Another Look at Quadratic Divergences

Dedicated to the memory of Bruno Zumino

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Hermann Nicolai
MPI für Gravitationsphysik, Potsdam
(Albert Einstein Institut)

Based on joint work with

Piotr Chankowski, Adrian Lewandowski and Krzysztof Meissner,
arXiv:1404.0548

The electroweak hierarchy problem

A main focus of BSM model building over many years:

$$m_R^2 = m_B^2 + \delta m_B^2 \ , \quad \delta m_B^2 \ \propto \ \Lambda^2 \quad \textbf{and } m_R^2 \ll \Lambda^2$$

But is this really a problem?

- **Not in renormalized perturbation theory because $\Lambda \to \infty$ and because renormalisation "does not care" whether an infinity is quadratic or logarithmic!** *(as exemplified by dimensional regularisation which does not even "see" quadratic divergences for $d = 4 + \varepsilon$)*.
- **Yes, if SM is embedded into Planck scale theory and Λ is a *physical* scale (cutoff) $\Rightarrow$ quadratic dependencies on cutoff imply extreme sensitivity of low energy physics to Planck scale physics.**

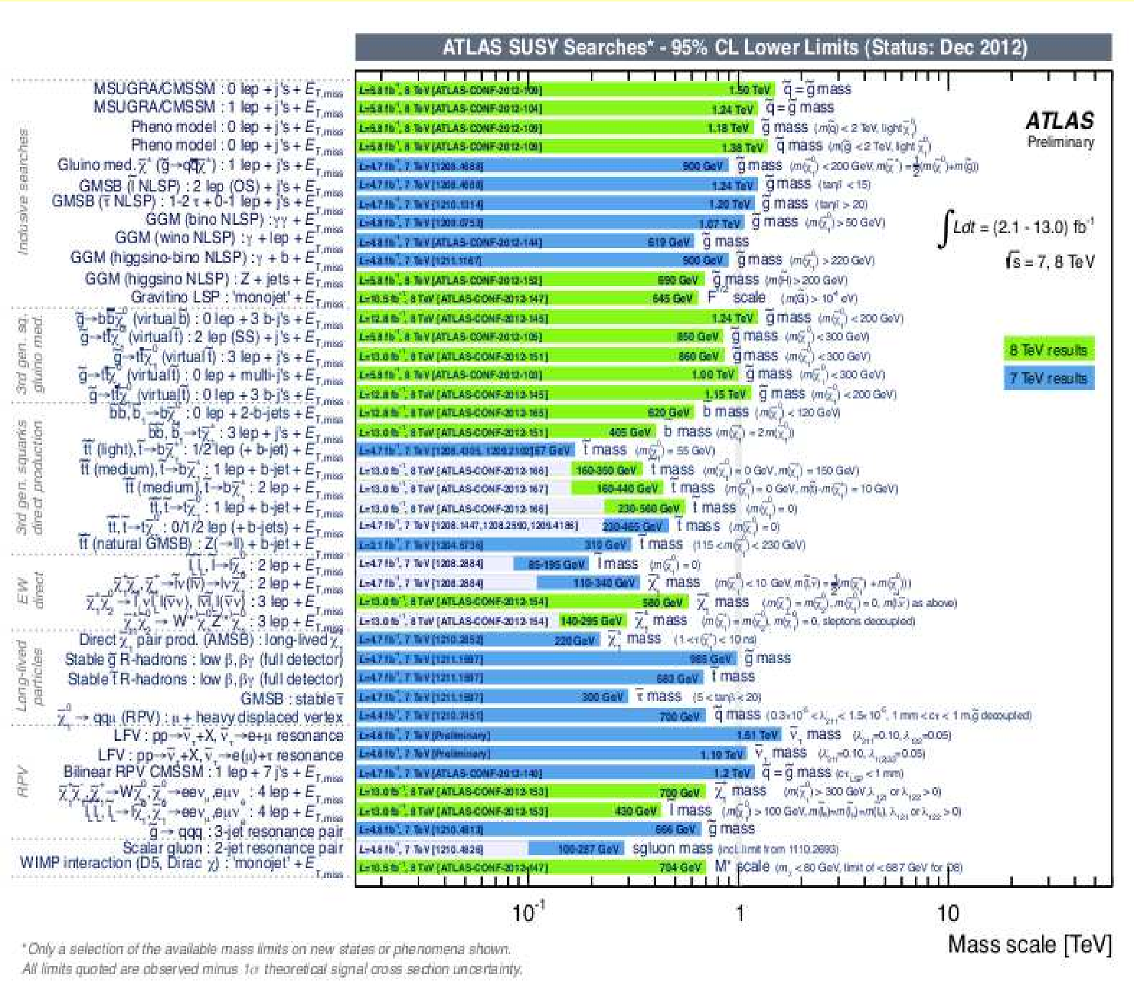
Two popular proposed solutions

- **Low energy supersymmetry:** exact cancellation of quadratic divergences by (softly broken) supersymmetry $\Rightarrow$ choice of cutoff Λ does not matter, can formally send $\Lambda \to \infty$ and adopt any convenient renormalisation scheme.
- **Technicolor** (motivated by QCD): no fundamental scalars $\Rightarrow$ no quadratic divergences $\Rightarrow$ H boson would have to be composite (could still be true...)

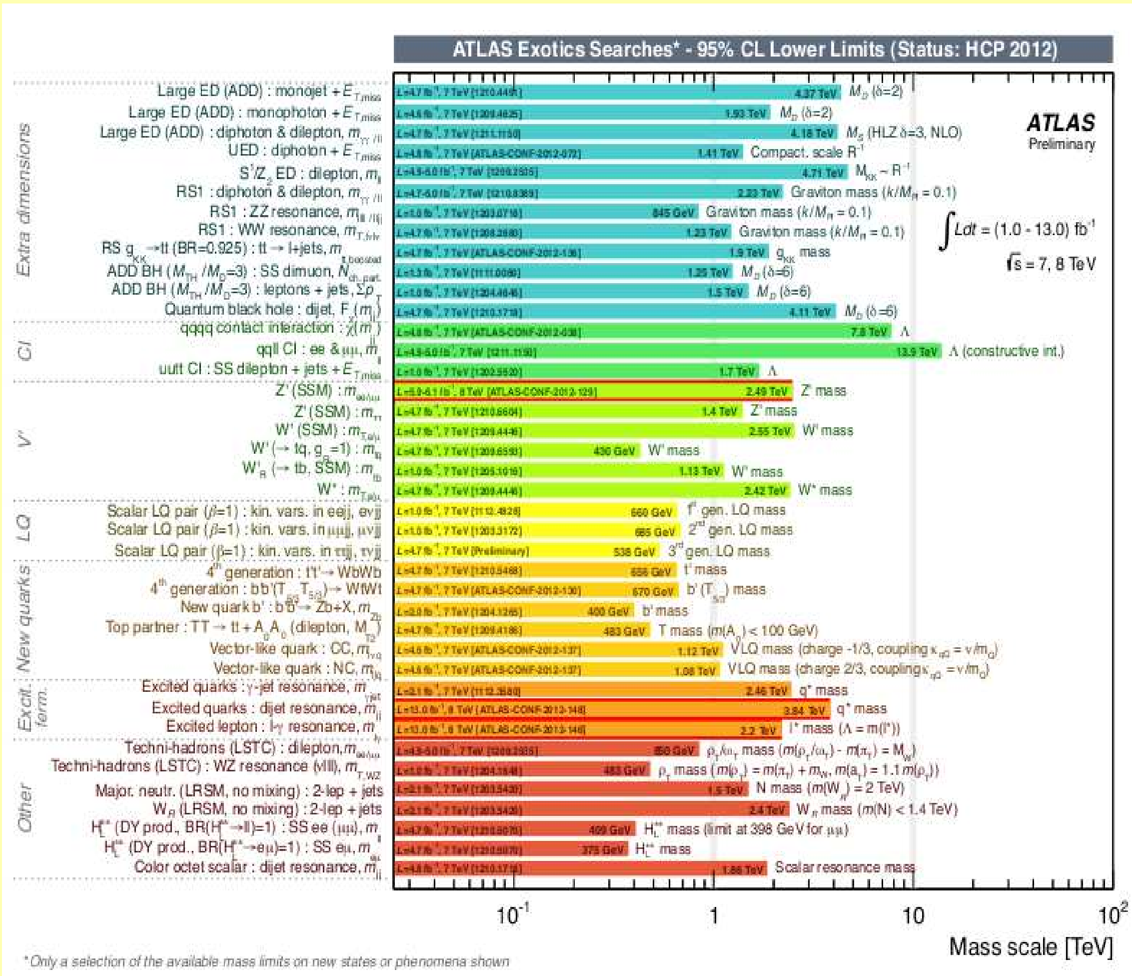
... as well as a number of other ideas

NB: these proposals would only solve the *technical part* of the hierarchy problem (= stabilising small numbers against large perturbative corrections), but would *not* explain the observed hierarchy of scales!

Low energy supersymmetry?



Low energy exotics?



Absence of any evidence (so far) from LHC for either of these options ⇒ explore alternative options ⇒

Can the SM survive all the way to Planck scale $M_{\rm PL}$?

In this talk: explore softly broken conformal symmetry (SBCS) for a *minimal extension of usual SM* as an alternative option.

NB: this proposal does without *low* energy supersymmetry, but supersymmetry is probably still essential for a finite and consistent theory of quantum gravity.

Realisation of such a scenario would move the SUSY breaking scale back up to the Planck scale, but make no explicit assumptions about Planck scale theory other than its UV finiteness (= UV completeness).

Reminder: the conformal group $SO(2,4)$

This is an old subject! [see e.g. H.Kastrup, arXiv:0808.2730]

Conformal group = extension of Poincaré group (with generators $M_{\mu\nu}, P_\mu$) by five more generators D and K_μ:

- **Dilatations (D) :** $\qquad x^\mu \to e^\alpha x^\mu$
- **Special conformal transformations (K^μ):**

$$x'^\mu = \frac{x^\mu - x^2 \cdot c^\mu}{1 - 2c \cdot x + c^2 x^2}$$

$e^{i\alpha D} P^\mu P_\mu e^{-i\alpha D} = e^{2\alpha} P^\mu P_\mu \;\; \Rightarrow$ ***exact*** **conformal invariance implies that one-particle spectrum is either continuous ($= \mathbb{R}_+$) or consists only of the single point $\{0\}$.**

Consequently, conformal group cannot be realized as an exact symmetry in nature.

Conformal Invariance and the Standard Model

Fact: Standard Model of elementary particle physics is conformally invariant at tree level except for explicit mass term $m^2\Phi^\dagger\Phi$ in potential $\Rightarrow$

Masses for vector bosons, quarks and leptons $\rightarrow$

Can 'softly broken conformal symmetry' ($\equiv$ 'SBCS') stabilize the electroweak scale w.r.t. the Planck scale?

Concrete implementation of this idea requires

- **consistency conditions (absence of Landau poles and of instabilities of effective potential), and**
- **absence of any intermediate mass scales between $M_{\rm EW}$ and $M_{\rm PL}$ ('grand desert scenario').**

Evidence for large scales other than M_{Pl}?

- **(SUSY?) Grand Unification:** $m_X \geq \mathcal{O}(10^{15}\,\mathrm{GeV})$**?**
  - **But: proton refuses to decay (so far, at least!)**
  - **SUSY GUTs: unification of gauge couplings at** $\geq \mathcal{O}(10^{16}\,\mathrm{GeV})$
- **Light neutrinos** $(m_\nu \leq \mathcal{O}(1\,\mathrm{eV}))$ **and heavy neutrinos** **→ most popular (and most plausible) explanation of observed mass patterns via seesaw mechanism:**

  [Gell-Mann,Ramond,Slansky; Minkowski; Yanagida]

$$m_\nu^{(1)} \sim \frac{m_D^2}{M}\,,\; m_D = \mathcal{O}(m_W) \;\Rightarrow m_\nu^{(2)} \sim M \geq \mathcal{O}(10^{12}\,\mathrm{GeV})?$$

- **Resolution of strong CP problem** $\Rightarrow$ **need *axion*** $a(x)$**.** **Limits e.g. from axion cooling in stars** $\Rightarrow$

$$\mathcal{L} = \frac{1}{4f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad \textbf{with } f_a \geq \mathcal{O}(10^{10}\,\mathrm{GeV})$$

NB: axion is (still) an attractive CDM candidate.

Conformal Invariance and Quantum Theory

Important Fact: classical conformal invariance is *generically* broken by quantum effects (unlike SUSY!) $\Rightarrow$

- **Impose anomalous Ward identity**

$$\Theta^{\mu}{}_{\mu} = \sum_n \beta^{(n)}(g)\mathcal{O}^{(n)}(x)$$

  [W. Bardeen, FERMILAB-CONF-95-391-T, FERMILAB-CONF-95-377-T]

  and try radiative symmetry breaking *à la* Coleman-Weinberg. But: quadratic divergences?

- **Admit soft breaking (=explicit mass terms) as is commonly done for MSSM like models, but insist on cancellation of quadratic divergenes**

NB: it is known that option (1) does not work for usual SM with one physical Higgs, but with one extra complex scalar (as in our model) there is more freedom.

SBCS Consistency Requirements

Assume existence of a UV complete and finite fundamental theory, such that Λ is a physical cutoff to be kept finite, and impose vanishing of quadratic divergences at particular distinguished scale Λ ($= M_{\rm PL}$?) :

- **Bare mass parameters should obey $m_B(\Lambda) \ll M_{\rm PL}$;**
- **there should be neither Landau poles nor instabilities for $M_{\rm EW} < \mu < \Lambda$ (manifesting themselves as the unboundedness from below of the effective potential depending on the running scalar self-couplings);**
- **all couplings $\lambda_R(\mu)$ should remain small (for the perturbative approach to be applicable and stability of the effective potential electroweak minimum).**

Furthermore use known SM values of couplings and masses as input parameters at $\mu = M_{\rm EW}$.

Bare vs. renormalized couplings

With cutoff Λ and normalization scale μ we have

$$\lambda_B(\mu, \lambda_R, \Lambda) = \lambda_R + \sum_{L=1}^{\infty} \sum_{\ell=1}^{L} a_{L\ell}\, \lambda_R^{L+1} \left(\ln \frac{\Lambda^2}{\mu^2} \right)^{\ell} ,$$

so that $\lambda_B = \lambda_R$ for $\mu = \Lambda$, and

$$m_B^2(\mu, \lambda_R, m_R, \Lambda) \; = \; m_R^2 \; - \; \hat{f}^{\rm quad}(\mu, \lambda_R, \Lambda)\, \Lambda^2 + \; m_R^2 \sum_{L=1}^{\infty} \sum_{\ell=1}^{L} c_{L\ell}\, \lambda_R^{L} \left(\ln \frac{\Lambda^2}{\mu^2} \right)^{\ell}$$

Crucial fact: coefficient of Λ^2 can be written as a function of the bare coupling(s) only, *i.e.* $\hat{f}^{\rm quad}(\mu, \lambda_R, \Lambda) \equiv f^{\rm quad}(\lambda_B(\mu, \lambda_R, \Lambda))$.

Thus, keeping the physical cutoff Λ finite we can set

$$f^{\rm quad}(\lambda_B) = 0$$

NB: this condition would not make sense if $\Lambda \to \infty$ where bare couplings are expected to become singular!

Quadratic divergences in Standard Model

[M. Veltman(1982);Y.Hamada,H.Kawai,K.Oda, PRD87(2013)5; D.R.T.Jones,PRD88(2013)098301]

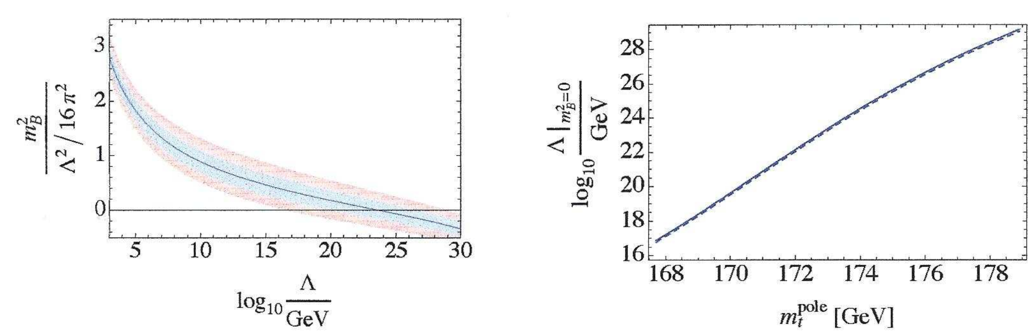


Figure 3: Left: The bare Higgs mass m_B^2 in units of $\Lambda^2/16\pi^2$ vs the UV cutoff scale Λ. The blue (narrower) and pink (wider) bands represent the one and two sigma deviations of m_t^{pole}, respectively. Right: The UV cutoff scale at which the bare mass m_B^2 becomes zero as a function of m_t^{pole}. The solid (dashed) line corresponds to the scale where m_B^2 ($m_{B,\,\text{1-loop}}^2$) becomes zero. In both panels, we have taken the central values $\alpha_s(m_Z) = 0.1184$ and $m_H = 125.7\,\text{GeV}$.

Only one scalar: $f^{\text{quad}}(\lambda_R(\mu)) = 0$ **for** $\mu \approx 10^{24}\text{GeV} \gg M_{\text{PL}}$ **!**

Is the Standard Model doomed?

[Y.Hamada,H.Kawai,K.Oda, PRD87(2013)5]

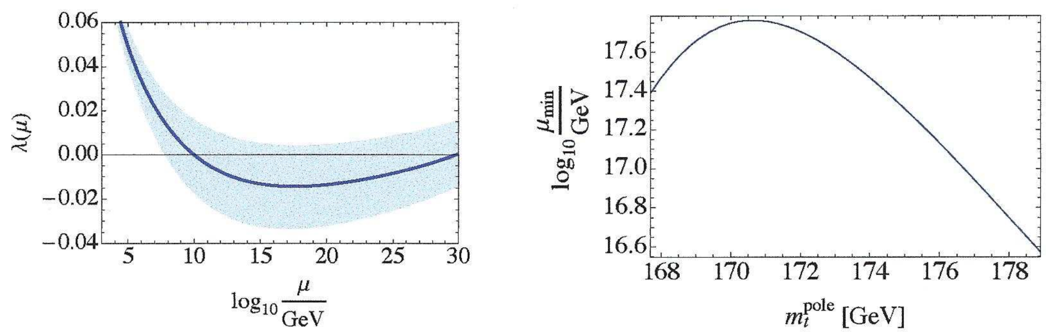


Figure 2: Left: $\overline{\text{MS}}$ running of the quartic coupling λ. The band corresponds to the 1σ deviation $m_t^{\text{pole}} = 173.3 \pm 2.8\,\text{GeV}$. Right: The scale μ_{min} at which $\lambda(\mu)$ takes its minimum value, as a function of m_t^{pole}. In both panels, low energy inputs are given by the central values $\alpha_s(m_Z) = 0.1184$ and $m_H = 125.7\,\text{GeV}$.

$\lambda_R(\mu)$ becomes negative for $\mu > 10^{10}\,\text{GeV}$ $\Rightarrow$ instability?

$\rightarrow$ might also be relevant to cosmology!

Minimal extension of SM = CSM

[K. Meissner,HN, PLB648(2007)312; Eur.Phys.J. C57(2008)493]

- **Start from conformally invariant (and therefore renormalizable) fermionic Lagrangian** $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}'$

$$\mathcal{L}' := \Big(\bar{L}^i \Phi Y^E_{ij} E^j + \bar{Q}^i \epsilon \Phi^* Y^D_{ij} D^j + \bar{Q}^i \epsilon \Phi^* Y^U_{ij} U^j + + \bar{L}^i \epsilon \Phi^* Y^\nu_{ij} \nu^j_R + \phi \nu^{iT}_R \mathcal{C} Y^M_{ij} \nu^j_R + \text{h.c.} \Big) - V(\Phi, \phi)$$

- **Besides usual $SU(2)$ doublet Φ: new scalar field $\phi(x)$**

$$\phi(x) = \varphi(x) \exp\left(\frac{ia(x)}{\sqrt{2}\mu} \right)$$

- **No fermion mass terms, all couplings dimensionless**
- Y^U_{ij}, Y^E_{ij}, Y^M_{ij} **real and diagonal:** $Y^M_{ij} = y_{N_i} \delta_{ij}$
  Y^D_{ij}, Y^ν_{ij} **complex** $\rightarrow$ **parametrize family mixing (CKM)**
- **Neutrino masses from usual seesaw mechanism (but with** $\langle \phi \rangle < \mathcal{O}(1\,\text{TeV})$ $\Rightarrow$ **no new large scale)**

Scalar Sector of CSM

Right-chiral neutrinos and one complex scalar $\Rightarrow$

$$V(\Phi,\phi) = m_H \Phi^\dagger\Phi + m_\phi^2|\phi|^2 + \lambda_1(\Phi^\dagger\Phi)^2 + 2\lambda_3(\Phi^\dagger\Phi)|\phi|^2 + \lambda_2|\phi|^4$$

where $\Phi = (\Phi_1, \Phi_2)$ is the $SU(2)_{\mathrm{EW}}$ doublet and ϕ is the extra gauge singlet. At the minimum

$$\sqrt{2}\langle\Phi_i\rangle = v_H\delta_{i2} \;\; , \quad \sqrt{2}\langle\phi\rangle = v_\phi$$

with mass eigenstates h^0 and φ^0

$$\begin{pmatrix} h^0 \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}(\Phi_2 - \langle\Phi_2\rangle) \\ \sqrt{2}\,\mathrm{Re}(\phi - \langle\phi\rangle) \end{pmatrix}, \qquad (1)$$

with masses $M_h < M_\varphi$ and $|\tan\beta| < 0.3$ (as dictated by existing experimental bounds if h^0 = SM Higgs-Boson).

Quadratic divergences in CSM

Two physical scalars $\Rightarrow$ two conditions (at one loop)

$$16\pi^2 f_1^{\rm quad}(\lambda, g, y) \;=\; 6\lambda_1 + 2\lambda_3 + \frac{9}{4}g_w^2 + \frac{3}{4}g_y^2 - 6y_t^2$$

$$16\pi^2 f_2^{\rm quad}(\lambda, g, y) \;=\; 4\lambda_2 + 4\lambda_3 - \sum_{i=1}^{3} y_{N_i}^2$$

- **Start from known values of electroweak couplings g_y, g_w, y_t at $\mu = M_{\rm EW}$ and evolve them to $\mu = M_{\rm PL}$.**
- **Choose λ_1, y_N and determine λ_2 and λ_3 from $f_k^{\rm quad} = 0$**
- **Evolve all couplings back to $\mu = M_{\rm EW}$ and check whether all consistency requirements are satisfied.**

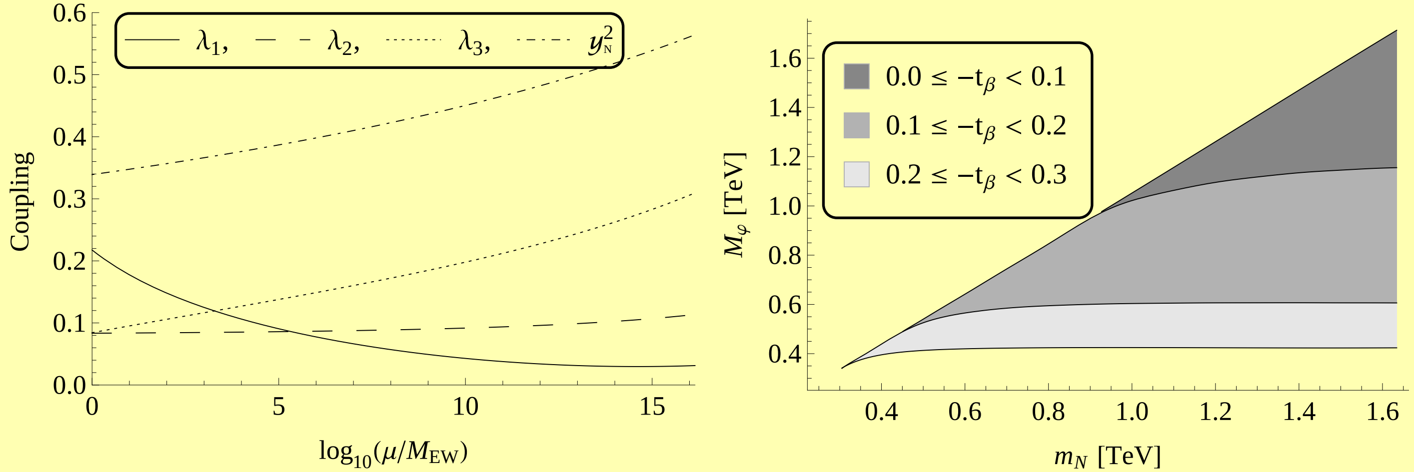
$\Rightarrow$ leads to a range of possible values for new heavy scalar φ^0 and heavy neutrinos (with $m_N < 1\,{\rm TeV}$).

β-functions at one loop

$$
\begin{aligned}
\tilde{\beta}^{(1)}_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_3^2 - 3\lambda_1\left(3g_w^2 + g_y^2 - 4y_t^2\right) + \\
&\quad + \frac{9}{8}g_w^4 + \frac{3}{4}g_w^2 g_y^2 + \frac{3}{8}g_y^4 - 6y_t^4 \\
\tilde{\beta}^{(1)}_{\lambda_2} &= 20\lambda_2^2 + 8\lambda_3^2 + 2\lambda_2\sum_{i=1}^{3} y^2_{N_i} - \sum_{i=1}^{3} y^4_{N_i} \\
\tilde{\beta}^{(1)}_{\lambda_3} &= \frac{1}{2}\lambda_3\left\{24\lambda_1 + 16\lambda_2 + 16\lambda_3 - \left(9g_w^2 + 3g_y^2\right) + 2\sum_{i=1}^{3} y^2_{N_i} + 12y_t^2\right\} \\
\tilde{\beta}^{(1)}_{g_w} &= -\frac{19}{6}g_w^3\,, \quad \tilde{\beta}^{(1)}_{g_y} = \frac{41}{6}g_y^3, \quad \tilde{\beta}^{(1)}_{g_s} = -7g_s^3, \\
\tilde{\beta}^{(1)}_{y_t} &= y_t\left\{\frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g_w^2 - \frac{17}{12}g_y^2\right\}, \\
\tilde{\beta}^{(1)}_{y_{N_j}} &= \frac{1}{2}y_{N_j}\left\{2y^2_{N_j} + \sum_{i=1}^{3} y^2_{N_i}\right\}
\end{aligned}
$$

where $\tilde{\beta} \equiv 16\pi^2\beta$**.**

Admissible parameter ranges



- **Couplings remain small (also at two loops)**
- M_φ **grows with decreasing mixing angle** β
- **Usual seesaw mechanism applies, such that**
  - **Small light neutrino masses with** $Y_\nu \sim 10^{-5} - 10^{-6}$
  - **'Heavy' neutrinos not extremely heavy:** $m_N < 1\,\mathrm{TeV}$**?**
- **Caveats: scheme dependencies, threshold effects?**

What might be observed

- h^0 **decay width is decreased:** $\Gamma_{h^0} = \cos^2\beta\,\Gamma_{SM} < \Gamma_{SM}$
- **Crucial relation for** φ^0 **decay width:**

$$\Gamma_{\varphi^0} = \sin^2\beta\,\Gamma_{SM} + \Gamma_{\varphi^0\to h^0h^0} + \Gamma_{\varphi^0\to\nu_R\nu_R}$$

  - **First term: narrow resonance ('shadow Higgs boson')**
  - **Second and third terms: decay of heavy scalar into two or three** h^0**'s, and two heavy neutrinos (if kinematically allowed, i.e.** $M_\varphi > 2m_N$ **for at least one species).**

- **Decay via two** h^0 **bosons might produce spectacular signatures with 5,...,8 leptons coming out of a single vertex. But: rates *vs.* background?**
- **Proposal can be easily discriminated against other extensions of SM that might produce similar signatures, but that would come with a lot of extra baggage (accompanying signatures).**

Outlook

- **Usual SM probably cannot survive to Planck scale $\Rightarrow$ requires *some* extension, if only to accommodate right-chiral neutrinos.**
- **A conformally motivated extension of the SM can in principle satisfy all consistency requirements, if properly embedded into a UV complete theory.**
- **Model accommodates axions naturally: $f_a \propto m_W^2/m_\nu$.**
- **Low energy supersymmetry may after all *not* be required for stability of electroweak scale...**
- **... but is probably needed for a UV complete theory of quantum gravity and quantum space-time.**

Conclusion: Nature is probably still a bit smarter than us, and may have a few more tricks up her sleeve!