

Chiral behavior of pion properties from lattice QCD

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Erice, June 28, 2013

Motivation

$SU(3)_c$ + two massless quarks (u,d)

$$SU(2)_R \times SU(2)_L \times U(1)_V \xrightarrow{S\chi SB} \begin{matrix} SU(2)_V \times U(1)_V \\ 3 \text{ Goldstone Bosons } (\pi^\pm, \pi^0) \end{matrix}$$

Real World: Quarks masses are small but different from zero

Pseudo Goldstone Bosons (PGB) with $M_\pi > 0$

Chiral Perturbation Theory (χPT)

- Effective theory of PGBs.
- ✓ Perturbative expansion in terms of PGB momenta and masses.
- ✗ Infinite new parameters.

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Chiral Perturbation Theory (χPT)

- Effective theory of PGBs.
- ✓ Perturbative expansion in terms of PGB momenta and masses.
- ✗ Infinite new parameters.
- ✓ Only a finite number at each order.
- ✓ All these parameters are determined by QCD.

SU(2) Chiral Perturbation Theory (χ PT)

Gasser, Leutwyler Nucl.Phys.B(85), Ann.Phys.158(84)

Leading Order (LO) $\mathcal{O}(p^2)$

$$F = \lim_{m_q \rightarrow 0} F_\pi \quad \text{and} \quad B = \lim_{m_q \rightarrow 0} \frac{\langle 0 | \bar{q}q | 0 \rangle}{F_\pi^2}$$

Next to Leading Order (NLO) $\mathcal{O}(p^4)$

Seven new terms in the Lagrangian $\mathcal{O}(p^4)$: \bar{l}_i , $i = 1, \dots, 7$

Next to Next to Leading Order (NNLO) $\mathcal{O}(p^6)$

Many other terms

Are we able to compute these numbers from QCD?

Quark-mass dependence of M_π^2 and F_π

Colangelo, Gasser, Leutwyler Nucl.Phys.B603(01)

x-expansion $\left(x = \frac{M^2}{(4\pi F)^2}\right)$, $M^2 = 2Bm_{ud}$

$$M_\pi^2 = M^2 \left(1 - \frac{x}{2} \log\left(\frac{\Lambda_3^2}{M^2}\right) + x^2 \left(\frac{17}{8} \log\left(\frac{\Lambda_M^2}{M^2}\right) + k_M\right) + \mathcal{O}(x^3)\right)$$

$$F_\pi = F \left(1 + x \log\left(\frac{\Lambda_4^2}{M^2}\right) + x^2 \left(-\frac{5}{4} \log\left(\frac{\Lambda_F^2}{M^2}\right) + k_F\right) + \mathcal{O}(x^3)\right)$$

$$\log\left(\frac{\Lambda_M^2}{M^2}\right) = \frac{1}{51} \left(4 \left(7 \log\left(\frac{\Lambda_1^2}{M^2}\right) + 8 \log\left(\frac{\Lambda_2^2}{M^2}\right)\right) - 9 \log\left(\frac{\Lambda_3^2}{M^2}\right) + 49\right)$$

$$\log\left(\frac{\Lambda_F^2}{M^2}\right) = \frac{1}{30} \left(2 \left(7 \log\left(\frac{\Lambda_1^2}{M^2}\right) + 8 \log\left(\frac{\Lambda_2^2}{M^2}\right)\right) + 6 \left(\log\left(\frac{\Lambda_3^2}{M^2}\right) - \log\left(\frac{\Lambda_4^2}{M^2}\right)\right)\right)$$

$$\bar{l}_i \equiv \log\left(\frac{\Lambda_i^2}{M_\pi^2}\right)$$

Lattice QCD (Wilson, 74)

Continuous

Lattice

$$x_\mu \in R^4$$

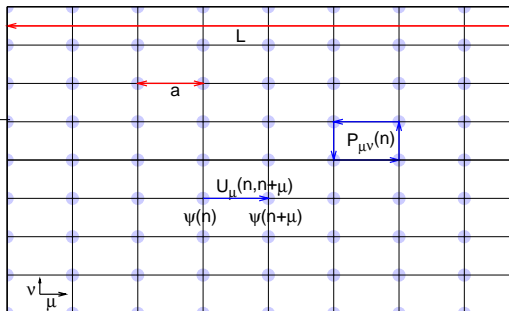
$$n_\mu \in L^4$$

$$\psi(x)$$

$$\psi(n)$$

$$A_\mu(x)$$

$$U_\mu(n) = Te^{i \int_x^{x+\mu} A_\mu}$$



- QCD Action

$$\int dx^4 \bar{\psi} (\not{D} + m) \psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

Lattice QCD (Wilson, 74)

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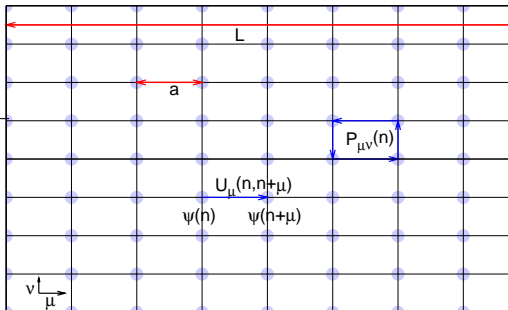
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- QCD Action

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- Lattice-QCD ActionS

- Gauge Actions: Wilson, Lüscher-Weisz, Symanzik, ...
- Fermionic Actions: Wilson, staggered, Neuberger, Domain Wall, ...

→ Correct continuum limit → QCD

- Expectation value

$$\langle \mathcal{O}(\bar{\psi}, \psi, A_\mu) \rangle = \mathcal{Z} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu \mathcal{O}(\bar{\psi}, \psi, A_\mu) e^{-S(\bar{\psi}, \psi, A_\mu)}$$

- On the Lattice:
 - Path integral is well define
 - Monte Carlo integration
 - Main goal: generate a representative ensemble of gauge configurations

$$\langle \mathcal{O}(\bar{\psi}, \psi, A_\mu) \rangle = \frac{1}{N} \sum_i^N \mathcal{O}(\bar{\psi}_i, \psi_i, A_{\mu,i})$$

Gauge Action

Lüscher, Weisz Phys.Lett.B158(85)

$$S_G = \frac{1}{\beta} \left[\frac{c_0}{3} \sum_{\text{plaq}} \text{ReTr}(1 - U_{\text{plaq}}) + \frac{c_1}{3} \sum_{\text{rect}} \text{ReTr}(1 - U_{\text{rect}}) \right]$$

Fermionic action

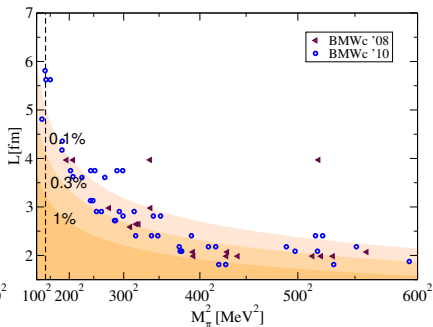
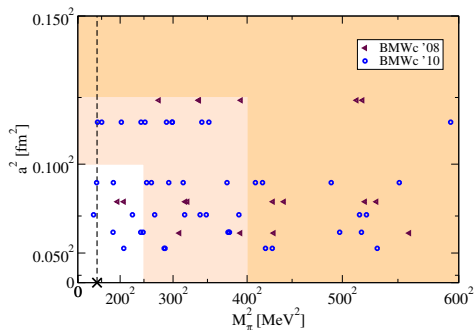
Sheikholeslami, Wohlert Nucl.Phys.B259(85)

$$S_F = S_{\text{Wilson}} - \frac{c_{\text{SW}}}{2} \sum_x \sum_{\mu < \nu} (\bar{\psi} \sigma_{\mu\nu} F_{\mu\nu}[V] \psi)(x)$$

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ and c_i are set to their tree level values.

- V refers to links which have undergone two HEX smearing.

Hasenfratz, Knechtli Phys.Rev.D64(01)
 Morningstar, Peardon Phys.Rev.D69(04)
 Capitani, Durr, Hölbling JHEP11(06)



- ⇒ 47 points at five different lattice spacing: $a^{-1} \sim (1.697, 2.131, 2.561, 3.026, 3.662)$ GeV.
- ⇒ Pion masses around the physical point: $M_\pi^{\text{MIN}} \sim (136, 131, 120, 182, 219)$ MeV.
- ⇒ Spatial volumes as large as $\sim (6 \text{ fm})^3$ and temporal extents up ~ 8 fm.

What do we need from the lattice to compute LEC?

Inputs for χ PT theory

- $(am_{ud}), (aM_\pi), (aF_\pi)$

Strange mass corrections

- (aM_{SS})

Scale Setting

- lattice spacing: a

Renormalization

- Z_S and Z_A

Improved ratio-different method

BMWc Phys.Lett.B701(11),JHEP 1108(11)

$$r = \frac{m_s^{PCAC}}{m_{ud}^{PCAC}}, \quad d = m_s^{\text{bare}} - m_{ud}^{\text{bare}}$$

$$r^{\text{imp}} = r, \quad d^{\text{imp}} = d \left(1 - \frac{d}{2} \frac{r+1}{r-1} \right)$$

$$am_{ud}^{\text{scheme}} = \frac{1}{Z_S^{\text{scheme}}} \frac{d^{\text{imp}}}{r^{\text{imp}} - 1}, \quad am_s^{\text{scheme}} = \frac{1}{Z_S^{\text{scheme}}} \frac{r^{\text{imp}} d^{\text{imp}}}{r^{\text{imp}} - 1}$$

$$am_1^{PCAC} + am_2^{PCAC} = \frac{\sum_{\vec{x}} \langle \partial_\mu [A_\mu(x)] \mathcal{O}(0) \rangle}{\sum_{\vec{x}} \langle P(x) \mathcal{O}(0) \rangle}$$

M_π and F_π constant from the Lattice

M_π and F_π are extracted from the combined fit of correlators

$$\sum_{\vec{x}} \langle (\bar{d}\gamma_5 u)(\vec{x}, t)(\bar{u}\gamma_5 d)(0) \rangle = C_{PP} \cosh \left((aM_\pi) \left(\frac{T}{2} - t \right) \right)$$

$$\sum_{\vec{x}} \langle (\bar{d}\gamma_0\gamma_5 u)(\vec{x}, t)(\bar{u}\gamma_5 d)(0) \rangle = C_{AP} \sinh \left((aM_\pi) \left(\frac{T}{2} - t \right) \right)$$

F_π

$$(aF_\pi) = \frac{C_{AP}}{\sqrt{2(aM_\pi)C_{PP}}} e^{\frac{(aM_\pi)T}{4}}$$

Strange quark effects and scale setting

$$M_{SS}^2 \equiv 2M_k^2 - M_\pi^2$$

$$\sum_{\vec{x}} \langle (\bar{s}\gamma_5 u)(\vec{x}, t)(\bar{u}\gamma_5 s)(0) \rangle = C'_{PP} \cosh \left((aM_K) \left(\frac{T}{2} - t \right) \right)$$

Scale setting

- $\frac{M_\pi}{M_\Omega}, \frac{M_K}{M_\Omega} \rightarrow$ Experimental value

Renormalization Z_S and Z_A

- RI/MOM scheme

Martinelli, Pittori, Sachrajda, Testa, Vladikas Nucl.Phys.B445(95)

Chiral Fits

LECs are computed by a combined correlated fit:

- x-expansion: $B_\pi \equiv \frac{M_\pi^2}{2m_{ud}}$ and F_π in terms of m_{ud}

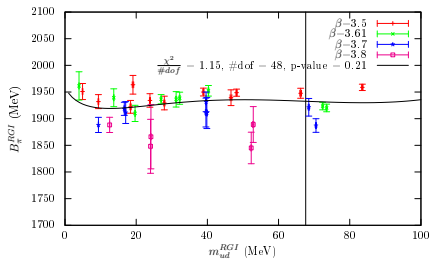
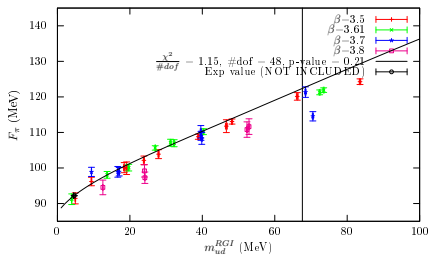
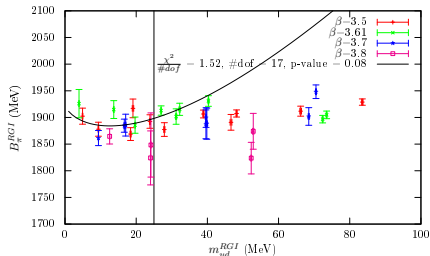
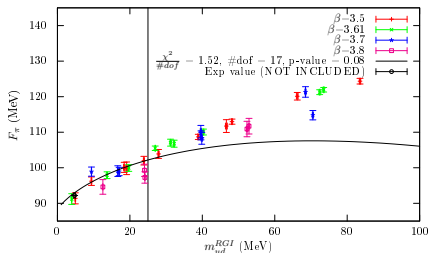
Statistical errors:

- 2000 bootstrap samples are used in each fit

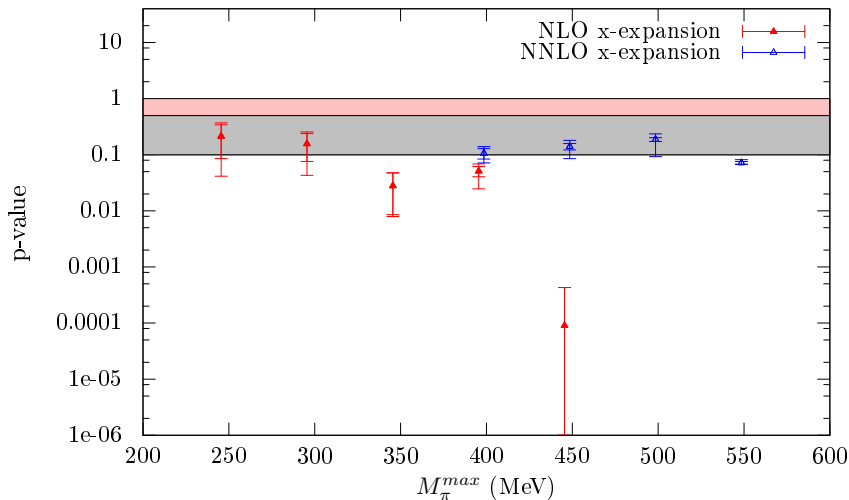
Systematics errors:

- 2 different time fitting ranges for correlators.
- 2 M_π cuts for interpolating scale fixing quantity M_Ω (380/480 MeV).
- 3 M_π cuts for chiral fits (250/300/350 MeV).

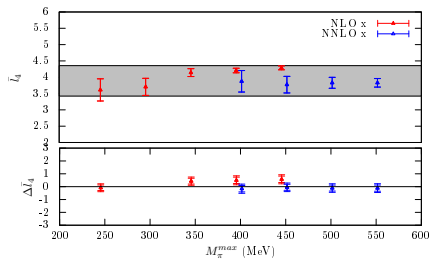
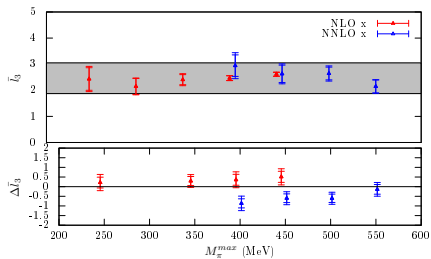
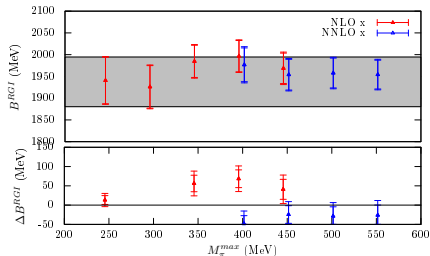
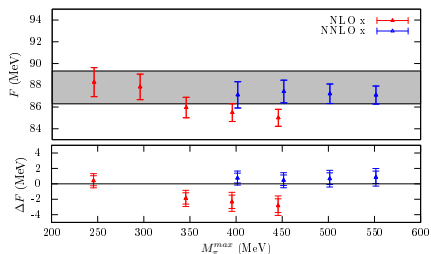
Typical Fit



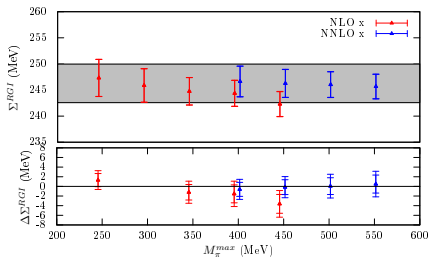
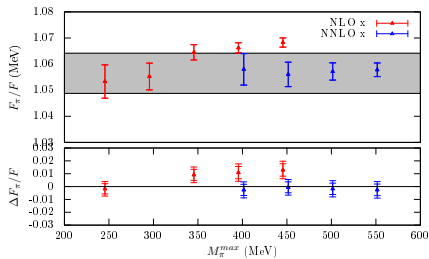
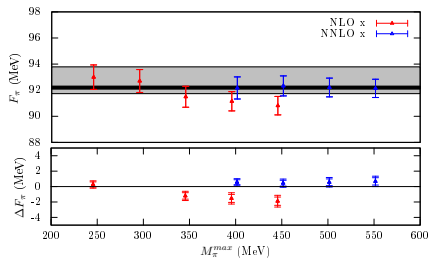
Range of validity of SU(2) χ PT NLO (preliminary results)



LO LEC in terms of M_π (preliminary results)



F_π and other interesting quantities in terms of M_π (preliminary results)



LECs from x-expansion (Preliminary results)

	x-expansion	FLAG + PDG
LO		
B^{RG1} [GeV]	$1.94 \pm 0.05 \pm 0.02$	
F [MeV]	$87.9 \pm 1.3 \pm 0.7$	86.4 ± 0.7
$[\Sigma^{RG1}]^{1/3}$ [MeV]	$247 \pm 3 \pm 1$	246 ± 14
NLO		
\bar{l}_3	$2.3 \pm 0.4 \pm 0.2$	3.2 ± 0.7
\bar{l}_4	$3.7 \pm 0.3 \pm 0.1$	4.4 ± 0.6
Other quantities		
F_π [MeV]	$92.7 \pm 0.9 \pm 0.4$	$92.2 \pm 0.02 \pm 0.14$
F_π/F	$1.055 \pm 0.006 \pm 0.003$	1.067 ± 0.009

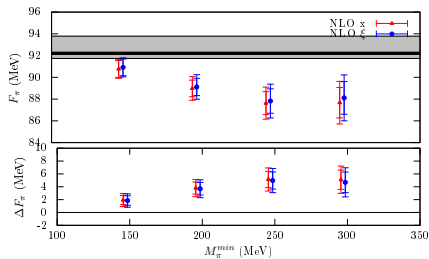
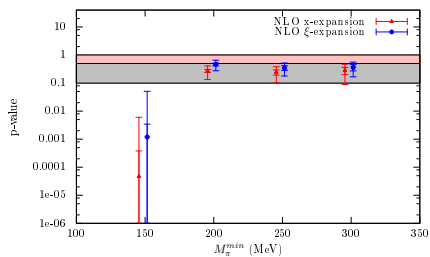
Conclusion and outlook

- SU(2) χPT has been compared to lattice QCD simulations all the way down to the physical pion mass point [see also [Borsanyi et al, arXiv:1205.0788](#) for an analysis with staggered fermion simulations]
- Preliminary results for χ -expansion look promising:
 - Within our O(1%) statistical errors, NLO SU(2) χPT describes the F_π and M_π^2 well up to $M_\pi \approx 350 \text{ MeV}$
 - LO and NLO LECs generally consistent with expectations, but errors are substantially reduced
- Still many checks and analyses to be made, but final results should significantly improve our knowledge of SU(2) χPT as applied to these fundamental observables
- The era of precision on Lattice QCD has started
- Ab-initio computations of QCD at low energy are possible now

thank you!



Fits for the physical limit (Preliminary results)



Log?

Fitting χ PT formulae with 2HEX data

x-expansion

$$\chi^2 = \sum_i^{N_{\text{points}}} X_i^T S_i^{-1} X_i + \sum_{\beta} \left(\frac{(a_{\beta}^p - a_{\beta})^2}{\sigma_{a_{\beta}}^2} + \frac{(Z_{S,\beta}^p - Z_{S,\beta})^2}{\sigma_{Z_{S,\beta}}^2} + \frac{(Z_{A,\beta}^p - Z_{A,\beta})^2}{\sigma_{Z_{A,\beta}}^2} \right)$$

$$X = \begin{pmatrix} am_{ud} - a^p Z_S^p m_{ud}^p (1 - \gamma_1 (a^p)^2) \\ (aM_{SS})^2 - (a^p)^2 (M_{SS}^p)^2 \\ \frac{(aM_{\pi})^2}{2(am_{ud})} - (a^p Z_S^p) (1 + \gamma_1^p (a^p)^2 + \gamma_2^p ((M_{SS}^p)^2 - M_{\Phi,SS}^2)) B_{\pi}(m_{ud}^p, B^p, F^p, \Lambda_3^p | l_{12}, k_M, k_F) \\ (aF_{\pi}) - \frac{a^p}{Z_a^p} (1 + \gamma_3^p ((M_{SS}^p)^2 - M_{\Phi,SS}^2) + \gamma_4^p xG(M_{\pi}L)) F_{\pi}(m_{ud}^p, B^p, F^p, \Lambda_4^p | l_{12}, k_M, k_F) \end{pmatrix}$$

$$S_{ab} = \sum_{m=1}^{N_{\text{BOOTS}}} (y_a^m - y_a^0) (y_b^m - y_b^0), \quad y^t = \left(am_{ud}, (aM_{SS})^2, \frac{(aM_{\pi})^2}{2(am_{ud})}, aF_{\pi} \right)$$

- A term, $\gamma_1 a^2 + \gamma_2 ((M_{SS}^p)^2 - M_{\Phi,SS}^2) + \gamma_3^p xG(M_{\pi}L)$, was tried for each component of X.
- $l_{12} = 7 \log(\Lambda_1^2) + 8 \log(\Lambda_2^2)$, k_M and k_F are included only for NNLO fits
- Fits are repeated for each bootstrap sample.