick scatterer
- NRP: x polarization -3.04×10^{-12}Pa m/V$|E_0|^2$
  y polarization, 2.40×10^{-12}Pa m/V$|E_0|^2$
- few percents of light pressure (and transparent)
- light pressure: $\sim 10\ \mu$Pa for sunlight

3. Force on the scatterer in 2D

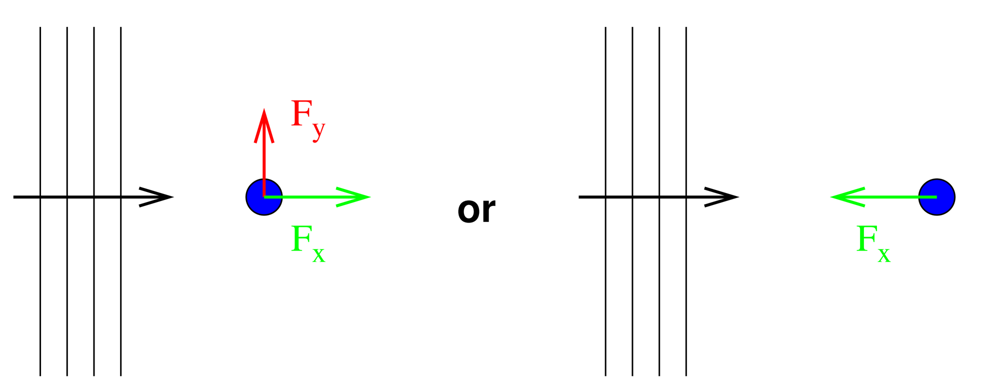

Figure 1: *Force components*

Force acting on the scatterer: **momentum flow**

$$\mathbf{F} = -\lim_{R\to\infty}\int_{-\pi}^{\pi}\mathbf{Te}_r R d\vartheta\,, \qquad (2)$$

assumptions: **free propagation** for $R\to\infty$. Inserting partial waves:

$$F = -4\sum_\ell\left\{A^\dagger S^\dagger_{\ell+1}KS_\ell A - A^\dagger KA\right\} \qquad (3)$$

where $F=F_x+iF_y$, $K=\mathrm{diag}(k_1,k_2)$: **wave numbers** of modes, $A=(A_1,A_2,\dots)^T$.

Also applicable for **Aharonov–Bohm scattering** [5]:

- $S_\ell=\exp(2i\delta_\ell)$, $\delta_\ell=\pi(\ell-\nu)/2$, $\nu^2=(\ell-A_0)^2$, where A_0 is **no. of flux quanta**
- reproduces known results: $F_x=-2k\left[\cos(2\pi A_0)-1\right]|A|^2$, $F_y=2k\sin(2\pi A_0)|A|^2$

4. Scattering off cosmic strings

Cosmic strings[6]

- String core: Higgs field zero
- $\Phi\sim v\exp(i\vartheta)$ (v: VEV, ϑ: angle)
- gauge covariant derivarive $\to 0$ for $r\to\infty$: $\mathbf{A}\propto\mathbf{e}_\vartheta/r$ (Aharonov-Bohm-like asymptotics)
- GUT string: vector potential inside string corresponds to broken gauge generator (X boson)

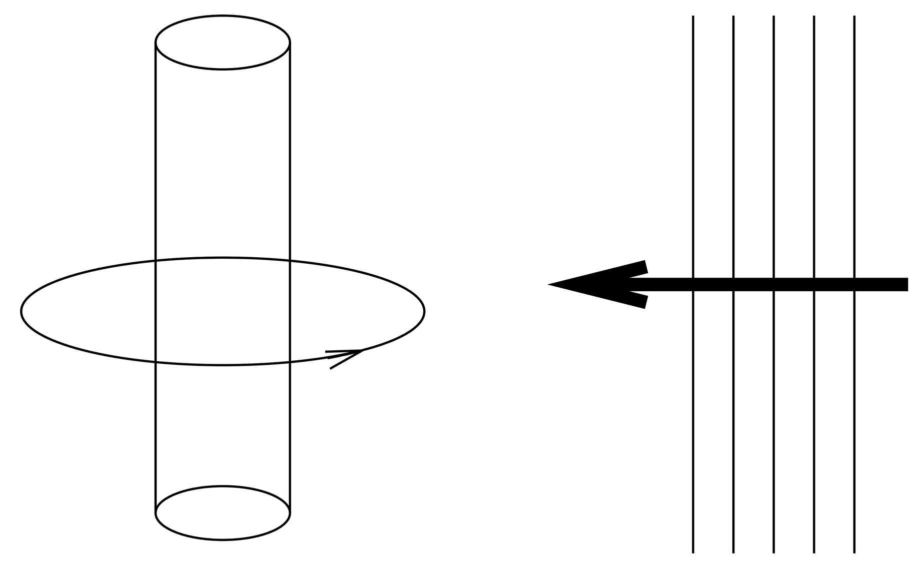

Figure 2: *Scattering off a vortex line/cosmic string*

Early Universe: friction dominated era of string network **Approximate force**: Aharonov–Bohm scattering [6]

- $F=-\sum_i 4n_i\mathbf{v}(1-\exp(2\pi i\nu_i\Phi))$
  where ν_i it the charge (coupling strength between particle species i and the X-boson)
- incoming flux, $|A_i|^2k_i=n_iv$, n_i is the number density of PS i

String induces baryon number violating processes[7]

$$B+\text{string}\to\ell+\text{string} \qquad (4)$$

Simplified model: ignore spins, one $U(1)$ gauge potential

$$\left(\nabla+i\mathbf{A}\frac{\sigma_2}{2}\right)^2\rho-K^2\rho=0\,,\quad \sigma_2=\begin{pmatrix} & -i\\ i & \end{pmatrix}\,, \qquad (5)$$

where

- $\mathbf{A}=\mathbf{e}_\vartheta/r$ gauge potential of the string
- $\rho=(u,d)$ wave function of two species, u heavy, d light
- fermionic boundary conditions $\rho(r,\vartheta+2\pi)=-\rho(r,\vartheta)$
- units: $m_u=2$, $m_d=1.5$, $\hbar=c=1$

Partial waves:

$$(u,d)=\sum_{\ell=-\infty}^{\infty}\mathrm{e}^{i(\ell+\gamma)\vartheta}(u_\ell(r),d_\ell(r))\,, \qquad (6)$$

with $\gamma=1/2$. The radial functions satisfy

$$\begin{aligned} u_\ell''+\frac{u_\ell'}{r}-\frac{\eta_u^2}{r^2}u_\ell+\frac{c}{r^2}d_\ell+k_u^2u_\ell&=0\,,\\ d_\ell''+\frac{d_\ell'}{r}-\frac{\eta_d^2}{r^2}d_\ell+\frac{c^*}{r^2}u_\ell+k_d^2d_\ell&=0\,, \end{aligned} \qquad (7)$$

where $\eta_u^2=\eta_d^2=(\ell+1/2)^2+1/4$ and $c=i(\ell+1/2)$.

5. Force acting on the cosmic string

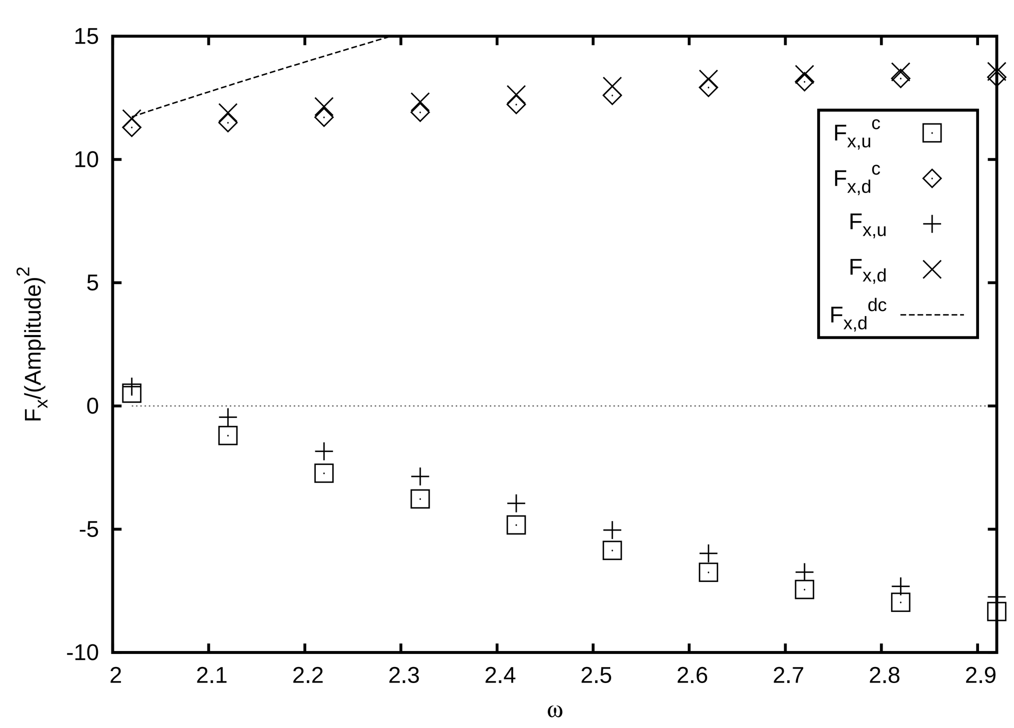

Figure 3: *The longitudinal force component F_x as a function of the frequency ω: the numerical result is compared to the decoupled, (dc) approximation, the latter only depicted for $2<\omega\le2.3$. For the incoming heavy mode u, the radiation pressure becomes negative at $\omega\approx2.1557$. The effect of the vortex core is also plotted: the longitudinal force F_x acting on a point vortex and on a vortex with a linear core (marked with 'c'). The existence of a vortex core enhances NRP.*

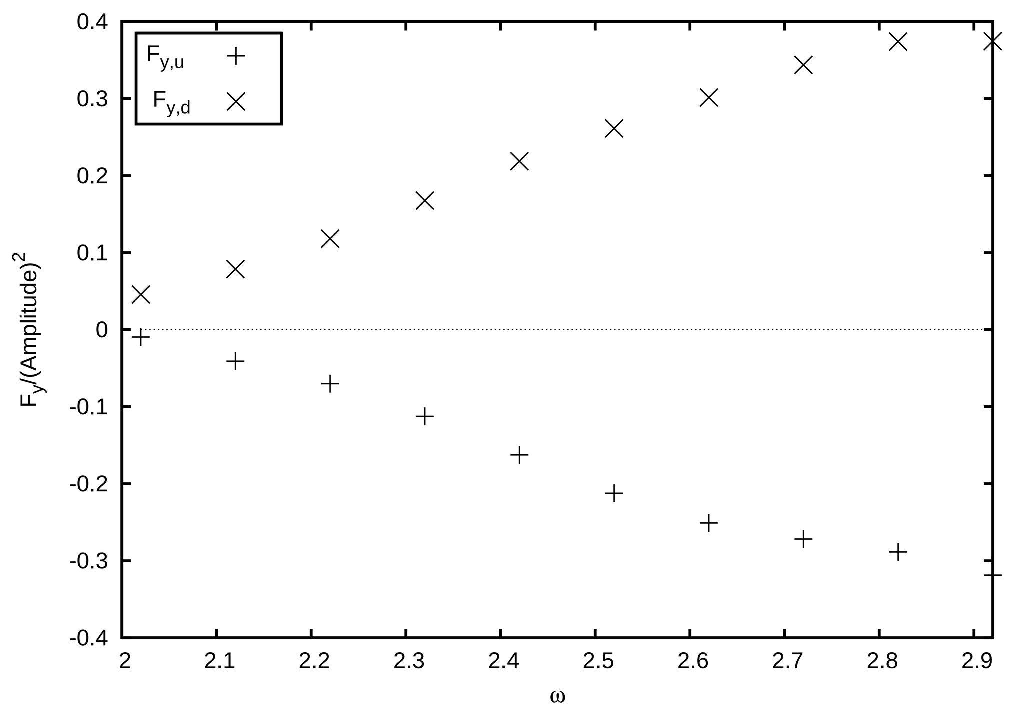

Figure 4: *The transversal force component F_y as function of the frequency ω.*

- numerical solution of eqns. (7)
- AB-approximation: OK for light mode,
  even the sign is wrong for heavy one
- Effect is robust: vortex core does not influence it much
- At $v=0.655c$ (typical value at the end of friction era)

$$F_x^u=-6.09|A|^2\,,\quad F_x^d=7.44|A|^2\,.$$

Total force: add up forces for each particle species

- amplitude: $|A_i|^2k_i\propto n_iv$
- scattering energy as $m_i/\sqrt{1-v^2}$

where v is the string velocity

NRP on cosmic strings

- Baryon number violating scattering on cosmic strings
- Heavy baryonic modes accelerate the string instead of acting as a medium with friction
- Longer path
- More baryon asymmetry washed away

6. Further application

Planar scattering situations

- Reproduced known results for superfluid vortices (one channel)
- vortices in superconductors
- preliminary results: two-channel scattering in two-band superconductors (e.g., MgB_2), scattering between channels influences force greatly, but no tractor beam

References

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