HIGHER CURVATURE SUPERGRAVITY,
SUPERSYMMETRY BREAKING AND INFLATION

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IN MEMORY OF MY FRIEND AND COLLEAGUE

BRUNO ZUMINO

(1923–2014)
PLAN OF LECTURES

**Motivation and Scenarios for Inflation**

Standard Model for Cosmology: The Basics

The Inflaton Component (as a Perfect Fluid)

Starobinsky (Planck) Versus Chaotic Inflation (BICEP2)

Embedding in Supergravity and Different Formulations

Minimal Models: Chiral Versus Vector Supermultiplets

Higher Curvature and Supersymmetry Breaking

Integrating Out the SGoldstino Multiplet:

Volkov-Akulov-Starobinsky Supergravity
This lecture is devoted to the application of higher curvature supergravity to a particular class of cosmological models for inflation in which the "inflaton" field is identified with the "scalaron," the latter being a purely gravitational mode which comes when we add to the Einstein-Hilbert action a term quadratic in the (scalar) curvature

\[ \mathcal{L}_{\text{modified}} = \mathcal{L}_E + \alpha R^2 = R + \alpha R^2 \]

This theory is "dual" to standard Einstein coupled to a scalar field
The revival of these models is motivated by the fact that recent experiments (Planck, BICEP) seem to favor simple one field cosmological models for inflation even if there is a tension between the two different experiments. In fact while for the slow roll parameter $n_s = 1 - 6\epsilon + 2\eta \approx 1 - \frac{2}{N} = 0.96$ (spectral index of scalar perturbation) the same formula agrees for the other slow roll parameter $\eta$ (ratio of tensor to scalar perturbations) different models seem to be favorite.
$z = \frac{12}{N^2} \left( \text{STAROBINSKY INFLATION}, \text{HIGGS INFLATION} \right)$

$z < 0.08 \left( \text{PLANCK} \right)$

or

$z = \frac{8}{N} \left( \text{CHAOTIC INFLATION} \right)$ \text{ LINDE} \leq 0.2$

The slow roll parameters $\epsilon, \eta$ and the number of e-folding $N$ before the end of inflation are defined in terms of the scalar potential $V(\phi)$ of a canonically normalized inflaton field with Lagrangian $\mathcal{L} = \mathcal{L}_E - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi)$

$\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2, \eta = \frac{V''}{V}, N = \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V_i} d\phi \quad (N = 50-60)$
\text{e-folding number} \quad N

\[ N = \int_{t_0}^{t_f} H(t) \, dt = \ln \frac{a_f}{a_o} \]

\[ a_f = a_o \exp N \]

\[ a_f = a_o \prod_i \exp N_i \]

Suppose \( N = \sum_i N_i \)

If \( H(t) \sim H_0 t \) \( \Rightarrow \)

\[ H_0 \Delta t = N \]
The standard model for cosmology describes the universe as made of different forms of energy densities, which act as sources of the gravitational field, which in turn is described by a Friedmann-Lemaître-Robinson-Walker geometry

\[ ds^2 = dt^2 + a(t)^2 \left[ \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2 \right] \]

where the three-dimensional slice is a maximally symmetric space \((R_{ij,kl}=K(g_{ij}g_{je}-g_{ie}g_{jk}))\) and \((K=0,\pm1)\) for \(K>0, K<0, K=0\) we refer to closed, open and flat universe. \(a(t)\) is the "scale factor," which tells us how big is the 3d-slice at (comoving) time \(t\).
The above assumptions are motivated by the "Copernican Principle," namely that our universe looks isotropic and homogeneous. Isotropy says that space looks the same in any direction. Homogeneity is that the metric looks the same everywhere. If a space is isotropic everywhere then it is homogeneous. If we have both of them then the 3D slice is a maximally symmetric space: \( \frac{SO(4)}{SO(3)} \) \( k>0 \), \( \frac{SO(3,1)}{SO(3)} \) \( k<0 \), \( \mathbb{E}_3 \) \( k=0 \).
The Einstein equations are
\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \quad (\kappa = R_p^4) \quad (\kappa = 8\pi G = \frac{1}{\hbar c^4}) \]

Or equivalently,
\[ R_{\mu\nu} = \kappa \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) \text{ which implies } T = T^\rho_\rho = -\frac{1}{\kappa} R \]

Note that if \( T_{\mu\nu} = -\Lambda g_{\mu\nu} \) (vacuum energy), then \( \kappa = 4\pi \Lambda = \text{const.} \) and we have a maximally symmetric space-time which is

\[ \kappa > 0 \text{ (De Sitter), } \Lambda < 0 \text{ (Antide Sitter), } \Lambda = 0 \text{ (Minkowski) } \]

What is the general form of \( T_{\mu\nu} \)?

We can answer to this question if we make the picture of the universe as being
MADE BY PERFFECT FLUIDS, DESCRIBED BY \((p, \rho)\)

ENERGY DENSITY AND PRESSURE WITH

\[
T_{\mu\nu} = (p+\rho) U_\mu U_\nu + \rho g_{\mu\nu} = (p+\rho)(U_\mu U_\nu + \frac{1}{4} g_{\mu\nu}) + \frac{1}{4} (3p-\rho) g_{\mu\nu}
\]

WHERE \(U_\mu\) IS THE RELATIVISTIC FOUR VELOCITY VECTOR.

IF WE IMPOSE AN "EQUATION OF STATE"

\[
p = W\rho \quad (\text{WHERE } p = p(t), \rho = \rho(t), W \text{ CONST})
\]

WE HAVE SEVERAL VALUES FOR \(W - |W| \leq 1\) NDEC

MOREOVER THE STRESS TENSOR CONSERVATION

\[
\nabla_\mu T_{\mu\nu} = 0 \quad \text{IMPLIES} \quad \dot{\rho} = -3(1+W) \frac{a}{a} \Rightarrow \]

\[
\rho \propto a^{-3(1+W)} \quad \text{- NOTE IN PARTICULAR THAT A}
\]

VACUUM ENERGY \(T_{\mu\nu} \propto g_{\mu\nu} \Rightarrow p+\rho = 0 (W = -1)\) WHILE A TRACELESS

STRESS TENSOR \(T_{\mu\nu}^{\mu\nu} = 0 \Rightarrow 3p-\rho = 0 \Rightarrow W = \frac{1}{3} \) (radiation)
The value of the scalar curvature is

\[ R = -kT_\mu^\mu = k(1-3W)\rho = \sum_i k_i (1-3W_i) \rho_i \]

For \( W = -1 \) we have dS or AdS depending whether \( \rho > 0 \)

The Einstein equations now read

\[ G_{00} = kT_{00} \]
\[ G_i^i = kT_i^i \]

Because of the symmetries of the FLRW geometry (one is first derivative & the other is second derivative)

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \quad k = \pm 1, 0 \]

By introducing the Hubble parameter \( H(t) = \frac{\dot{a}}{a} \)

And noticing that \[ \frac{\ddot{a}}{a} = H + H^2 \]

We can rewrite the Einstein Eqs. as
\[ H^2 = \frac{8\pi G}{3} (\rho - \frac{K}{a^2}) \quad \dot{H} = -4\pi G (\rho + p) + \frac{K}{a^2} \]

where we can define \( \rho_k = \frac{-3K}{8\pi G a^2} \), \( \omega_k = -\frac{1}{3} \) so that

\[ H^2 = \frac{8\pi G}{3} (\rho + \rho_k) \quad \dot{H} = -4\pi G (1 + \omega) \rho - 4\pi G (1 - \frac{4}{3}) \rho_k \]

and in more genera way, if \( \rho = \sum_i \rho_i \), \( \rho_i = \omega_i \rho_i \)

we have

\[ H^2 = \frac{8\pi G}{3} \sum_i \rho_i \quad \dot{H} = -4\pi G \sum_i (1 + \omega_i) \rho_i \]

(where \( \rho_i \) include \( \rho_k \))

by dividing the first by \( H^2 \) and defining the density parameter \( \Omega = \frac{8\pi G \rho}{3H^2} \) we have

\[ \Omega - 1 = \frac{K}{a^2} \] defining \( \rho_{\text{crit}} = \frac{3H^2}{8\pi G} \) we have

\[ \Omega = \frac{\rho}{\rho_{\text{crit}}} \quad \sum_i \Omega_i = 1 \quad \text{with} \quad \Omega_i = \frac{\rho_i}{\rho_{\text{crit}}} \quad \text{(including} \ \rho_k) \]
The second Friedmann equation can be written in terms of the "deceleration parameter"

$$q = -a \frac{\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2} (H + H^2) = \frac{4\pi G}{3H^2} (\rho + 3p) = \frac{4\pi G}{3H^2} (1+3W)\rho$$

Again for several components of energy densities

$$q = \frac{4\pi G}{3H^2} \sum_i (1+3W_i)\rho_i = \sum_i \frac{\Omega_i}{2} (1+3W_i)$$

Note that $\Omega_k$ does not contribute to $q$

So the two Einstein Eq's give

$$\sum_i \Omega_i = 1 \quad , \quad q = \sum_i \frac{\Omega_i}{2} (1+3W_i) \leftrightarrow (W = \frac{1}{3}, \text{does not contribute})$$

Values of $W$ are:
- $W = 0$ (dust $\rightarrow$ baryonic matter)
- $W = -\frac{1}{3}$ (curvature contribution)
- $W = 1$ (fast roll scalar field)

Note that $R \geq 0$ when $W \leq \frac{1}{3}$ (only for $W = 1$ is negative)
Whenever a single term dominates we say that the universe is dominated by that component of \((p_i, \rho_i) \Rightarrow (w_i = \frac{p_i}{\rho_i}, \rho_i)\).

\(W = 0\) (Matter dominated), \(W = \frac{1}{3}\) (Radiation dominated)

\(W = -1\) (Vacuum dominated), \(W = -\frac{1}{3}\) (Curvature dominated)

When only one component dominates the Friedmann Eq's can be integrated easily:

\[
\left( \frac{a'}{a} \right)^2 \propto \frac{8\pi G}{3} a^{-3(1+W)} \Rightarrow (W \neq -1) \quad a \sim t^{\frac{2}{3(1+W)}}
\]

\(W = -1\) \(a \sim e^{Ht}\) (H constant)

Not surprisingly the two empty matter solutions \((W = -1, -\frac{1}{3})\) correspond to deSitter and Minkowski space (region of)
A SCALAR FIELD (INFLATION) AS A PERFECT FLUID COMPONENT OF THE UNIVERSE

\[ T_{\mu \nu} = -\frac{\partial L}{\partial g_{\mu \nu}} + g_{\mu \nu} \mathcal{L} \; ; \; \mathcal{L} = -\frac{1}{2} g^{\phi \phi} \dot{\phi}^2 - V(\phi) \]

WE COMPARE THE \( T_{\lambda \mu}^\phi \) WITH \( T_{\mu \nu} \) OF A PERFECT FLUID

\[ T_{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \; , \; T_{ij} = \rho g_{ij} \]

\[ \rho + p = \dot{\phi}^2 \]
\[ \rho - 3p = 2(\dot{\phi}^2 - V(\phi)) \]
\[ p - \rho = 2V \]
\[ T_{\lambda \mu} = \dot{\phi}^2 - 4V \]

IN GENERAL

\[ \frac{1}{2} \dot{\phi}^2 (1 - w) = V(1 + w) \]

THE MATTER EQUATION GIVES:

\[ \frac{\partial L}{\partial \phi} - \partial_\mu \partial_\lambda \phi = 0 \]
The full sets of EoS become

\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \]

\[ \dot{\phi} - \frac{k}{a^2} = -4\pi G \dot{\phi}^2 \]

\[ \ddot{\phi} + 3H \dot{\phi} + V_\phi = 0 \]

Inflation claims to solve the flatness and horizon problems. The flatness problem is the explanation for having today \( \Omega \approx 1 \) even if \( \Omega_{\text{matter}} \ll \Omega_k \). (Why \( k \approx 0 \))

The regime of inflation is \( \dot{\phi}^2 \ll V(\phi), |\dot{\phi}| \ll |3H\dot{\phi}|, |V'\) \( \)

(Sufficient period where potential energy dominates the kinetic energy)

For this to happen the slow roll parameters \( \epsilon, \eta \ll 1 \)
\[-3H \dot{\varphi} = V_\varphi \quad (\dot{\varphi} \ll 1)\]

\[H^2 = \frac{8\pi G \cdot V}{3} \quad (\dot{\varphi} \ll 1)\]

\[V = \frac{3H^2}{K} \Rightarrow \frac{V}{V_\varphi} = -\frac{3H^2}{K} \cdot \frac{1}{3H \varphi} = -\frac{H}{K}\]

\[H = -k \frac{V}{V_\varphi} \varphi^2 \Rightarrow \int_{t_0}^{t_{\text{end}}} H + dt = \int_{\varphi_0}^{\varphi_{\text{end}}} \frac{V}{V_1} d\varphi = H_0 \Delta t\]
THE STAROBINSKY MODEL: \( R + R^2 \) THEORY

Dual (conformally equivalent) to standard (Whitt) gravity coupled to a scalar field with a potential giving rise to inflation

\[
L = -\frac{1}{2} R + \alpha R^2 \quad (M_P = 8\pi G = 1)
\]

\[
L = -\frac{1}{2} R + \frac{1}{2} \sigma (\Lambda - R) + \alpha \Lambda^2 = -\frac{1}{2} R (1 + \sigma) + \frac{1}{2} \sigma \Lambda + \alpha \Lambda^2
\]

Now observe that we have a Jordan frame function \((1 + \sigma)\). Going to the Einstein frame through the change of variable \( g_{\mu\nu}^e = g_{\mu\nu} (1 + \sigma)^{-1} \)

We get, in terms of \( \phi \): \( 1 + \sigma = \exp \sqrt{\frac{\lambda}{3}} \phi \)

\[
L = -\frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{16\alpha} \left[ 1 - \exp -\sqrt{\frac{\lambda}{3}} \phi \right]^2
\]
OFF-SHELL COMPONENTS OF THE GRAVITY FIELD

GIVE EXTRA MASSIVE MODES IN HIGHER CURVATURE SUPERGRAVITY

\[ L^1 = L_E + L_{R^2} + L_{\text{Weyl}} \]

\[ \frac{1}{k} R + L_{R^2} + \beta W_{\mu \nu}^2 \]

\[ g_{\mu \nu} \rightarrow 10 - 4 = \Phi \text{ OFF-SHELL COMPONENTS} \]

\[ G_{\mu \nu} \rightarrow 10 - 4 = \Phi \text{ OFF-SHELL COMPONENTS} \]

\[ (\text{diffeomorphisms}) \]

\[ s \rightarrow 1 \quad + \quad S_{5} \quad + \quad 5 \quad \text{Spin} \; 2 \]

\[ M_{\mu}^2 \rightarrow \frac{1}{k} \]

\[ M_{2}^2 \rightarrow (\gamma \frac{1}{k \beta} \]
THE "SCALARON" POTENTIAL IN $R + R^2$ THEORY

$V(\phi) = \frac{\partial V}{\partial \phi} = 0 \rightarrow \text{UNBROKEN SUPERSYMMETRY}$

INFLATIONARY PHASE: $\frac{\partial V}{\partial \phi} \neq 0 \rightarrow \text{BROKEN SUPERSYMMETRY}$
By reinserting $M_{Pl}$ we get $\alpha^{-1} \propto H$.

The Chaotic Model:

$$\Lambda = -\frac{1}{2} R - \frac{1}{2} \left( \partial_\mu \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \rightarrow m \sim H$$

By rewriting normalized fields $\frac{\phi}{M_{Pl}}$ the de-Sitter cosmological constant $\Lambda$ during inflation is in all cases $\Lambda \sim H^2 M_{Pl}^2$ ($H/M_{Pl} \sim 10^{-5}$).
Here we present the supergravity embedding of these two models which is minimal in two respects: it uses the minimal set of multiplets needed to describe the models. It also uses the minimal off-shell representation of the underlying local supersymmetry algebra. The latter introduces new fields which are "auxiliary" (not propagating) in the standard Einstein supergravity but become propagating when higher curvature terms are introduced.
The minimal supergravity extension of such a model was derived in the late eighties (CECOTTI, CECOTTI, SF, PORRATI, SABHARWAL) in two different forms depending on two different off-shell completion of the supergravity multiplet:

- $V^a, \psi^I, A_\mu, S, P$

- $V^a, \psi^I, A_\mu, b_{\mu\nu}, A_\mu \rightarrow A_\mu + \partial_\mu a, b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$

The six bosonic degrees of freedom which make the gravity multiplet to have the same number of bosons and fermions $(12b + 12f)$ give two different supersymmetric extension of the Starobinsky M.
OFF-SHELL COMPONENTS OF (N=1) SUPERGRAVITY
FIELDS GIVE EXTRA MASSIVE SUPERMULTIPLETS
IN HIGHER CURVATURE SUPERGRAVITY

Graviton: $g_{\mu\nu} = 1_0 + 5_2$
$A_\mu, S, D = 6 \left( \frac{3}{2} + 1_0 + 1_0 + 1_0 \right)$

Gravitino: $\psi_{\mu
= 16 - 4 = 12 = 2 \times \frac{3}{2} + 2 \times \frac{1}{2}$

(LOCAL SUPERSYMMETRY)
OLD MINIMAL)
PHYSICAL MASSIVE EYRAL MULTIPLETS $2 \left( \frac{1}{2}, 1_2 \right) 4b + 4f$
GHOST SPIN 2 MULTIPLETS $2 2 \left( 3_2 \right), 1$

NEW MINIMAL) $A_\mu, b_\mu \rightarrow 3 + 3$
PHYSICAL MASSIVE VECTOR MULTIPLETS $\left( 1, 2 \left( \frac{1}{2} \right), 0 \right) 4b + 4f$

(S.F., GRISARU, VAN NIEUWENHUIZEN)
The "dual," standard supergravity action contains, in the a) formulation two "matter," chiral (massive) multiplets $T, S \ (46 + 4f)$ while in the b) formulation contains a "massive," vector (or tensor) multiplet $V \ (46 + 4f)$. The main difference is that in the a) theory we are in presence of a "four-field," model, in the b) theory we have a "single-field," inflaton model.

Since the other three bosonic degrees of freedom combine in a massive vector, standard supergravity formulae allow to describe the a) theory in terms of a
Kahler potential $K$ and a superpotential $W$.

It turns out that their form is

$$K = -3 \log (1 + T + \bar{T} - h(s, \bar{s})),$$  
$$W = \lambda T$$

($\lambda$ is a constant related to the $\alpha$ parameter).

And $h(s, \bar{s})$ is an arbitrary real function which starts with

$$h(s, \bar{s}) = s \bar{s} + O(s^3)$$

terms.

It is possible to choose the function $h(s, \bar{s})$ to make the inflationary trajectory stable.

Here, the "inflaton" is identified with the $R e T$ scalar, while the other three scalars are "extremized." The potential for

$$R e T = \exp(-\sqrt{2} \phi)$$

is the Starobinsky potential.
It can be shown that this theory, for any \( h(s, \bar{s}) \)

is "dual" to a higher curvature supergravity theory.

The scalar supercurvature \( (s, f, zuhino) \) is a chiral superfield \( \overline{D}_2 R = 0 \) and \( h(s, \bar{s}) \) corresponds to terms of the form \( h(R, \bar{R}) \) in the supergravity side. It is important to notice that the inflaton potential is an "F term" potential, which means it comes from the standard expression

\[
V(T, S) = e^K \left( \partial_i W \partial_j \overline{W} K^{ij} - 3 |W|^2 \right) \quad (i, j = S, T)
\]

and the inflaton potential is

\[
V(\phi) = V(T, S) \left|_{\partial V / \partial s = 0, \partial V / \partial \text{Im} T = 0} \right.
\]
It happens that all supersymmetric models for the inflaton potential considered in the literature (Kallosh, Linde; Ellis, Navopoulos, Olive; Kallosh, Linde, Roest, ...) are mostly deformation of the previous model with modification of $K CT, T, S, S$ and of $W CT, S$ but still keeping the same $(S, T)$ chiral multiplet content.

It is in fact possible to show that at least two multiplets are needed to get an inflationary potential. In fact former theories with higher supercurvature terms of $F$ term type with chiral function $f(R)$ ($\bar{D}f = 0$)
WERE CONSIDERED IN THE PAST (KETOV) BUT WERE
SHOWN (ELLIS, NANOPOULOS, OLIVE; F.F., KEHAGIAS, PORRATI)
NOT TO PRODUCE AN INFLATIONARY POTENTIAL.

AN IMPORTANT DEFORMATION OF THE (S,T) MODEL
FROM WHICH THE CONCEPT OF "ATTRAFTORS" CAKE
FROM (KALLOSH, LINDE, ROEST) IS A SUPERPO/TENTIAL OF
THE TYPE $W(S,T) = S f(t)$ WHICH ALLOWS BOSONIC
POTENTIALS CONTAINING ARBITRARY FUNCTIONS OF THE
INFLATION $f(\tanh \frac{\phi}{V_0})$. THESE THEORIES ARE NO
LONGER EQUIVALENT TO PURE HIGHER CURVATURE
SUPERGRAVITY BUT IN CERTAIN CASES, TO HIGHER
CURVATURE COUPLED TO A (SINGLE) CHIRAL MULTIPLE
For instance, taking $K$ as before

$$K = -\frac{3}{2} \log (1 + T + \overline{T} - h(S, \overline{S})) \quad \text{but now} \quad W = S f(T)$$

The 'dual,' higher derivative supergravity is

a 'matter coupled theory' with \((\text{ECOTT, KALLOSH})\)

$$\Phi = e^{-\frac{1}{3} K} = 1 + T - \frac{f(T)}{f'(T)} + \overline{T} - \frac{\overline{f}(T)}{\overline{f}'(T)}$$

and a term $\frac{1}{H'(T)} \overline{RR}$

AND BOTH TERMS BECOME $T$ INDEPENDENT IF $f(T) = a T$. 
The formulation gives directly a single-field inflation model where a \(^{3D}\) term potential for the massive superfield is generated.

The most general self-interaction of such massive vector multiplet with spin content \((1, 2(\frac{1}{2}), 0)\) resides on a "real function" \(J\) of a "real variable", \(C^1 : J(C)\) (van Proeyen)

The bosonic part of the supergravity action is

\[
\mathcal{L} = -\frac{1}{2} R - \frac{i}{4} F_{\mu\nu}(B) F^{\mu\nu}(B) + \frac{g^2}{2} J''(C) B_\mu B^{\mu} + \frac{i}{2} J'(C) (\partial_\mu C)^2 - \frac{g^2}{2} J'(C)^2
\]

So the potential is \(V(C) = \frac{g^2}{2} J'(C)^2\)

Note that the Lagrangian only depends on
$J', J''$ so a linear term in $J$ shifts $J'$ by a constant but leaves $J''$ invariant. This constant is the so-called Fayet-Iliopoulos term.

By using the Stueckelberg trick one writes this Lagrangian as a gauge theory by shifting $A_\mu = B_\mu + \frac{i}{3} \partial_\mu a$ so that

$$\frac{g^2}{2} J''(C) B_\mu B^\mu = \frac{g^2}{2} J''(C) (A_\mu + \frac{i}{3} \partial_\mu a)^2$$

In the limit $g \to 0$ the theory becomes

$$\mathcal{L} = -\frac{1}{2} R - \frac{1}{4} F_{\mu\nu} (A')^2 + \frac{i}{2} J''(C) (\partial_\mu a)^2 + \partial_\mu C)^2$$

The $(a, C)$ variables can be complexified

$z = iC - a$ and the $J$ function can be interpreted as Kahler potential $J = -\frac{1}{2} K(\text{Im} z)$
The higher curvature supergravity in the formulation is "dual to a self-interacting massive vector multiplet with a very precise choice of \( J(C) = \frac{3}{2} (2g - C' + C') - (S, F, C, \beta, S) \)

Computation of the potential, for a canonically normalized field \( C' = -\exp \sqrt{\frac{2}{3}} \phi \) one obtains \( (S, F, K, \beta, \omega, \rho, \lambda) \) (Kallosh, Linde, Parni"

The Starobinsky potential and Lagrangian (Farakos, Kehagias, Riotto)

\[
L = -\frac{1}{2} (\partial \phi)^2 - \frac{9}{8} g^2 (1 - \exp -\sqrt{\frac{2}{3}} \phi)^2
\]

So the supersymmetric generalization just reproduces the single-field Starobinsky model with \( \alpha = \frac{1}{g^2} \). It is interesting to observe that the particular form of \( J(C) \) corresponds to
An SU(1,1)/U(1) symmetric Kähler manifold with a parabolic isometry being gauged. For a Kähler potential \( K = -3 \alpha \log \text{Im} z \) the curvature is

\[
R(C) = \frac{J''''(C) - J''(C) J''''(C)}{2 J''(C)^2} = -\frac{2}{3\alpha}
\]

And for \( \alpha \to \infty \) \( R(C) \to 0 \).

The \( \alpha \) dependent potential becomes (S.F. Kallab, Linde, Pomati)

\[
V(\varphi) = \frac{g}{\alpha} \varphi^2 \left( 1 - \exp \sqrt{\frac{2}{3\alpha}} \varphi \right)^2 = \frac{g}{\alpha} \varphi^2 \varphi(C)^2
\]

Not that the canonical variable \( \varphi \) is related to the \( C \) variable by the equation (\( \varphi(C) = J'(C) \))

\[
J''(C) = \left( \frac{d\varphi}{dC} \right)^2 = \varphi'(C)
\]
It then follows

\[ P'(c) = P'(\phi) \frac{d\phi}{dc} = \left( \frac{d\phi}{dc} \right)^2 \Rightarrow P'(\phi) = \frac{d\phi}{dc} \quad (P(\phi) = P[c(\phi)]) \]

\[ c'(\phi) = \int d\phi \frac{dc}{d\phi} = \int d\phi \frac{1}{P'(\phi)} \]

\[ J(c) = \int dc J'(c) = \int P(\phi) \frac{dc}{d\phi} d\phi = \int \frac{P(\phi)}{P'(\phi)} d\phi \]

The kinetic term of the Kahler manifold is

\[ \left( \frac{1}{2} J''(c) \right) \left( c_{\phi} c_{\phi}^2 + (c_{\phi})^2 \right) = \frac{1}{2} \left[ (c_{\phi}\phi)^2 + (P'(\phi))^2 (c_{\phi}^2) \right] \]

The previous equations allow us to compute

\[ c(\phi) \quad \text{once} \quad P'(\phi) = \frac{d\phi}{dc} \quad \text{is solved} \]

The curvature in the \( \phi \) variable is

\[ R(\phi) = -4 \frac{P''''(\phi)}{P'(\phi)} \]
The one-field supergravity model for inflation can be deformed in two ways:

1) Simply change JCC → change the Kähler manifold

2) Don't change the manifold but change its gauged isometry.

For the case of symmetric spaces this procedure generates five models—three with constant curvature depending whether a parabolic, elliptic or hyperbolic isometry is gauged—two with vanishing curvature where the parabolic or elliptic isometry is gauged. (SF, FRE, JORM)
Chaotic Inflation: In a $F^*$ term multi-field potential term it is hard to obtain (at least in some directions of the field space) a quadratic potential. One way is to impose a shift symmetry on the Kähler potential (Kawasaki, Yamaguchi, Yanagida) (Kallosh, Linde, Westphal) (Ellis, Garcia, Nanopoulos, Olive).

In terms of the $(T, S)$ chiral fields this exchange the role of $(\text{Im} T, \text{Re} T)$ since it is now $\text{Im} T$ which plays the role of inflaton. It is then natural, in the supergravity dual, to call this scenario imaginary Starobinsky model ($S^F, \text{Kehoe}, \text{es}, \text{Riotto}$), even if a coupling to matter is needed in order to stabilize the $\text{Re} T$ component.
Chaotic inflation in the $6j$ single field supergravity minimal embedding.

In this case an exact model is possible since we can take a flat-Kahler space where we gauge a parabolic isometry (translation). The alternative gauging of an elliptic isometry would give a quartic potential.

For this case $\mathcal{J}^{(1)} = \text{const.}$. $\mathcal{J}(C) = \frac{1}{2} C^2 + \delta C$

But the FI term is irrelevant in this case. Then $\mathcal{E}(\phi) = \phi$ and

$V(\phi) = \frac{1}{2} m^2 \phi^2$
This model can also be obtained from the constant (α) curvature case by taking the limit $\alpha \to \infty$, $g^2 \to \infty$ with $M^2 \alpha \frac{g^2}{\alpha}$ fixed.

$$g^2 \left(1 - e^{-\frac{2}{3\alpha}}\Phi^2\right)^2 \to \frac{m^2}{2} \Phi^2.$$  

Supersymmetry breaking:

We observe that the above potential is a $D^2$ term so during inflation $D$ is large and the gaugino is the Goldstino. The decoupling of the other (chiral) component occurs when $J^{(c)}(c) \to 0$ and we get an unbroken gauge symmetry in DeSitter space (Friedman model).
Supergravity formulations in different conformal gauges (different Jordan functions):

OF INFLATION (Kallosh-Linde) (type a) formulation

Three basic fields: $S$ (conformon) \quad T (scalaron) \quad S (Goldstino)

The superpotential $W(S,T) = ST$ has $F$-terms

$\frac{\partial W}{\partial S} = T \neq 0$ during inflation

$\frac{\partial W}{\partial T} = S = 0$ during inflation (and later)

$S|_\theta=0$ is the Goldstino (partner of the Goldstino since supersymmetry is linearly realized)
In fact this explains why in the a) formulation two chiral superfields are needed.

A new effective Lagrangian, since supersymmetry is badly broken during inflation, can be obtained replacing the $S$ Goldstino multiplet by the Volkov-Akulov superfield $X$ with $X^2 = 0$

$$X = \frac{G^a G_a}{2F_X} + \sqrt{2}\theta^a G_a + \bar{\theta}^a \theta X F_X$$

Casalbuoni et al. Komargodskii, Seiberg

In the "dual" supergravity theory this corresponds to the "chiral" scalar supercurvature $R$ to become nilpotent $R^2 = 0$ (Antoniadis, Dudas, S.F. Sagnotti)
SUPERSYMMETRY BREAKING

NON LINEAR REALIZATION OF SUPERSYMMETRY

VOLKOV-AKULOV

\[ \mathcal{L} = \mathcal{L}_E + \frac{i}{f} \mathcal{G} \bar{\psi} \gamma^\mu \partial_{\mu} \mathcal{G} \]

\( \mathcal{G}_i \in \text{Grassmann valued} \)

\[ \mathcal{L}_{\text{VA}} = f^2 \text{det} \mathcal{V}_{\alpha \mu} \]

\[ \mathcal{V}_{\alpha \mu} = \delta_{\alpha \mu} + \frac{i}{f^2} \mathcal{G} \gamma_{\alpha} \partial_{\mu} \mathcal{G} \]

IN SUPERSPACE

\[ L = X \bar{X}_D + f X_F \quad (X^2 = 0) \]

\( G_0(x) \) GOLDSTINO

\( f(M^2) \) SUSY BREAKING PARAMETER

EA TRANSF PAR
When coupled to supergravity, one gets a theory of a massive gravitino coupled to gravity:

\[ K = -3 \ln (1 - xx) = 3xx, \quad W = fx + W_0 \]

Vacuum energy: \[ V_0 = \frac{1}{5} f^2 - 3 |W_0|^2 \] \[ \text{Gravitino mass} \quad m_{3/2} = |W_0| \]

Noether current: \[ J^\mu = \text{fg} \mu G + \ldots \]

Now we couple V-A to supergravity and to the scalaron multiplet. The spin 3/2 massive action is of the form:

\[ \frac{i}{2} \left[ R + \psi \gamma \psi + m_{3/2} \psi \gamma \psi + \mathcal{K} \psi \gamma \psi - V(f, W_0) \right] \]
THE CONSTRAINED V-A SUPERFIELD $X$ IS THEN COUPLED TO THE SCALARON GIVING RISE TO AN ALMOST STANDARD SUPERGRAVITY WITH KÄHLER AND SUPERPOWNTENTIAL

$$K = -3 \ln(T + \bar{T} - XX) , \quad W = MXT + fX + W_0, X^2 = 0$$

THIS GIVES THE $T$, POTENTIAL

$$V = \frac{|MT + f|^2}{3(T + \bar{T})^2}, \quad \text{BY DEFINING} \quad T = e^{-\frac{\sqrt{2}}{3} \phi} + i a \sqrt{\frac{2}{3}}$$

ONE GETS (ADFS) $V > 0$ (NO SCALE STRUCTURE)

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - \frac{M^2}{12} \left(1 - e^{-\frac{\sqrt{2}}{3} \phi}\right)^2 - \frac{1}{2} e^{-2\phi \sqrt{\frac{2}{3}}(\partial \alpha)^2} - \frac{M^2}{18} e^{-2\phi \sqrt{\frac{2}{3}} \alpha^2}$$

$$= \mathcal{L}^{\text{Starobinsky}} + \mathcal{L}^{\text{Axion}} + (\text{fermionic terms})$$
The dual supergravity action is

\[ L(S_0, R) = - \left[ S_0 \bar{S}_0 - \frac{R \bar{R}}{M^2} \right] + (W_0 + \frac{3R}{S_0}) S_0^3 + 6 R^2 S_0 \]

The bosonic part of this action can be obtained by dualizing the previous action \( L^\star \) + \( L^{\text{axion}} \). Having set \( e^{1/2} = 1 + 2x \) and Weyl rescaled \( g \rightarrow (1 + 2x)g \) so that

\[ L = \frac{1}{2} \left( 1 + 2x \right) R - \frac{1}{2} \left( \Theta g \right)^2 - \frac{M^2}{3} \left( x^2 + \frac{q^2}{6} \right) \]

We now replace the axion part by

\[ L^{\text{axion}} = - \frac{M^2}{18} q^2 + A^m \delta m A + \frac{1}{2} \left( 1 + 2x \right) A_m^2 \]

The dual Lagrangian is obtained by integrating over \( A \) (the original one over \( A_m \))

\[ L = \frac{1}{2} \left( 1 + 2x \right) (R + A_m^2) - \frac{M^2}{3} x^2 + \frac{q}{2M^2} (\Theta - A)^2 \]
AND WE FINALLY GET \((A_m - \sqrt{2/3}A_m)\) AFTER INTEGRATING OVER \(\chi\)

\[
L = \frac{1}{2} \left( R + \frac{2}{3}A_m^2 \right) + \frac{3}{4M^2} \left( R + \frac{2}{3}A_m^2 \right)^2 + \frac{3}{M^2} (\nabla \cdot A)^2
\]

THIS IS THE \(R + R^2\) LAGRANGIAN WITH \(S = P = 0\)

NOTE THAT THE AXION FIELD IS MUCH HEAVIER THAN \(\phi\) DURING INFLATION WHERE \(\phi_0 > 0\) AND LARGE

\[
m^2_\phi \sim \frac{M^2}{g} e^{-2\phi_0 \sqrt{2/3}} \ll m^2_a = \frac{M^2}{g}
\]