Problems with Ultrahigh-energy Neutrino Interactions

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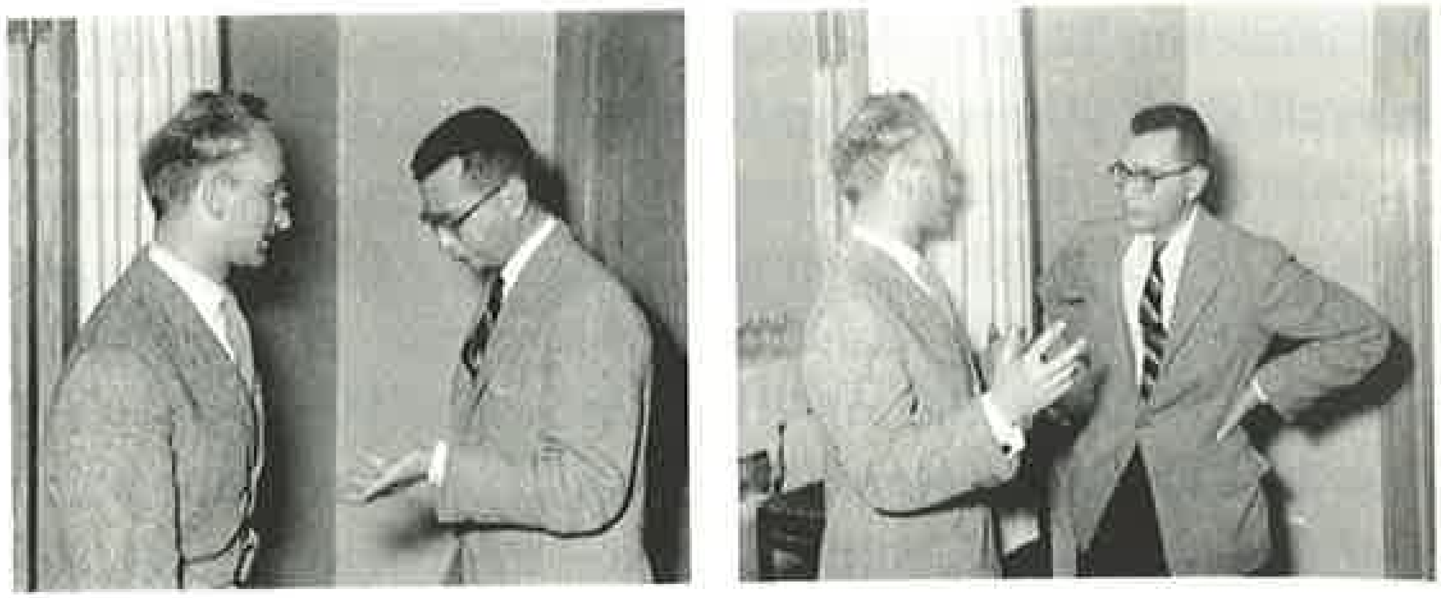


Abbildung 3.3: Murray Gell-Mann reichte manchmal die Sprache allein nicht für die Kommunikation.

Heisenberg: Aber wo sind denn Ihre Quarks, wo existieren die?

Ich: Nun, eben im Proton.

Heisenberg: Das kann man doch nicht Existieren nennen.

Ich: Anfangs konnte man das Atom auch nicht zerlegen und hat doch an das Atommodell geglaubt.

Heisenberg: Aber da hatte man wenigstens das Elektron gesehen, aber Quark hat man noch keines gesehen. Ich glaube, alle Ihre Erfolge des Quarkmodells sind nur Zufallstreffer, da steckt nichts Reales dahinter.

Walter Thirring
Kosmische Impressionen
Seifert Verlag, 2008
page 102

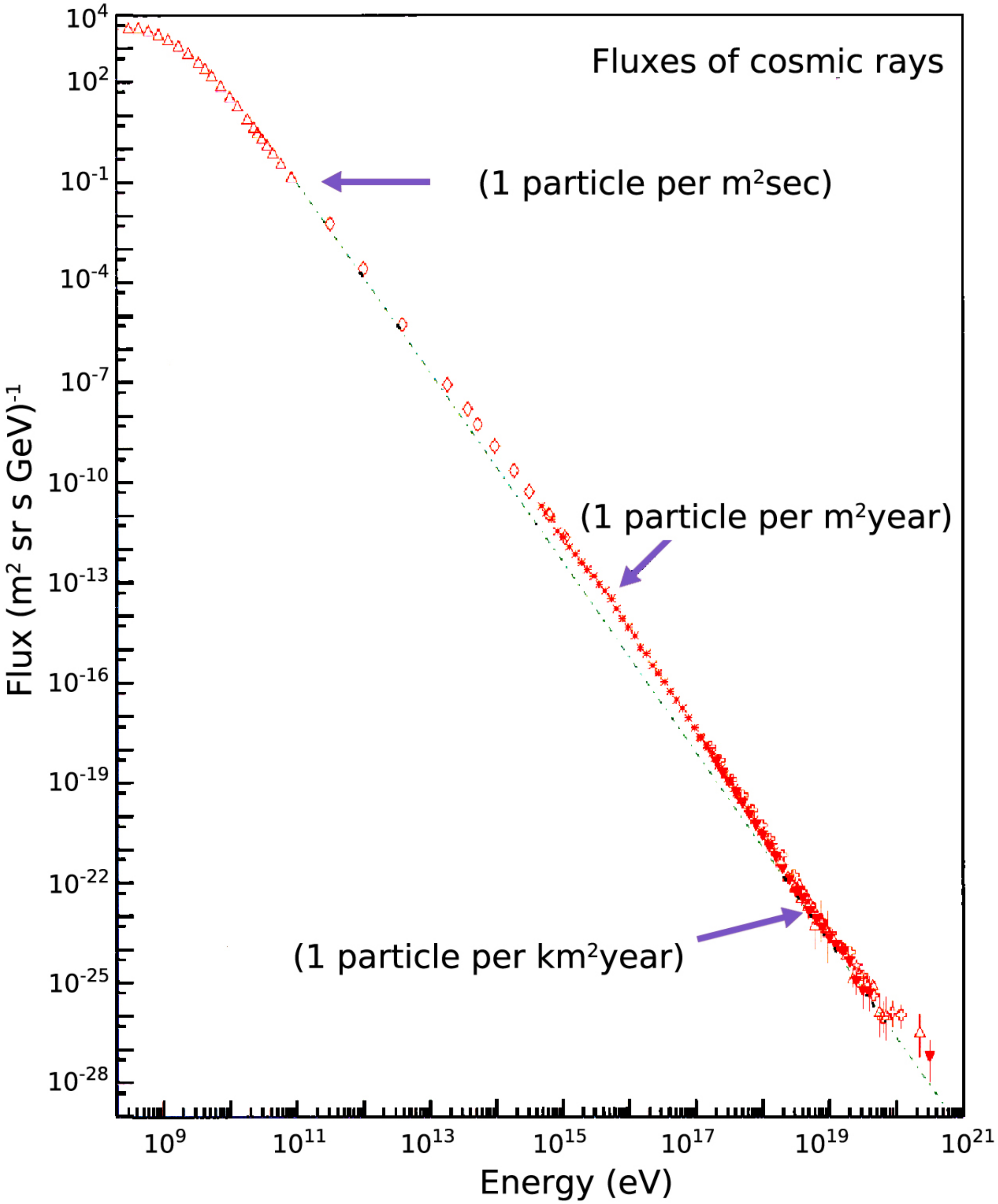
1. The Ice Cube Experiment

$\nu(\bar{\nu}) + X \rightarrow l^{\pm} +$ anything
$\nu(\bar{\nu}) + X \rightarrow \nu(\bar{\nu}) +$ anything

Discovery 2013:

2 events $E_\nu \cong 1\ PeV = 10^3\ TeV$
26 events $50\ TeV \lesssim E_\nu \lesssim 1\ PeV$
zero events above $2\ PeV$.

Galactic or extragalactic origin
($\sim$ 4.35 away from atmospheric ν background)



Fluxes of cosmic rays
(1 particle per m²sec)
(1 particle per m²year)
(1 particle per km²year)
Flux (m² sr s GeV)⁻¹
Energy (eV)
10^4
10^2
10^-1
10^-4
10^-7
10^-10
10^-13
10^-16
10^-19
10^-22
10^-25
10^-28
10^9
10^11
10^13
10^15
10^17
10^19
10^21

Interpretation: See e.g. recent review:

"Cosmic Neutrino Pevatrons: A Brand New Pathway to Astronomy, Astrophysics, and Particle Physics".
arXiv:1312.6587v3 (January 2014)
Anchordoqui et al., 80 pages

- Galactic, Extragalactic Models
- Cosmic Probes of Fundamental Physics

  e.g. Superheavy Dark Matter

  Leptoquark $\nu_\tau + q \rightarrow LQ \rightarrow \tau + q$

  Exotic ν-properties:

  $\nu_e e^- \rightarrow W^- \rightarrow$ anything, (Glashow 1960)

  $E_\nu \simeq 6.3 PeV$

  not seen.

  Speculation: $E^\nu_{Max} = \frac{m_\nu}{\sqrt{1-\beta_\nu^2}} = \gamma_\nu m_\nu$

  $\gamma_\nu = \frac{E^\nu_{Max}}{m_\nu} < \infty$ in violation of Lorentz invariance.

  (Barger et al. arxiv: 1404.0622)

2. The Ultra-High-Energy Cosmic-Neutrino-Nucleon Cross Section.

2.1 Introduction

For present and future search and investigation of ultra-high-energy cosmic neutrinos a prediction of the neutrino-nucleon cross section is indispensable.

Large extension of the kinematic range where experimental data available is necessary.

e.g.
Quigg et al. (1986)
Gandhi et al. (1996, 1998)
Cooper-Sarkar and Sarkar (2008, 2011)

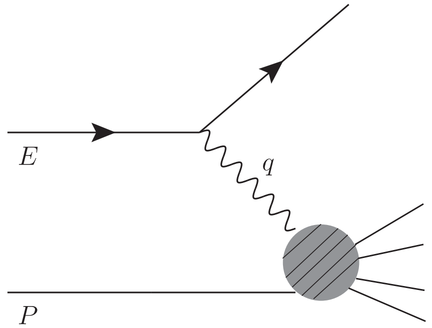
2.2 Neutrino-Nucleon Cross Section

$$\sigma_{\nu N}(E) = \frac{G_F^2}{2\pi} \int_{Q^2_{min.}}^{s-M_p^2} dQ^2 \left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \int_{M_p^2}^{s-Q^2} \frac{dW^2}{W^2} \sigma_r(x, Q^2).$$

e.g. Goncalves and Hepp (2011)

$$s = 2M_pE + M_p^2 \cong 2M_pE,$$

$$x = \frac{Q^2}{2qP} = \frac{Q^2}{W^2 + Q^2 - M_p^2} \cong \frac{Q^2}{W^2},$$



$$\sigma_r(x, Q^2) = \frac{1 + (1-y)^2}{2} F_2^\nu(x, Q^2) - \frac{y^2}{2} F_L^\nu(x, Q^2) + y(1 - \frac{y}{2}) x F_3^\nu(x, Q^2).$$

$$y = \frac{Q^2}{2M_pEx} \cong \frac{W^2}{s}.$$

For $s \gg M_W^2 \approx 10^4 \mathrm{GeV}^2$,

dominant contribution from $Q^2 \cong M_W^2$,

$$x \cong \frac{M_W^2}{s} \ll 0.1,$$

2.3 Connection to ep deep inelastic scattering (DIS)

HERA (1990 to 2007): DIS at low values of

$$x \equiv x_{bj} \simeq \frac{Q^2}{W^2}, \text{ where}$$
$$5 \cdot 10^{-4} \leq x \leq 10^{-1}$$
$$0 \leq Q^2 \leq 100 \text{GeV}^2$$

For n_f actively contributing quark flavors:

$$\frac{1}{n_f} F_{2,L}^{\nu N}(x, Q^2) = \frac{1}{\sum_q Q_q^2} F_{2,L}^{eN}(x, Q^2);$$

$$F_{2,L}^{\nu N}(x, Q^2) = \frac{n_f}{\sum_q^{n_f} Q_q^2} F_{2,L}^{eN}(x, Q^2), \quad \text{with } \frac{n_f}{\sum_q^{n_f} Q_q^2} = \frac{5}{18} \quad (\text{for } n_f = 4).$$

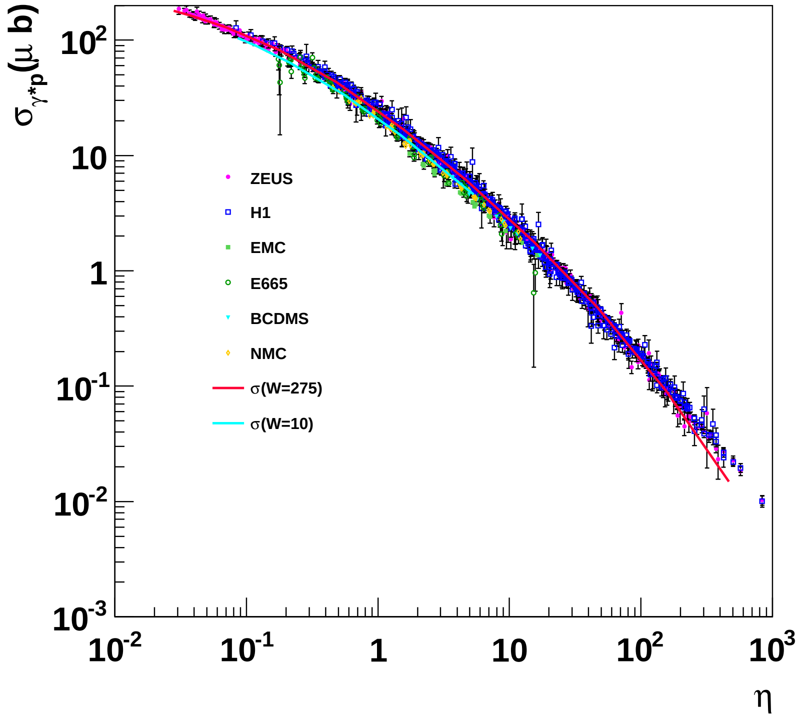
$$F_2^{ep}(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha} \sigma_{\gamma^* p}(W^2, Q^2).$$

2.4. DIS – Empirical Results

Low-x Scaling

Empirically : $\eta(W^2, Q^2) \equiv \dfrac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)}$,

$$\Lambda_{sat}^2(W^2) \sim (W^2)^{C_2}$$



Schildknecht, Surrow, Tentyukov (2000)

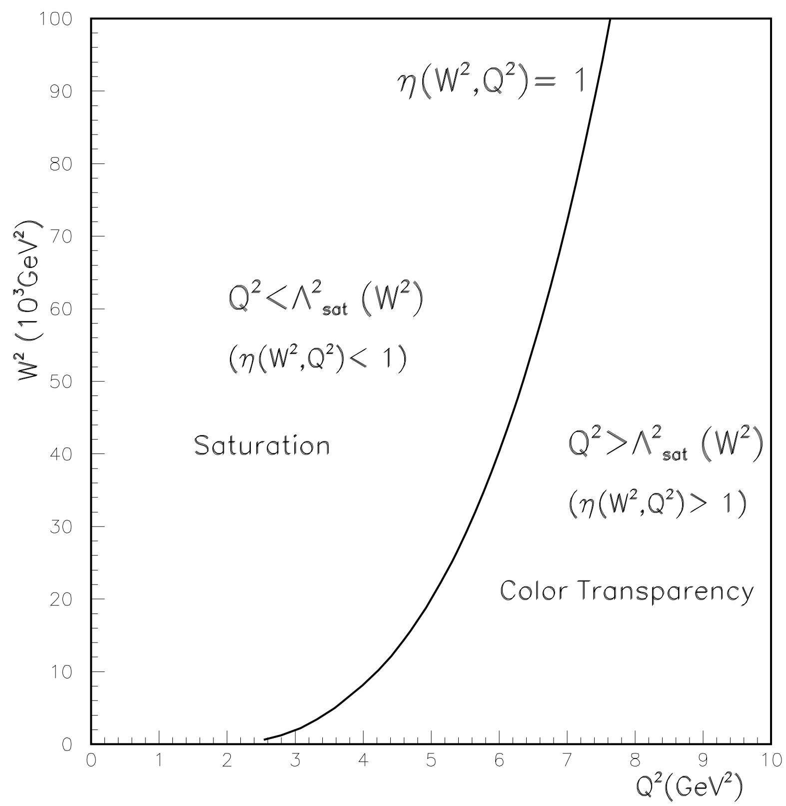
$$\sigma_{\gamma^* p}(W^2, Q^2) \;=\; \sigma_{\gamma^* p}(\eta(W^2, Q^2))$$
$$\sim\; \sigma^{(\infty)} \begin{cases} ln\frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2,Q^2) \ll 1 \\ \frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2,Q^2) \gg 1 \end{cases}$$

The limit of $\eta(W^2, Q^2) \to 0$, or $W^2 \to \infty$ at Q^2 fixed

$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma^* p}(\eta(W^2,Q^2=0))} = \lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\ln\left(\frac{\Lambda^2_{sat}(W^2)}{m_0^2}\frac{m_0^2}{(Q^2+m_0^2)}\right)}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1 + \lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\ln\frac{m_0^2}{Q^2+m_0^2}}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1.$$

$$\sigma_{\gamma^* p}\left(\eta(W^2,Q^2=0)\right) = \sigma_{\gamma p}(W^2)$$

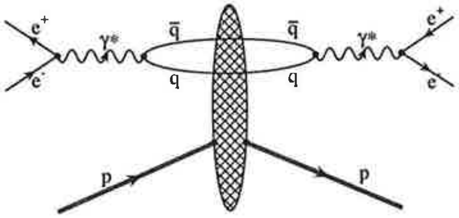
D. Schildknecht, DIS 2001 (Bologna)



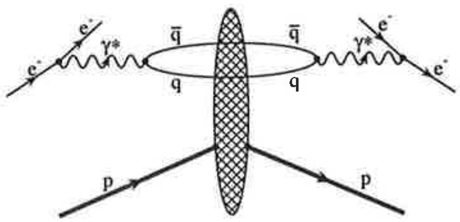
$Q^2[GeV^2]$	$W^2[GeV^2]$	$\frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma p}(W^2)}$
1.5	2.5×10^7	**0.5**
	1.26×10^{11}	**0.63**

2.5 The Color Dipole Picture (CDP).

The longitudinal and the transverse photoabsorption cross section

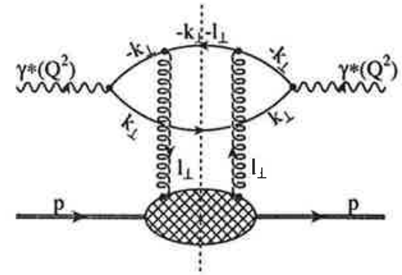


(a)



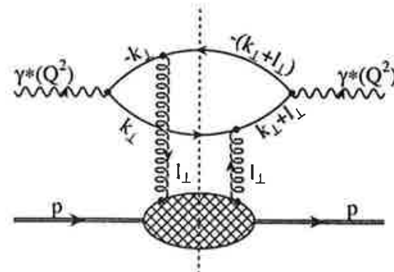
(b)

channel 1:

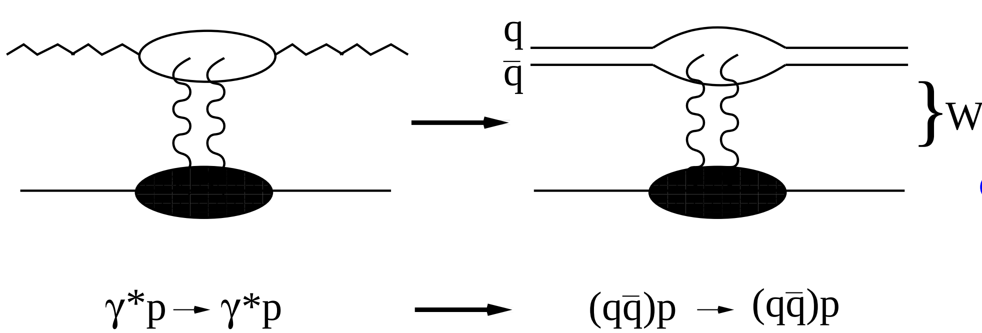


(a)

channel 2:



(b)

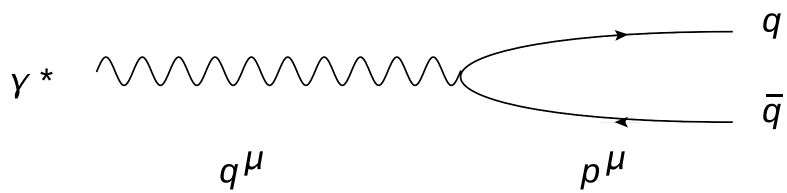


Generalized Vector Dominance

Sakurai, Schildknecht (1972)

Low (1975)

$$\gamma^* \longrightarrow (\rho^0, \omega, \phi) \; + \; \text{continuum}$$
$$\sim (q\bar{q}) \text{ continuum } (M_{q\bar{q}}),$$



Lifetime $\quad \tau = \frac{1}{\Delta E} = \frac{2M_p\nu}{Q^2+M^2_{q\bar{q}}}\frac{1}{M_p} \gg \frac{1}{M_p}.$

A) $\quad \sigma_{\gamma^*_{L,T}}(W^2, Q^2) = \int dz \int d^2\vec{r}_\perp |\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|^2$

$$\times \, \sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)$$

Remarks: i) $|\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|$: Probability for $\gamma^*_{L,T} \rightarrow q\bar{q}$ fluctuation

ii) $\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)$: $(q\bar{q})p$ cross section dependent on W^2

(not on $x \equiv \frac{Q^2}{W^2}$)

Gauge-invariant two-gluon coupling

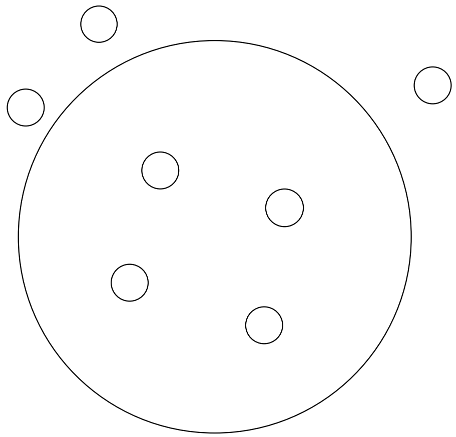
B) $\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) = \int^{\vec{l}_\perp^{\,2}{}_{Max}(W^2)} d^2\vec{l}_\perp \tilde{\sigma}\left(\vec{l}_\perp^{\,2}, z(1-z), W^2\right)\left(1 - e^{-i\vec{l}_\perp \vec{r}_\perp}\right)$

i) $\vec{r}_\perp \vec{l}_\perp < r_\perp l_{\perp\ Max}(W^2) \ll 1$

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) \sim \vec{r}_\perp^{\,2} \int^{\vec{l}_\perp^{\,2}{}_{Max}(W^2)} d\vec{l}_\perp^{\,2} \vec{l}_\perp^{\,2} \tilde{\sigma}(\vec{l}^{\,2}, z(1-z), W^2)$$

$$= \vec{r}_\perp^{\,2} \Lambda^2_{sat}(W^2)$$

Nikolaev, Zakharov (1991)



"Color Transparency"

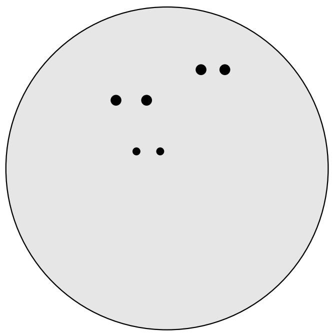
$$\Lambda^2_{sat}(W^2) \ll \frac{1}{\vec{r}_\perp^{\,2}}$$

$$\frac{Q^2}{\Lambda^2_{sat}(W^2)} \cong \eta(W^2, Q^2) \gg 1$$

ii) $r_\perp l_{\perp\ Max}(W^2) \gg 1$

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) \sim \int^{\vec{l}_\perp^{\,2}\ Max(W^2)} d\vec{l}_\perp^{\,2} \tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2)$$

$$= \sigma^{(\infty)}(W^2)$$



"saturation"

$q\bar{q}$ dipole $\cong$ hadron

$$\Lambda^2_{sat}(W^2) \gg \frac{1}{\vec{r}_\perp^{\,2}}$$

$$\frac{Q^2 + m_0^2}{\Lambda^2_{sat}(W^2)} \equiv \eta(W^2, Q^2) \ll 1$$

Photoabsorption Cross Section

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_{\gamma^* p}(\eta(W^2, Q^2)) =$$

$$= \frac{\alpha}{\pi} \sum_q Q_q^2 \begin{cases} \frac{1}{6}(1+2\rho)\sigma_L^{(\infty)}(W^2)\frac{1}{\eta(W^2,Q^2)} & , \quad (\eta(W^2,Q^2) \gg 1) \\ \sigma_T^{(\infty)}(W^2) \ln \frac{1}{\eta(W^2,Q^2)} & , \quad (\eta(W^2,Q^2) \ll 1) \end{cases}$$

$q\bar{q}$ dipole interaction *implies*

i) scaling in $\eta(W^2, Q^2)$

ii) color transparency and saturation.

On the Gluon Distribution

$$\sigma_{\gamma^* p}(\eta(W^2,Q^2)) \sim \begin{cases} \sigma^{(\infty)} \frac{\Lambda^2_{sat}(W^2)}{Q^2} & , \quad (\eta(W^2,Q^2) \gg 1) \\ \sigma^{(\infty)} \ln \frac{\Lambda^2_{sat}(W^2)}{Q^2+m_0^2} & , \quad (\eta(W^2,Q^2) \ll 1) \end{cases}$$

$$F_2(x,Q^2) \sim \begin{cases} \sigma^{(\infty)} \Lambda^2_{sat}(W^2) & \sim \quad \alpha_s(Q^2) xg(x,Q^2)\big|_{x=\frac{Q^2}{W^2}} \\ Q^2 \sigma^{(\infty)} \ln \frac{\Lambda^2_{sat}(W^2)}{Q^2+m_0^2} & \sim \quad Q^2 \sigma^{(\infty)} \ln \frac{\alpha_s xg(x,Q^2)}{\sigma^{(\infty)}(Q^2+m_0^2)}\Big|_{x=\frac{Q^2}{W^2}} \end{cases}$$

Saturation: Transition from $F_2 \sim \alpha_s xg(x,Q^2)$

to $\quad F_2 \sim Q^2 \sigma^{(\infty)} \ln \frac{\alpha_s xg(x,Q^2)\big|_{x=\frac{Q^2}{W^2}}}{\sigma^{(\infty)}(Q^2+m_0^2)}$

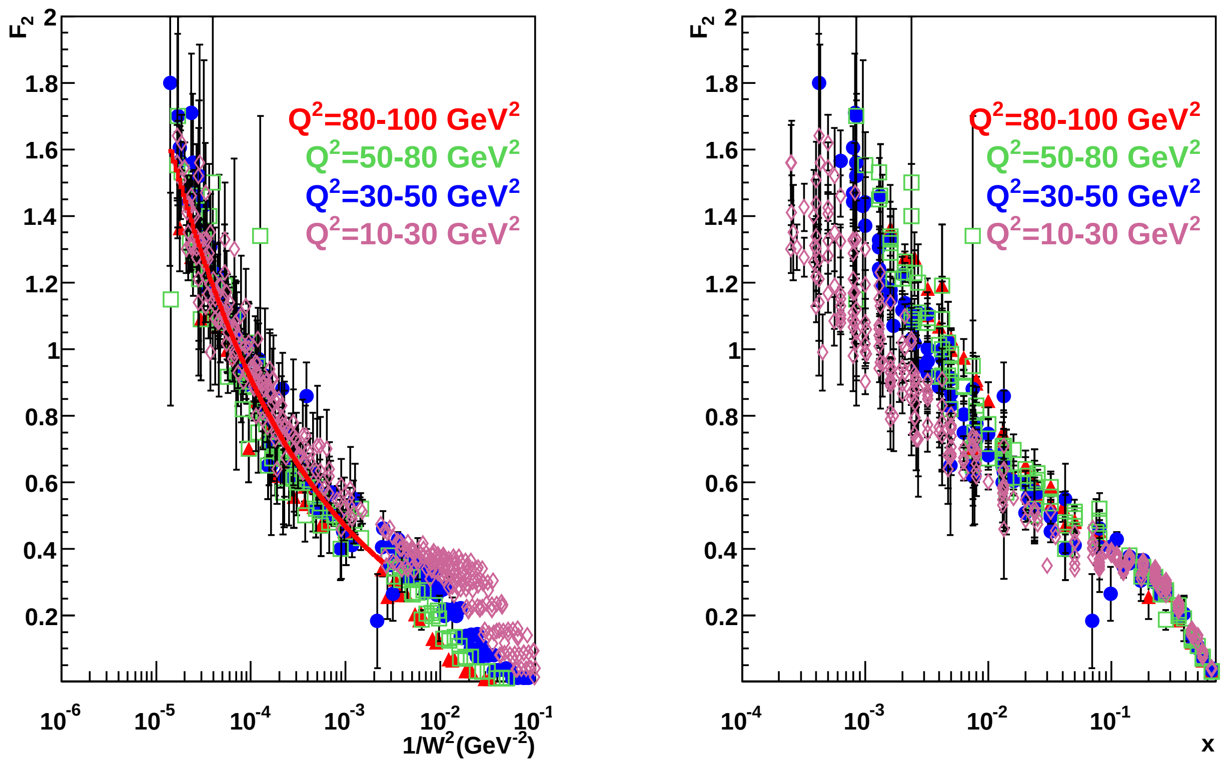
Measurement of $\Lambda^2_{sat}(W^2)$ (i.e. gluon distribution)

for $\eta(W^2,Q^2) \gg 1$

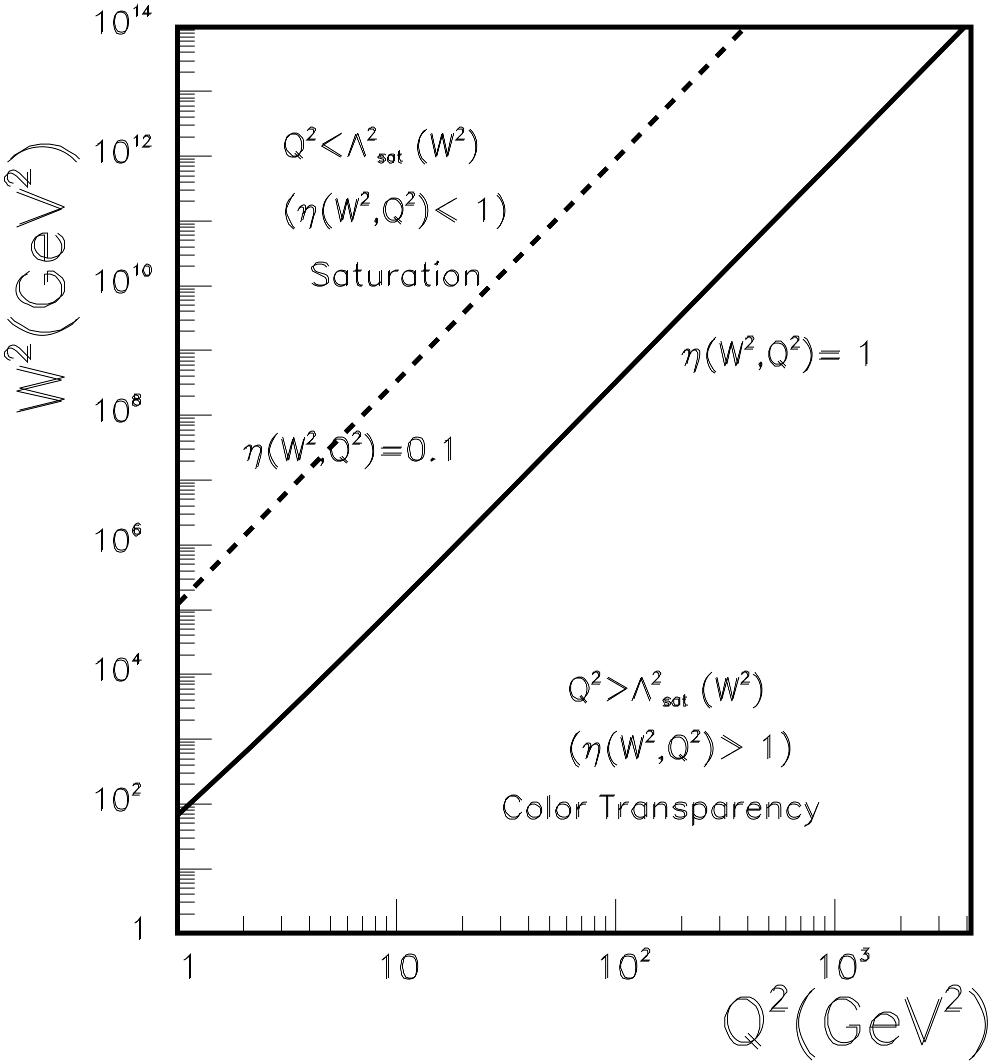
determines saturation behavior for $\eta(W^2,Q^2) \ll 1$.

$$F_2(W^2) = f_2 \cdot \left(\frac{W^2}{1\mathrm{GeV}^2}\right)^{C_2=0.29}$$

$$f_2 = 0.063 \quad \text{(fitted parameter)}$$



Experimental evidence for $F_2(x, Q^2) = F_2(W^2 \cong Q^2/x)$ and for the prediction of $C_2 = 0.29$.



$Q^2 < \Lambda^2_{sat}\ (W^2)$
$(\eta(W^2,Q^2) < 1)$
Saturation
$\eta(W^2,Q^2)=0.1$
$\eta(W^2,Q^2)= 1$
$Q^2 > \Lambda^2_{sat}\ (W^2)$
$(\eta(W^2,Q^2) > 1)$
Color Transparency
$W^2(GeV^2)$
$Q^2(GeV^2)$
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
1
1
10
10^{2}
10^{3}

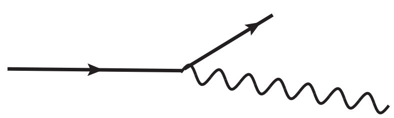
Conventional argument:

$$\sigma_{\gamma^* p}(\eta) \sim \sigma^{(\infty)} \frac{1}{\eta} \sim \frac{\alpha_s(Q^2) x g(x, Q^2)}{Q^2}$$

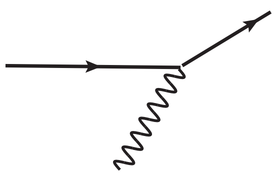
assumes validity of $1/\eta$ dependence for $\eta \to 0$

gluon distribution $\sim F_2 \to \infty$

saturation due to non-linear gluon evolution.



Quark splitting



recombination

ignores $q\bar{q}$ as color dipoles.

CDP: Saturation $\cong$ hadronic cross section,

direct consequence of $q\bar{q}$ dipole interaction.

Non-linear evolution not needed in DIS.

Explicit expression for $\sigma_{\gamma^* p}(W^2 Q^2)$ in the CDP:

Cvetic, Schildknecht, Surrow, Tentyukov (2000/2001)

Ansatz: $\sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'_\perp \Lambda_{\rm sat}(W^2)) = \sigma^{(\infty)}(W^2)(1 - J_0(r'_\perp \Lambda_{\rm sat}(W^2)))$

$$\sigma_{\gamma^* p}(W^2, Q^2) = \frac{\alpha R_{e^+e^-}}{3\pi} \sigma^{(\infty)}(W^2) I_0(\eta(W^2, Q^2)) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right),$$

$$R_{e^+e^-} = 3 \sum_q Q_q^2, \quad \Lambda^2_{\rm sat}(W^2) = C_1 \left(\frac{W^2}{1 {\rm GeV}^2}\right)^{C_2}.$$

$$I_0(\eta(W^2, Q^2)) = \frac{1}{\sqrt{1 + 4\eta(W^2, Q^2)}} \ln \frac{\sqrt{1 + 4\eta(W^2, Q^2)} + 1}{\sqrt{1 + 4\eta(W^2, Q^2)} - 1} \cong$$

$$\cong \begin{cases} \ln \frac{1}{\eta(W^2, Q^2)} + O(\eta \ln \eta), & \text{for } \eta(W^2, Q^2) \to \frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}, \\ \frac{1}{2\eta(W^2, Q^2)} + O\left(\frac{1}{\eta^2}\right), & \text{for } \eta(W^2, Q^2) \to \infty, \end{cases}$$

Photoproduction limit:

$$\sigma^{(\infty)}(W^2) = \frac{3\pi}{R_{e^+e^-} \alpha} \frac{1}{\ln \frac{\Lambda^2_{sat}(W^2)}{m_0^2}} \begin{cases} \sigma_{\gamma p}^{Regge}(W^2), \\ \sigma_{\gamma p}^{PDG}(W^2). \end{cases} \quad \text{where} \begin{cases} \sigma_{\gamma p}^{Regge}(W^2) & \sim (W^2)^{0.0808} \\ \sigma_{\gamma p}^{PDG}(W^2) & \sim (\ln W^2)^2 \end{cases}$$

For $\eta(W^2, Q^2) \gg 100$, exclusion of high-mass $q\bar{q}$ fluctuations:

$$m_1^2(W^2) = \xi \Lambda_{\rm sat}^2(W^2), \text{ where empirically } \xi = 130.$$

$$\sigma_{\gamma^* p}(W^2, Q^2) = \frac{\alpha R_{e^+e^-}}{3\pi} \sigma^{(\infty)}(W^2) I_0(\eta(W^2, Q^2)) \times \frac{\frac{\xi}{\eta(W^2,Q^2)}}{1 + \frac{\xi}{\eta(W^2,Q^2)}} + O\left(\frac{m_0^2}{\Lambda_{\rm sat}^2(W^2)}\right)$$

where

$$\frac{\frac{\xi}{\eta(W^2,Q^2)}}{1 + \frac{\xi}{\eta(W^2,Q^2)}} \cong \begin{cases} 1, & \text{for } \eta(W^2, Q^2) \ll \xi = 130 \\ \frac{\xi}{\eta(W^2,Q^2)}, & \text{for } \eta(W^2, Q^2) \gg \xi = 130 \end{cases}$$

Parameters:

$$\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda_{\text{sat}}^2(W^2)};$$

$m_0^2 = 0.15\ \text{GeV}^2 \quad (m_0^2 < m_\rho^2)$

$$\Lambda_{\text{sat}}^2(W^2) = C_1 \left(\frac{W^2}{1\ \text{GeV}^2}\right)^{C_2},$$

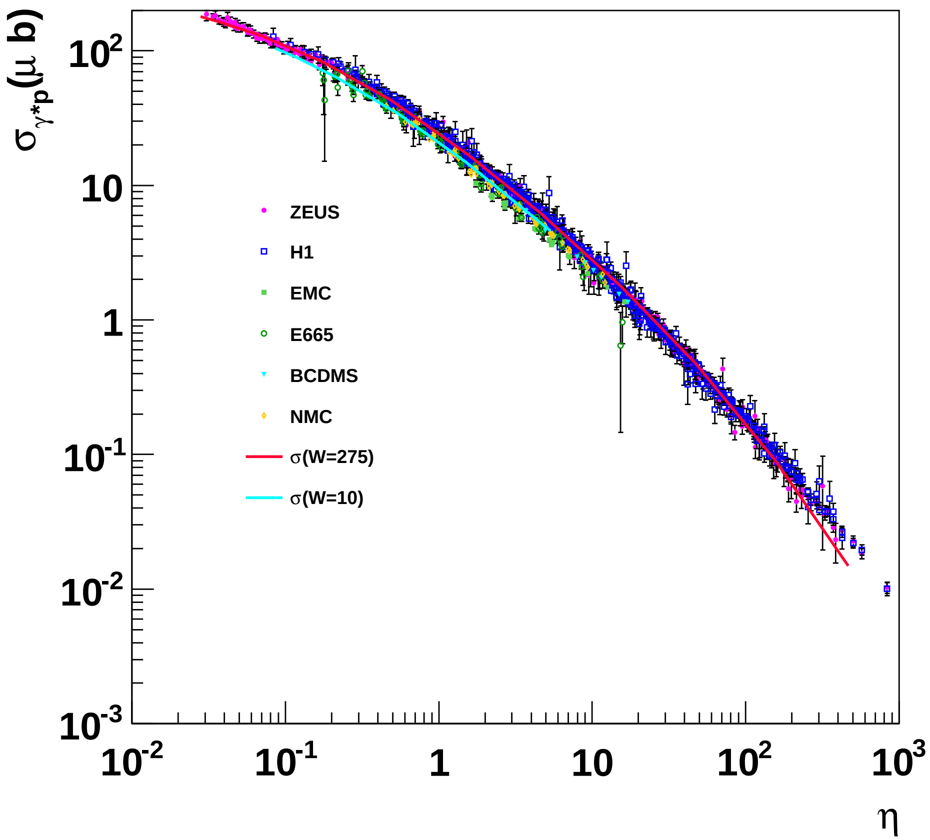
$C_1 = 0.34\ \text{GeV}^2$

$C_2 = 0.29$

$$M_{q\bar{q}}^2 \leq m_1^2(W^2) = \xi \Lambda_{\text{sat}}^2(W^2);$$

$\xi = 130$

The photoabsorption cross section $\sigma_{\gamma^* p}(\eta(W^2, Q^2))$



$$\eta = \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)}$$

3. The Neutrino-Nucleon Cross Section in the CDP

$$\sigma_{\nu N}(E) = \frac{G_F^2 M_W^4}{8\pi^3\alpha} \frac{n_f}{\sum_q Q_q^2} \int_{Q^2_{Min}}^{s-M_p^2} dQ^2 \frac{Q^2}{(Q^2+M_W^2)^2} \times \int_{M_p^2}^{s-Q^2} \frac{dW^2}{W^2} \frac{1}{2}(1+(1-y)^2)\sigma_{\gamma^* p}(\eta(W^2,Q^2)).$$

Kuroda, Schildknecht, arXiv:1305.0440v3, Phys. Rev. D88 (2013) 053007

$$r(E) = \frac{\sigma_{\nu N}(E)_{\eta(W^2,Q^2)<1}}{\sigma_{\nu N}(E)}.$$

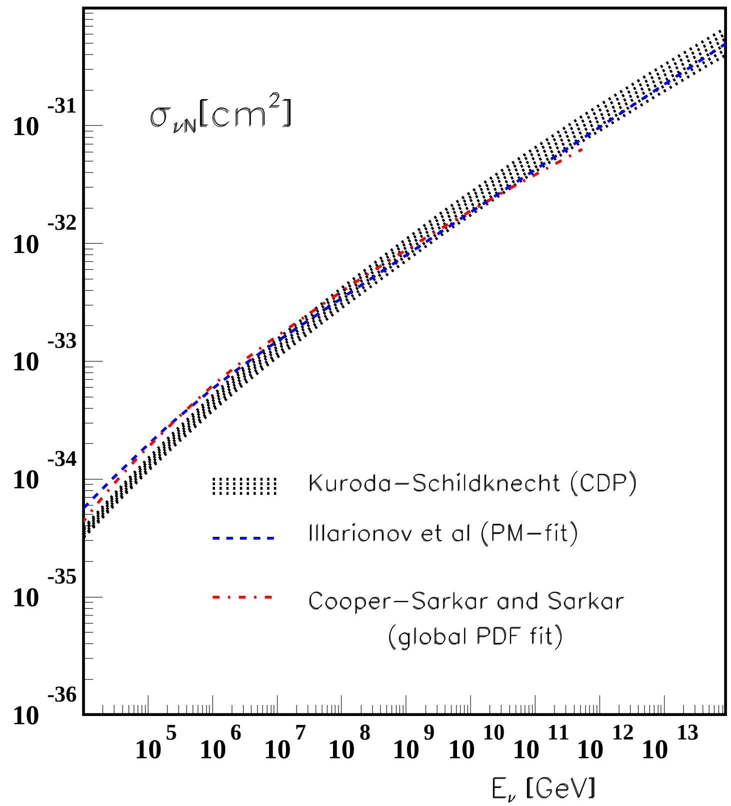
Contribution from saturation region

$$r(E) < \bar{r}(E),$$

$$\bar{r}(E) = \frac{2\int_{Q^2_{Min}}^{Q^2_{Max}(s)} dQ^2 \frac{Q^2}{(Q^2+M_W^2)^2} \int_{W^2(Q^2)_{Min}}^{s-Q^2} \frac{dW^2}{W^2} \ln \frac{1}{\eta(W^2,Q^2)}}{\int_{Q^2_{Min}}^{s-M_p^2} dQ^2 \frac{Q^2}{(Q^2+M_W^2)^2} \int_{M_p^2}^{s-Q^2} \frac{dW^2}{W^2} \frac{1}{2\eta(W^2,Q^2)}}.$$

$$r(E) < \bar{r}(E) = \frac{1}{2}\frac{\Lambda^2_{sat}(s)}{M_W^2} = \begin{cases} 1.74 \times 10^{-3} & \text{for } E = 10^6\text{GeV} \\ 2.51 \times 10^{-2} & \text{for } E = 10^{10}\text{GeV} \\ 3.63 \times 10^{-1} & \text{for } E = 10^{14}\text{GeV} \end{cases}.$$

The (charged-current) neutrino-nucleon cross section, $\sigma_{\nu N}(E)$, (based on $\sigma_{\gamma^* p}(\eta(W^2, Q^2))$ from CDP) as a function of the neutrino energy $E_\nu(GeV)$.



4. Comparison with "Froissart-inspired" ansatz

Heisenberg (1953):

Lorentz-contracted sphere with exponentially decreasing edge
Minimum "blackness" necessary for particle production.
Collision radius then given by radius of "suffiently black" region.
$\sigma_{hN}(W^2) \sim (\ln W^2)^2$

Froissart (1961):

From unitarity and analyticity, upper bound,
$\sigma_{hN}(W^2) < (\ln W^2)^2$.

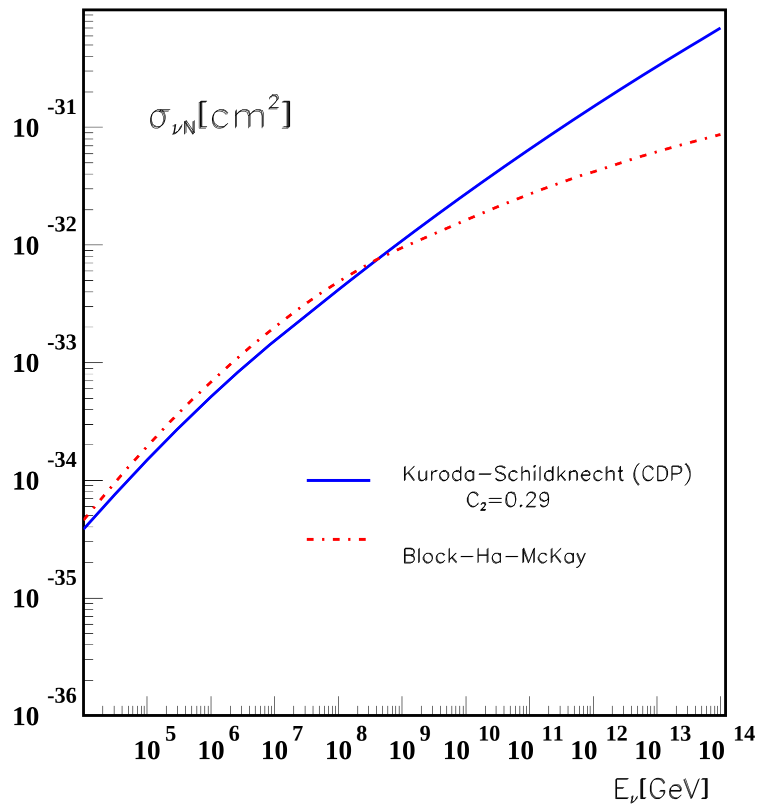
"Froissart-inspired ansatz":

$$F_2^{ep}(x_1, Q^2) \sim \sum_{n,m=0,1,2} a_{nm} (\ln Q^2)^n (\ln(1/x))^m$$

Block et al. (2006 to 2013)

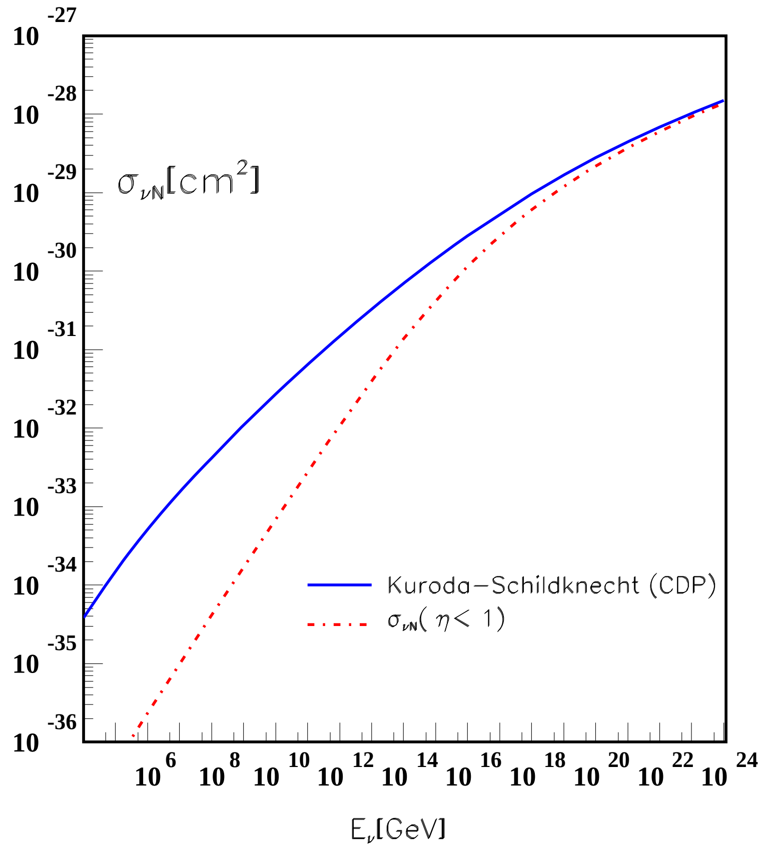
Fit to HERA low-x data with seven fit parameters.

Comparison of $\sigma_{\nu N}(E)$ from the CDP with the results from the "Froissart-inspired" ansatz



On suppression see also
e.g. Machado (2011)

The neutrino-nucleon cross section in the CDP for (unrealistic!) ultra-ultra-high energies, $E \leq 10^{24}$GeV. Reduced growth of cross section for $E \gg 10^{12}$ GeV



$$\sigma_{\nu N}^{(c)}(E) = \sigma_{\nu N}^{(c)}(E)_{\eta(W^2,Q^2)<1} + \sigma_{\nu N}(E)^{(c)}_{\eta(W^2,Q^2)>1}$$

5. Conclusions

- **Inspired by cosmic-neutrino-search experiments.**
  Thorough examination of neutrino-nucleon cross section important for future results on ultra-high cosmic neutrinos.

- **Predictions for the charged-current neutrino-nucleon cross section based on the CDP are consistent with results obtained from pQCD fits ($E \leq 10^{12}$ GeV).**

- **The results based on the CDP *disagree* with results from the "Froissart-inspired" ansatz that predicts a strong suppression of the neutrino-nucleon cross section for energies $E \geq 10^{9}$ GeV**

- **A suppression of the cross section, i.e., a weaker growth with increasing energy, only occurs at unrealistically high ("ultra-ultra-high") energies of $E \gtrsim 10^{12}$ GeV**