Solution The Basic Cosmological Problems By Using The Holographic Principle

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The traditional point of view assumed that space filling fields constitute the dominant part of degrees of freedom in our World. However, it became clear soon that such estimate much hardened the development of the quantum gravity theory: for the latter to make sense one had to cutoff on small distances all the integrals appeared in the theory. Consequently our World was described on a three-dimensional discrete lattice with cell size of order of Planck length.

Recently some physicists came up with even more radical point of view: complete description of Nature required only two-dimensional lattice, situated on space boundary of our World, instead of the three-dimensional one. Such approach is based on so-called “holographic principle”.
The term comes from the optical holography, which represent nothing but two-dimensional recording of three-dimensional objects. The holographic principle is composed of the two main statements:

1. all information contained in some region of space can be "recorded" (presented) on boundary of that region;
2. the theory contains at most one degree of freedom per Planck area on boundaries of the considered space region

\[ N \leq \frac{Ac^3}{G\hbar}. \] (1)

Therefore central place in the holographic principle is occupied by the assumption that all information about the Universe can be coded on some two-dimensional surface — the holographic screen. Such approach leads to possibly new interpretation of cosmological acceleration and to completely novel concept of gravity. The density of information on the holographic screen is limited to \(10^{69} \text{bit/m}^2\).

In any effective quantum field theory defined in space region with typical length scale $L$ and using the ultraviolet cutoff $\Lambda$, the system entropy takes the form $S \propto \Lambda^3 L^3$. For instance, fermions, placed in nodes of space lattice with characteristic size $L$ and period $\Lambda^{-1}$, occupy one of the number $2^{(L\Lambda)^3}$ states. Therefore entropy of such system is $S \propto \Lambda^3 L^3$. According to the holographic principle, this quantity must obey the inequality

$$L^3 \Lambda^3 \leq S_{BH} \equiv \frac{1}{4} \frac{A_{BH}}{l_{Pl}^2} = \pi L^2 M_{Pl}^2,$$

where $S_{BH}$ is the black hole entropy and $A_{BH}$ stands for its event horizon area, which in the simplest case coincides with the surface of sphere with radius $L$.

This reasoning shows that magnitude of the infrared cutoff cannot be chosen independently of the ultraviolet one. Thus we formulate the important result: in frames of holographic dynamics the infrared cutoff magnitude is strictly linked to the ultraviolet one. In particular, if the inequality (2) holds, then one gets

$$L \sim \Lambda^{-3} M_{Pl}^2.$$
Effective field theories with UV-cutoff (3) obviously include many states with the gravitational radius exceeding the region where the theory was initially defined. In other words, for arbitrary cutoff parameter one can find sufficiently large volume where entropy in an effective field theory exceeds the Bekenstein limit.

In order to get rid of that difficulty an even more strict limitation is imposed on the infrared cutoff $L \sim \Lambda^{-1}$, which excludes all the states localized within limits of their gravitational radius. Taking into account all above mentioned and using the expression (4)

$$\rho_{\text{vac}} \approx \frac{\Lambda^4}{16\pi^2},$$

(4)

the condition (2) can be presented in the form

$$L^3 \rho_{\Lambda} \leq L M_{\text{Pl}}^2 \equiv 2M_{\text{BH}},$$

(5)

where $M_{\text{BH}}$ is the black hole mass with gravitational radius $L$. 

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Holography in Cosmology
In particle physics, cosmological constant naturally arises as the vacuum energy density, which can be estimated as the sum of the zero-point oscillations of quantum fields with mass \( m \).

\[
\rho_{\text{vac}} = \frac{1}{2} \int_0^\infty \frac{dk}{(2\pi)^3} \sqrt{k^2 + m^2} \approx \frac{1}{4\pi^2} \int_0^\Lambda k^2 dk \sqrt{k^2 + m^2}, \text{if } m \ll \Lambda \approx M_{Pl}, \quad \rho_{\text{vac}} \approx 10^{120} \rho_{\text{obs}}
\]  

(6)

Applying \((L^3 \rho_\Lambda \leq L M_{Pl}^2)\) this relation to the Universe as whole it is naturally to identify the IR-scale with the Hubble radius (simplest case) \( H^{-1} \). Then for the upper bound of the energy density one finds

\[
\rho_\Lambda \sim L^{-2} M_{Pl}^2 \sim H^2 M_{Pl}^2.
\]  

(7)

We will below denote its density as \( \rho_{DE} \). Accounting that \( M_{Pl} \approx 1.2 \times 10^{19} \text{GeV}; \quad H_0 \approx 1.6 \times 10^{-42} \text{GeV} \), one finds

\[
\rho_{\text{obs}} \sim 10^{-46} \text{GeV}^4 \text{ observed value} \quad \iff \quad \rho_{DE} \approx 3 \times 10^{-47} \text{GeV}^4.
\]  

(8)

Therefore the holographic dynamics is free from the cosmological constant problem.
Figure: The change of the vacuum energy density parameter, $\Omega_\Lambda$, as a function of the scale factor $a$, in a universe with $\Omega_{\Lambda 0} = 0.7$, $\Omega_{m0} = 0.3$. Scale factors corresponding to the Planck era, electroweak symmetry breaking (EW), and Big Bang nucleosynthesis (BBN) are indicated, as well as the present day. The spike reflects the fact that, in such a universe, there is only a short period in which $\Omega_\Lambda$ is evolving noticeably with time.

While a cosmological constant is by definition time-independent, the matter energy density is diluted as $1/a^3$ as the Universe expands. Thus, despite evolution of $a$ over many orders of magnitude, we appear to live in an era during which the two energy densities are roughly the same.

**Cosmic Coincidence Problem**

Why the densities of dark energy and dark matter are comparable today?
The holographic dark energy with cutoff $L$ is

$$\rho_{\text{DE}} = 3c^2M_{\text{Pl}}^2L^{-2}. \quad (9)$$

The coefficient $3c^2$ ($c > 0$) is introduced for convenience, and $M_{\text{Pl}}$ further stands for reduced Planck mass: $M_{\text{Pl}}^{-2} = 8\pi G$.

Setting $L = H^{-1}$ in the above bound and working with the equality (i.e., assuming that the holographic bound is saturated) it becomes $\rho_{\text{hde}} = 3c^2M_{\text{Pl}}^2H^2$. Combining the last expression with Friedmann’s equation for a spatially flat universe, $3M_{\text{Pl}}^2H^2 = \rho_{\text{hde}} + \rho_m$, results in

$$\rho_m = 3\left(1 - c^2\right)M_{\text{Pl}}^2H^2. \quad (10)$$

Now, the argument runs as follows: The energy density $\rho_m$ varies as $H^2$, which coincides with the dependence of $\rho_{\text{hde}}$ on $H$. So theirs ratio is constant and has the form

$$\frac{\rho_m}{\rho_{\text{DE}}} = \frac{1 - c^2}{c^2}. \quad (10)$$

Thus, the dark energy behaves as pressureless matter. Obviously, pressureless matter cannot generate accelerated expansion, which seems to rule out the choice $L = H^{-1}$. 

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Holography in Cosmology
Holographic Dark Energy: the different cutoff scales

The attempts to choose the different cutoffs scales $L$

- **Particle horizon** $L = R_p$, $R_p = a \int_0^a \frac{da'}{H(a') a'^2}$;

  $$w = -\frac{1}{3} + \frac{2}{3c} > -\frac{1}{3} \Rightarrow q > 0, \text{deceleration.} \quad (11)$$

  Thus one can see that the above described component does not deserves the name of “dark energy” in proper sense, as it cannot serve its main purpose — to provide the accelerated expansion of universe.

- **Cosmological event horizon** $L = R_h$, $R_h = a(t) \int_t^\infty \frac{dt'}{a(t')}$;

  $$w_{hde} = \frac{p_{hde}}{\rho_{hde}} = -\frac{1}{3} \left(1 + \frac{2}{c} \sqrt{\Omega_{DE}}\right) \Rightarrow q < 0, \text{acceleration.} \quad (12)$$

  The holographic dark energy with IR-cutoff on the event horizon still leaves unsolved problems connected with the causality principle: **according to the definition of the event horizon the holographic dark energy dynamics depends on future evolution of the scale factor. Such dependence is hard to agree with the causality principle.**

- **Age of the Universe** $L = T$, $T = \int_0^a \frac{da}{Ha}$ (agegraphic dark energy). This kind of dark energy we study in more detail.
The existence of quantum fluctuations in the metric directly leads to the following conclusion, related to the problem of distance measurements in the Minkowski space: the distance $t^a$ cannot be measured with precision exceeding the following

$$\delta t = \beta t_{Pl}^{2/3} t^{1/3},$$

(13)

where $\beta$ is a factor of order of unity. This expression so-called Karolyhazy uncertainty relation. Following we can consider the result as the relation between the UV and IR scales in frames of effective quantum field theory, which correctly describes the entropy features of black holes. This ratio can be interpreted as: if lifetime (age) of some spatial region of linear size $t$ equals $t$, then there exists a minimal cell $\delta t^3$, with energy that cannot be less than

$$E_{\delta t^3} \sim t^{-1}.$$  

(14)

From (13) and (14) it immediately follows that due to the energy-time uncertainty principle the energy density of quantum fluctuating metric in the Minkowski space equals

$$\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_{Pl}^2 t^2}.$$  

(15)

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$a$recall that we use the system of units where the light speed equals $c = \hbar = 1$, so that $L_{Pl} = t_{Pl} = M_{Pl}^{-1}$
The relation (15) allows to introduce an alternative model for holographic dark energy, which uses the age of Universe $T$ for IR-cutoff scale. In such a model
\begin{equation}
\rho_q = \frac{3n^2M^2_{Pl}}{T^2},
\end{equation}
where $n$ is a free parameter of model, and the number coefficient 3 is introduced for convenience. So defined energy density (16) with $T \sim H_0^{-1}$, where $H_0$ is the current value of the Hubble parameter, leads to the observed value of the dark energy density with the coefficient $n$ value of order of unity. Thus in SCM, where $H_0 \simeq 72 \text{ km sec}^{-1}\text{Mpc}^{-1}$, $\Omega_{DE} \simeq 0.73$, $T \simeq 13.7 \text{ Gyr}$, one finds that $n \simeq 1.15$.

Suppose that the Universe is described by the Friedmann equation
\begin{equation}
H^2 = \frac{1}{3M^2_{Pl}} (\rho_q + \rho_m).
\end{equation}

The state equation for the dark energy is
\begin{equation}
w_q = -1 + \frac{2}{3n} \sqrt{\Omega_q}.
\end{equation}

So such universe will be accelerated expanded, and would be similar to $\Lambda CDM$. 

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Thus the holographic model for dark energy with IR-cutoff scale set to the Universe age, allows the following:

1. to obtain the observed value of the dark energy density;
2. provide the accelerated expansion regime on later stages of the Universe evolution;
3. resolve contradictions with the causality principle.

The first successes of holographic principle application from one hand awoke hopes to create on that basis an adequate description of the Universe dynamics, free of a number of problems which the traditional approach suffers from.

Observations challenge

Starobinsky with co-authors, based on independent observational data, including the brightness curves for SNe Ia, cosmic microwave background temperature anisotropy and baryon acoustic oscillations (BAO), were able to show, that the acceleration of Universe expansion reached its maximum value and now decreases. In terms of the deceleration parameter it means that the latter reached its minimum value and started to increase (this is one of the possibilities).
Figure: The deceleration parameter dependence $q(z)$ reconstructed from independent observational data, including the brightness curves for SN Ia, cosmic microwave background temperature anisotropy and baryon acoustic oscillations (BAO). The red solid line shows the best fit on the confidence level $1\sigma$ CL.

Thus the main result of the analysis is the following: SCM is not unique though the simplest explanation of the observational data, and the accelerated expansion of Universe presently dominated by dark energy is just a transient phenomenon.
Current literature usually considers the models where the required dynamics of Universe is provided by one or another, and always only one, type of dark energy. As was multiply mentioned above, in order to explain the observed dynamics of Universe, the action for gravitational field is commonly complemented, besides the conventional matter fields (both matter and baryon), by either the cosmological constant, which plays role of physical vacuum in SCM, or more complicated dynamical objects — scalar fields, $K$-essence and so on. In the context of holographic cosmology, the latter term is usually neglected, restricting to contribution of the boundary terms. However such restriction has no theoretical motivation.

We consider the cosmological model which contains both volume and surface terms. The role of former is played by homogeneous scalar field in exponential potential, which interact with dark matter. The boundary term responds to holographic dark energy in form of (16).

$$\rho_q = \frac{3n^2M^2_{Pl}}{T^2},$$
The Model

Considering the flat Friedmann-Robertson-Walker Universe with the agegraphic dark energy, substance with an arbitrary equation of state \( w \) and energy density \( \rho_w \) and scalar field \( \rho_\phi \) the corresponding Friedmann equation is

\[
H^2 = \frac{1}{3M_p^2}(\rho_q + \rho_w + \rho_\phi). \tag{19}
\]

The conservation laws of the scalar field and matter are respectively

\[
\dot{\rho}_w + 3H(1 + w)\rho_w = Q, \tag{20}
\]
\[
\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = -Q, \tag{21}
\]

where \( Q \) denotes the phenomenological interaction term. We consider the most general of above cited types of interaction

\[
Q = 3H(\alpha \rho_\phi + \beta \rho_w). \tag{22}
\]

The evolution of scalar field is described by the Klein-Gordon equation, which in the case of interaction between the scalar field and matter takes the following form:

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = \frac{Q}{\dot{\phi}}. \tag{23}
\]

We consider the case when the interaction parameter \( Q \) is a linear combination of energy density for scalar field and dark energy

\[
Q = 3H(\alpha \rho_\phi + \beta \rho_m), \tag{24}
\]

where \( \alpha, \beta \) are constant parameter.
We consider the case with the interaction parameter of the form (24) with \( \beta = 0 \).

Figure: Behavior of \( \Omega_\varphi \) (dot line), \( \Omega_q \) (dash line) and \( \Omega_m \) (solid line) as a function of \( N = \ln a \) for \( n = 3, \alpha = 0.005 \) and \( \mu = -5 \) (left side). Evolution of deceleration parameter for this model (center) and the deceleration parameter \( q(z) \) reconstructed from independent observational data (right side).

Thus, this model can explain the nonmonotonic dependence of some cosmological parameters
In quantum information science, quantum entanglement is a central concept and a precious resource allowing various quantum information processing such as quantum key distribution. The entanglement is a quantum nonlocal correlation which cannot be prepared by local operations and classical communication.

### Entanglement entropy

For pure states the entanglement entropy $S_{\text{Ent}}$ is a good measure of entanglement. For a bipartite system $AB$ described by a full density matrix $\rho_{AB}$, $S_{\text{Ent}}$ is the von Neumann entropy

$$S_{\text{Ent}} = -\text{Tr}(\rho_A \ln \rho_A)$$

for a reduced density matrix

$$\rho_A \equiv \text{Tr}_B \rho_{AB}$$

obtained by partial tracing part B.

The Basic Conjecture of the Entanglement entropy are

- Quantum entanglement of matter or the vacuum in the universe increases like the entropy;
- There is a new kind of force - quantum entanglement force associated with this tendency;
- Gravity and dark energy are types of the quantum entanglement force associated with the increase of the entanglement, similar to Verlinde’s entropic force linked with the increase of the entropy.
The surface $\Sigma$ has the entanglement entropy $S_{\text{ent}} \propto r^2$ and entanglement energy

$$E_{\text{ent}} \equiv \int_{\Sigma} T_{\text{ent}} dS_{\text{ent}}.$$ 

If there is a test particle with mass $m$, it feels an effective attractive force in the direction of increase of entanglement.

In general, the vacuum entanglement entropy of a spherical region with a radius $r$ with quantum fields can be expressed in the form

$$S_{\text{ent}} = \frac{\beta r^2}{b^2},$$  \hspace{1cm} (27)

where $\beta$ is an $O(1)$ constant that depends on the nature of the field and $b$ is the UV-cutoff.

$S_{\text{Ent}}$ has a form consistent with the holographic principle, although it is derived from quantum field theory without using the principle.

Thus, from different and independent physical assumptions, we come to equal physical consequence. We can use both of this ideology with equal success and equivalent effect.
Why are we considering the quantum entanglement as an essential concept for cosmology?

1. There are interesting similarities between the holographic entropy and the entanglement entropy of a given surface. Both are proportional to its area in general and related to quantum nonlocality.

2. There is a gravitational force, there is always a Rindler horizon for some observers, which acts as information barrier for the observers. This can lead to ignorance of information beyond the horizons, and the lost information can be described by the entanglement entropy. The space-time should bend itself so that the increase of the entanglement entropy compensate the lost information of matter.

3. If we use the entanglement entropy of quantum fields instead of thermal entropy of the holographic screen, we can understand the microstates of the screen and explicitly calculate, in principle, relevant physical quantities using the quantum field theory in the curved space-time. The microstates can be thought of as just quantum fields on the surface or its discretized oscillators. Finally, identifying the holographic entropy as the entanglement entropy could explain why the derivations of the Einstein equation is involved with entropy, the Planck constant and, hence, quantum mechanics.

All these facts indicate that quantum mechanics and gravity has an intrinsic connection, and the holographic principle itself has something to do with quantum entanglement.
Thank you for your attention!