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Intercepts of π-meson correlation functions in μ-Bose gas model

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The model of usual harmonic oscillator in quantum mechanics is distinguished as fundamental model applicable to many physical systems. But, when applying the harmonic oscillator for description of real systems, these is some discrepancy between theoretical results and experimental data. Therefore, to make description of a physical system more successful, it is natural to explore modifications of the model. Thus, we deal with deformed oscillators.

The appearance of deformed oscillators is connected with the names of Biedenharn* and Macfarlane**. They established that the quantum algebra $SU_q(2)$ can be realized by two modes of q-analog of bosonic oscillator with commutation relation

$$aa^\dagger - qa^\dagger a = q^{-N},$$ where q is deformation parameter.

$$[a^\pm, a^\pm] = 0, \quad [N, a^\pm] = \pm a$$

In q-analog of Fock space:

$$a|0\rangle = 0, \quad |n\rangle = \frac{(a^\dagger)^n}{\sqrt{[N]_q!}}|0\rangle, \quad [N]_q = \frac{q^N - 1}{q - 1}, \quad N|n\rangle = n|n\rangle,$$

where $[N]_q! = [N]_q \cdot [N-1]_q \cdot ... \cdot [1]_q$, $[0]_q! = 1$.

The action of operators $a^\dagger, a$ on the state n is defined by formulas

$$a|n\rangle = \sqrt{[n]_q}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{[n+1]_q}|n+1\rangle.$$

*Biedenharn L.J., Phys. A: Math. Gen. **22** L873 (1989).

Mcfarlane A., J. Phys. A: Math. Gen. **22 4581 (1989).

Hamiltonian: $H = \frac{1}{2}(aa^\dagger + a^\dagger a), \quad \hbar\omega = 1, \qquad H|n\rangle = E_n|n\rangle.$

The connection between $a^\dagger$, a and the operators $b^\dagger$, b of usual quantum oscillator:

$$a^\dagger = \sqrt{\frac{[N]_q}{N}}\, b^\dagger, \quad a = b\sqrt{\frac{[N]_q}{N}}, \quad \text{where} \quad [N]_q = \frac{q^N - q^{-N}}{q - q^{-1}}.$$

Coordinate representation os $a^\dagger$, a :

$$a = \frac{e^{-2i\alpha x} - e^{i\alpha d/dx} e^{-i\alpha x}}{-i\sqrt{1 - e^{-2\alpha^2}}}, \quad a^\dagger = \frac{e^{2i\alpha x} - e^{i\alpha} e^{i\alpha d/dx}}{i\sqrt{1 - e^{-2\alpha^2}}}, \quad \text{де} \quad \alpha = \sqrt{-\ln q/2}.$$

The corresponding operators of coordinate and momentum in q-deformed quantum mechanics:

$$\hat{x} = \frac{a + a^\dagger}{\sqrt{2}} = \sqrt{\frac{2}{1 - e^{-2\alpha^2}}}\left(\sin(2\alpha x) - e^{\alpha^2/2} \sin\left(\alpha x + \frac{\alpha^2}{2}\, i\right) e^{i\alpha d/dx}\right),$$

$$\hat{p} = \frac{a - a^\dagger}{i\sqrt{2}} = \sqrt{\frac{2}{1 - e^{-2\alpha^2}}}\left(\cos(2\alpha x) - e^{\alpha^2/2} \cos\left(\alpha x + \frac{\alpha^2}{2}\, i\right) e^{i\alpha d/dx}\right).$$

When studying deformed oscillators it is convenient to use the concept of structute function of deformation $\varphi(N)$:

$$a^\dagger a = \varphi(N), \qquad aa^\dagger = \varphi(N+1).$$

For the ordinary quantum oscillator: $a^\dagger a = N$, $aa^\dagger = N+1$.

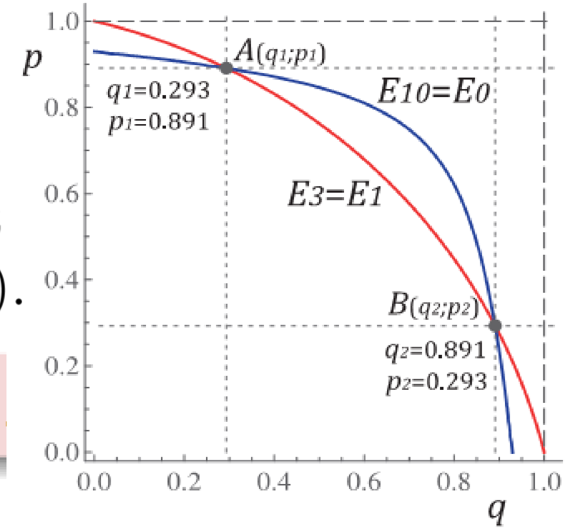
The structure function defines the **model** of deformed oscillator:

Structure function	Energy spectrum	
$\varphi_n^{\mathrm{AK}} = \frac{q^n - 1}{q - 1}$	$E_n^{\mathrm{AK}} = \frac{1}{2}\left(\frac{q^{n+1}-1}{q-1} + \frac{q^n - 1}{q-1}\right)$	Arik-Coon model
$\varphi_n^{\mathrm{BM}} = \frac{q^n - q^{-n}}{q - q^{-1}}$	$E_n^{\mathrm{BM}} = \frac{1}{2}\left(\frac{q^{n+1}-q^{-(n+1)}}{q-q^{-1}} + \frac{q^n - q^{-n}}{q-q^{-1}}\right)$	Biedenharn-Macfarlane model
$\varphi_n^{(\mathrm{p,q})} = \frac{q^n - p^n}{q - p}$	$E_n^{(\mathrm{p,q})} = \frac{1}{2}\left(\frac{q^{n+1}-p^{n+1}}{q-p} + \frac{q^n - p^n}{q-p}\right)$	p,q-oscillator model
$\varphi_n^{\mathrm{TD}} = nq^{n-1}$	$E_n^{\mathrm{TD}} = \frac{1}{2}\left((n+1)q^n + nq^{n-1}\right)$	Tamm-Dancoff model
$\varphi_n^{\mu} = \frac{n}{1+\mu n}$	$E_n^{\mu} = \frac{1}{2}\left(\frac{n}{1+\mu n} + \frac{n+1}{1+\mu(n+1)}\right)$	Jannussis μ-oscillator

Properties of deformed oscillators

Dedormed oscillators have unusual properties compared with the ordinary deformed oscillator. One of them is energy level degeneracy.

- Usual quantum oscillator: there is no level degeneracy in the energy spectrum ($d = 1$).
- In more general and complicated cases different types of energy level can exist (V.N. Zakhariev).



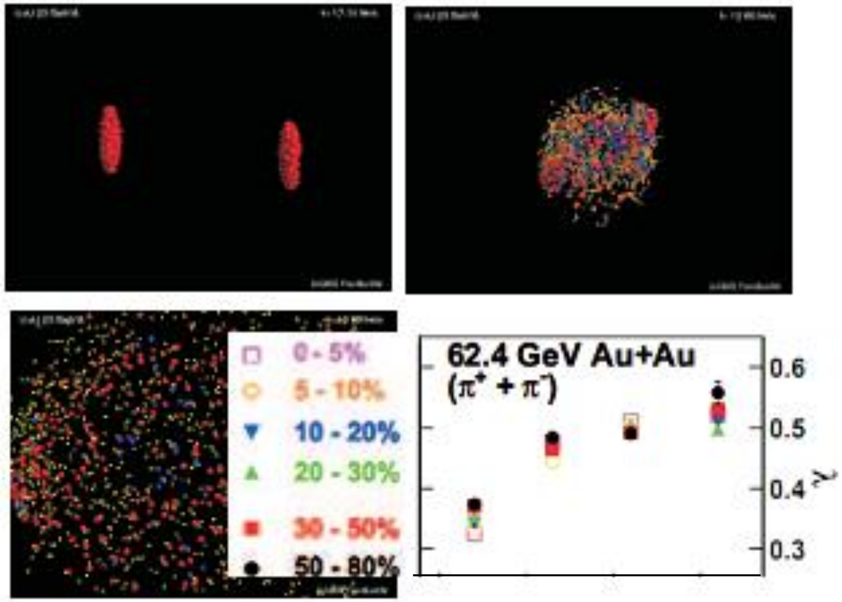
Spectrum of p, q-oscillator, $0 < p \leq 1, \quad 0 < q \leq 1$

$$E_n = E_0, \quad E_n = E_{n+1}, \quad E_n = E_{n+2}, n \geq 2.$$

Application of deformed oscillators

Deformed oscillators have application in different fields of physics: molecular and nuclear spectroscopy, integrable systems theory, quantum optics, statistical mechanics.

Quantum algebras or algebras of deformed oscillator have effective application in phenomenological investigation of the properties of elementary particles and theoretical aspects of relativistic nuclear collisions.



In the experiments of relativistic heavy ion collisions, as the result of collisions, the secondary particles (e.g. π-mesons) are produced and then registered.

Two-particle momentum correlation function:

$$C^{(2)}(k_1, k_2) = \gamma \frac{P_2(k_1, k_2)}{P_1(k_1)P_1(k_2)},$$

where one-particle and two-particle distributions are

$$P_1(p_i) = E_i \frac{dN}{d^3p_i} \qquad P_2(p_i, p_j) = E_i E_j \frac{dN}{d^3p_i d^3p_j}.$$

Correlation function can be rewritten in variables $Q = k_1 - k_2,\ K = (k_1 + k_2)/2$:

$$C^{(2)}(Q, K) \xrightarrow{k_1 = k_2} C^{(2)}(Q{=}0, K) = 1 + \lambda^{(2)}(m, \mathbf{K}),$$

$\lambda^{(2)}$ - intercept of two-particle correlation function.

If assume the particles are bosons, then $\lambda^{(2)} = 1$

Basing on the set of Jannussis μ-oscillators we develop the respective deformed analog of Bose gas model (μ-Bose gas).

The physical meaning of deformation parameter μ

- can be connected with the compositeness of particles (their substructure);
- or interaction between them.

Intercept can be rewritten in terms of operators $a^\dagger$, a:

$$\lambda^{(2)}(K) = \frac{\langle a^\dagger a^\dagger a a \rangle}{\langle a^\dagger a \rangle^2} - 1 = \frac{\langle [N]_\mu [N-1]_\mu \rangle}{\langle [N]_\mu \rangle^2} - 1,$$

where $a^\dagger a = \varphi_\mu(N) = [N]_\mu \equiv \frac{N}{1+\mu N}$.

Statistical average for a system with Hamiltonian H:

$$\langle N \rangle = \frac{Tr N e^{-\beta \sum_k H_k}}{Tr e^{-\beta \sum_k H_k}} = \frac{1}{e^{\beta \varepsilon} - 1}$$

where $\beta = \frac{1}{T}$, $k = 1$. Analogously one can obtain $\langle N^r \rangle$, $r \geq 2$.

The permutation relations are used:

$$a f(N) = f(N+1) a, \quad a^\dagger f(N) = f(N-1) a^\dagger$$

We choose the Hamiltonian in the form $H = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} N_{\mathbf{k}}$

The energy of particles: $\omega = (m_\pi^2 + \mathbf{K}^2)^{1/2}$.

Intercept of two-particle correlation function:

$$\lambda_\mu^{(2)} = \left\{X^{-1} - \left(\frac{1}{\mu}+\frac{1}{\mu^2}\right)\Phi(e^{-\beta},1,\mu^{-1}) - \left(\frac{1}{\mu}-\frac{1}{\mu^2}\right)\Phi(e^{-\beta},1,\mu^{-1}-1)\right\}\times$$

$$\times\left(X^{-1} - \mu^{-1}\Phi(e^{-\beta},1,\mu^{-1})\right)^{-2} X^{-1} - 1, \quad (1-e^{-\beta}) = X$$

Here Φ is Lerch transcedent: $\Phi = \sum_{n=0}^{\infty} z^n/(n+\alpha)^s$.

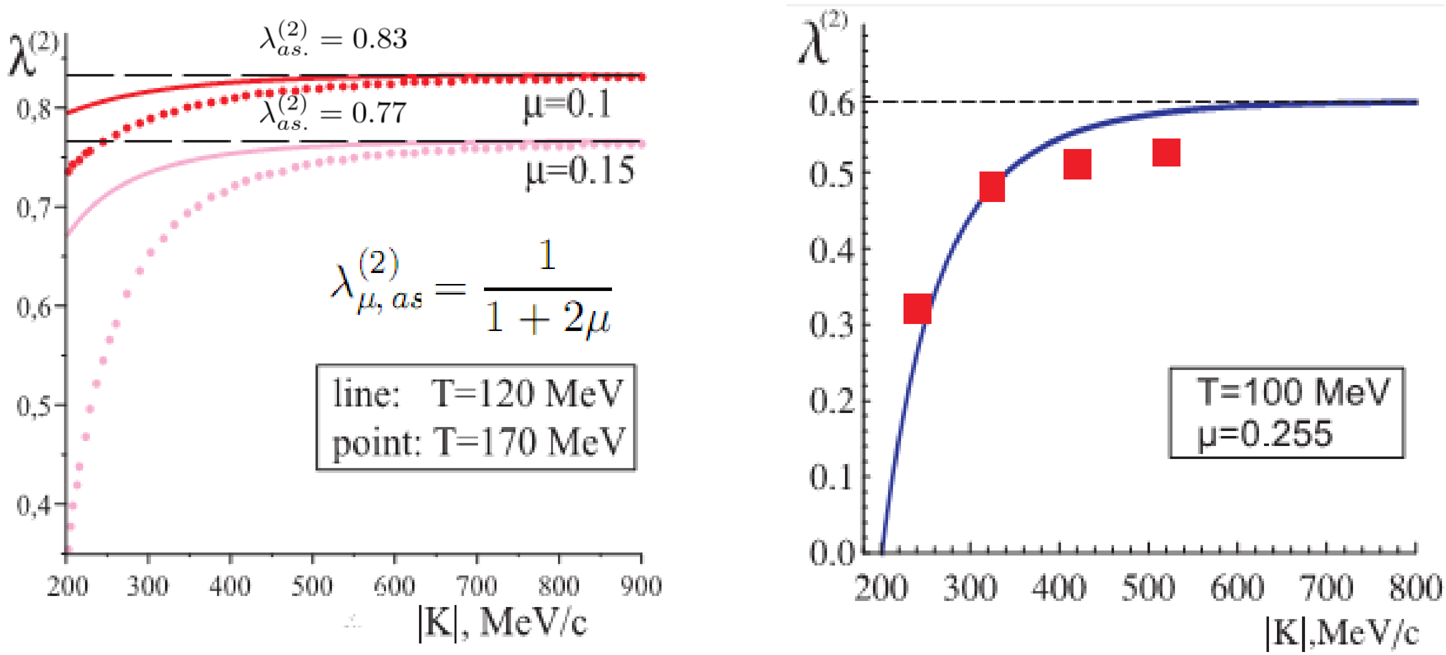


Fig.3. a) Dependence of $\lambda_\mu^{(2)}$ on the momentum K.
b) Comparison of theoretical results for $\lambda_\mu^{(2)}$ with experimental data from B.I. Abelev *et al.* (STAR Collab.), Phys. Rev. C **80**, 024905 (2009).