Proton structure functions
and determination of the PDFs at HERA

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**HERA collider (Hamburg, Germany)**

- **Photon virtuality**: $Q^2$
- **Inelasticity**: $y$
- **Bjorken Variable**: $x$

**Periods of operation**:

- **Integrated luminosity**: \( \sim 0.5 \text{ fb}^{-1} \) per experiment
Probe of proton structure

**Neutral Current** DIS cross section can be written via structure functions $F_2$, $xF_3$, $F_L$:

$$
\frac{d^2 \sigma_{\text{NC}}^\pm}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} [Y_+ F_2 \mp Y_- xF_3 - y^2 F_L]
$$

where $Y_\pm = 1 \pm (1 - y)^2$

**Proton structure functions:**
- $F_2$ - dominant, sensitive to sea quarks
- $xF_3$ - sensitive to valence quarks, essential at high $Q^2$
- $F_L$ - sensitive to gluon, essential at high $y$

At the leading order:

$$
F_2 = x \sum e_q^2 [q(x) + \bar{q}(x)]
$$

$$
xF_3 = x \sum 2e_q a_q [q(x) - \bar{q}(x)]
$$

At LO, **Charge Current** DIS $e^+ p$ and $e^- p$ cross sections are sensitive to different quark densities:

$$
\sigma_{CC}^+ \sim x[\bar{u} + \bar{c}] + x(1 - y)^2[d + s]
$$

$$
\sigma_{CC}^- \sim x[u + c] + x(1 - y)^2[\bar{d} + \bar{s}]
$$
Probe of proton structure

The preliminary combination of all inclusive DIS cross sections measured by the H1 and ZEUS collaborations at HERA

Neutral current

Charge current

- data in the kinematic range $0.01 < Q^2 < 3 \cdot 10^4 \text{GeV}^2$, $10^{-6} < x < 0.65$
- minimization of the $\chi^2$-function taking into account the correlated systematic uncertainties
- for $20 < Q^2 < 100 \text{ GeV}^2$ the typical total uncertainty $\sim 1\%$
Inclusive HERA data are the basis for any QCD analysis and extracting PDFs.

Series of PDF sets have been released by H1 and ZEUS collaborations.

HERAPDF2.0 set has just been preliminary released.

Features of HERAPDF2.0:

- the latest (and preliminary) PDF set from HERA
- combined all neutral and charge current inclusive HERA data are used
- NLO and NNLO QCD fits are performed for analysis
- The uncertainties are decreased at low-\(x\)
- The PDFs are consistent with previous official HERAPDF1.5
Extraction of the PDFs

The PDFs are extracted from QCD fit to the combined HERA data

**Fit settings:**

- Fitted distributions: 
  \[ x_g, x_{u}, x_d, \bar{x}_U = x_u, \bar{x}_D = x_d + x_s \]
- The simple functional form

\[
x_f(x) = A x^B (1 - x)^C (1 + Dx + Ex^2)
\]

- **A** – normalization
- **B** – low-x behaviour, **C** – high-x behaviour
- **D,** **E** – medium behaviour

- starting scale \( Q_0^2 = 1.9 GeV^2 \) below the charm mass threshold
- **NLO** and **NNLO** DGLAP
  - fit with heavy flavour schemes (RT, ACOT)
  - fit is performed for \( Q^2 \geq Q_{min}^2 = 3.5 GeV^2 \)
  - factorization and renormalization scales = \( Q^2 \)
  - fixed \( \alpha_s(M_Z) = 0.1176 \)
PDF uncertainties:

\[ \Delta \chi^2 = 1 \]

- Correlations of the sys uncertainties are properly taken into account
- Experimental, model and parametrization uncertainties:
  - model uncertainties obtained by variation \( Q_{\text{min}}^2, m_c, m_b, f_s \)
  - parametrization uncertainties are estimated by including additional parameters into the functional form and variation of \( Q_0^2 \)

\[ x_f(x) = Ax^B(1-x)^C(1+Dx+Ex^2) \]
NNLO fit demands fewer gluon at low-\(x\)
So, low-\(x\) still needs to be studied
The proton structure functions $F_2(x, Q^2)$, $F_L(x, Q^2)$ measured by the H1 and ZEUS collaborations at HERA.

The curves represent the prediction from the NLO QCD fit.
H1: Measurement of the Structure function $F_L$

HERA-II data of proton energy $E_p = 460, 575, 920$ GeV are used

The low-$x$ ($x \leq 0.002$) measurements of the structure function $F_L$ are extended to $Q^2 \geq 1.5$ GeV$^2$

(first measurements of $F_L$ in the region $120 \leq Q^2 \leq 800$ GeV$^2$

Within uncertainties, all predictions describe data reasonably well

Within uncertainties, all predictions describe data reasonably well
– gluon dominance is assumed
– valence contribution is ignored, however this contribution is substantial reaching 10% for $x \sim 0.01$.

Factorization:

$$\sigma_{T,L}(x, Q^2) = \int d^2r \int_0^1 dz |\Psi_{T,L}(z,r)|^2 \sigma(x, r^2)$$

GBW (Golec-Biernat, Wüsthoff) dipole model:

$$\sigma(x, r^2) = \sigma_0 \left(1 - e^{\left[-\frac{r^2}{4R_0^2(x)}\right]}\right), \quad R_0^2(x) = \left(\frac{x}{x_0}\right)^\lambda$$

IIM (Iancu, Itakura, Munier) dipole model: approximate solution BK eq. produces $\sigma(x, r^2)$

B-SAT (Kowalski, Motyka, Watt) dipole model:

$$\sigma(x, r^2) = \sigma_0 \left(1 - \exp\left[-\frac{\pi r^2 \alpha_s(\mu^2) xg(x, \mu^2)}{3\sigma_0}\right]\right)$$

$$xg(x, Q_0^2) = A_g x^{-\lambda_g} (1 - x)^{C_g}$$
DGLAP: RT scheme, ACOT scheme

Describe the data by the DGLAP and Dipole models in the kinematic region $x < 0.01$ and $Q^2 > 3.5 \text{ GeV}^2$.

IIM dipole model leads to the best $\chi^2$.

Estimate the valence quark contribution using DGLAP.

$Q^2 \geq 10 \text{ GeV}^2$: agreement

$Q^2 < 10 \text{ GeV}^2$: RT and ACOT predict lower $F_L$

<table>
<thead>
<tr>
<th>Fit</th>
<th>GBW</th>
<th>IIM</th>
<th>B-SAT</th>
<th>ACOT (NLO)</th>
<th>RT (NLO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole +DGLAP$_{\text{valence}}$</td>
<td>559.7/252</td>
<td>259.4/252</td>
<td>261.7/252</td>
<td>248.3/249</td>
<td>288.8/249</td>
</tr>
</tbody>
</table>

**Table:** Quality of fits in terms of $\chi^2/N_{\text{dof}}$ for GBW, IIM and B-SAT dipole model as well as ACOT and RT DGLAP schemes for various fit conditions.
**HERAFitter package**

is the open source QCD Fit Package used to determine the PDFs

- different data can be analysed: DIS $e^p$, Drell-Yan $pp$, $p\bar{p}$ and jet data.
- the package is based on MINUIT minimization
- theory predictions can be calculated by
  - VFNS from R. Thorne
  - ACOT from F. Olness
  - QCDNUM from M. Botje
  - APPLGRID code
  - FastNLO code
  - Dipole models
- PDF parametrizations: HERA, CTEQ, Chebyshev
- estimation of uncertainties: Hessian method, MC method

http://herafitter.org/
Summary

- The combined H1 and ZEUS cross section data precisely constraint the parton density functions.
- HERAPDF2.0 is the last preliminary PDF set released by the H1 and ZEUS collaborations. HERAPDF1.5 is the currently used set.
- The proton structure functions $F_2$ and $F_L$ have been measured at HERA.
- The dipole models describe the data at low $x$, the IIM dipole model is the best.
- The publicly available HERAFitter http://herafitter.org/ package is a powerful tool for determination of PDFs using different data.

Thank you so much for your attention!
Backup
Comparison of the measured $e$ charge asymmetry to the predictions of different PDF models for electron $p_T > 35$ GeV.

Measured $W$ charge asymmetry as a function of lepton pseudorapidity $|\eta|$ compared with theoretical predictions calculated to NNLO.
Deep Inelastic Scattering

The structure of the proton is studied in Deep Inelastic $e^\pm p$ Scattering. The Lorentz invariant variables:

- **Negative four-momentum transfer squared:**
  \[ Q^2 = -q^2 = -(k - k')^2 \]

- **Bjorken-$x$ variable:**
  \[ x = \frac{Q^2}{2P \cdot q} \]

- **Inelasticity:**
  \[ y = \frac{P \cdot q}{P \cdot k} \]

The center-of-mass energy squared
\[ s = (k + P)^2 \]

Neutral Current DIS cross section can be written via structure functions $F_2$, $F_L$, $xF_3$:

\[
\frac{d^2 \sigma_{NC}^{\pm}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ Y_+ F_2(x, Q^2) - y^2 F_L(x, Q^2) \mp Y_- x F_3(x, Q^2) \right]
\]

where $Y_\pm = 1 \pm (1 - y)^2$. 
DGLAP Evolution

Evolution of the PDFs along $Q^2$ is described in pQCD by DGLAP evolution equations.

Starting scale $Q_0^2 = 1.9 \text{ GeV}^2$

Scale $Q^2 = 10 \text{ GeV}^2$

DGLAP evolution equations allow us to obtain structure functions at any scale $Q^2 \geq Q_0^2$.
Low-\(x\) physics

- low-\(x\) region is characterised by the high gluon density
- DGLAP evolution equations may be unable to correctly describe low-\(x\) region
- since at NLO (\(\alpha_s^1\)) \(F_L\) gets contribution from the interaction of the virtual photon with
  - a quark emitting the gluon
  - a gluon as a parton of the proton
So, \(F_L\) is sensitive to gluon distribution

\[
xg(x, Q^2) \sim 1.77 \frac{3\pi}{2\alpha(Q^2)} F_L(x, Q^2)
\]

As a result, experimental input is crucially important for understanding the processes at low-\(x\)
Measurement of $F_L$

Experimental input is crucially important for understanding the processes at low-$x$

- testing DGLAP
- constraints to PDFs
- predictions for LHC
- testing Dipole models

- direct determination of $F_L$

The NC reduced cross section (at low $Q^2$):

$$\sigma_r(x, Q^2, y) = F_2(x, Q^2) - \left[ \frac{y^2}{1 + (1 - y)^2} \right] F_L(x, Q^2)$$

Since

$$Q^2 = x y s$$

For fixed $x$, $Q^2$:

different $y \Rightarrow$ different center-of-mass energies $s$

Direct $F_L$ measurement is possible only at colliding experiments with different beam energies
Measurement of $F_L$

Parametrized PDFs at the starting scale
$x_g, x_{u\nu}, x_{d\nu}, x\bar{U} = x\bar{u}, x\bar{D} = x\bar{d} + x\bar{s}$,

**Parametrization**

$$x_g(x) = A_g x^{B_g} (1 - x)^{C_g} - A'_g x^{B'_g} (1 - x)^{25}$$
$$x_{u\nu}(x) = A_{u\nu} x^{B_{u\nu}} (1 - x)^{C_{u\nu}} (1 + D_{u\nu} x + E_{u\nu} x^2)$$
$$x_{d\nu}(x) = A_{d\nu} x^{B_{d\nu}} (1 - x)^{C_{d\nu}}$$
$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}}$$
$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}}$$

**Additional constraints:** $A_g, A_{u\nu}, A_{d\nu}$ are from sumrules
$A_{\bar{U}} = A_{\bar{D}} (1 - f_s)$, $B_{\bar{U}} = B_{\bar{D}}$, $\rightarrow x\bar{u} \rightarrow x\bar{d}$ for low-$x$

**Parametrization choices for each HERAPDF set:**

<table>
<thead>
<tr>
<th>HERAPDF</th>
<th>$D_{u\nu}$</th>
<th>$E_{u\nu}$</th>
<th>$A'_g$</th>
<th>$B'_g$</th>
<th>$B_{u\nu}$</th>
<th>$\sum \text{par}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0</td>
<td>free</td>
<td>0</td>
<td>0</td>
<td>$B_{u\nu} = B_{d\nu}$</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
<td>free</td>
<td>0</td>
<td>0</td>
<td>$B_{u\nu} = B_{d\nu}$</td>
<td>10</td>
</tr>
<tr>
<td>1.6</td>
<td>free</td>
<td>free</td>
<td>free</td>
<td>free</td>
<td>free</td>
<td>14</td>
</tr>
<tr>
<td>1.7</td>
<td>0</td>
<td>free</td>
<td>free</td>
<td>free</td>
<td>free</td>
<td>13</td>
</tr>
</tbody>
</table>

**Note:** flexible gluon parametrization is used when $A'_g$ and $B'_g$ are free
In addition to the HERA-I DIS data, HERAPDF1.5 includes High $Q^2$ HERA-II data. It increases the statistics:

- by 2.5 times for $e^+ p$
- by about 10 times for $e^- p$

For HERAPDF1.5 the precision of the valence distributions at high $x$ is improved.
HERAPDF1.0 → HERAPDF1.5

IIM fit: $\chi^2 / n_{dof} = 397.6 / 352$ (plots in paper draft)
Data for $x > 0.01$ are not included in the fit.

Description of the data is much improved at high $x$ due to the addition of the valence contribution.

Overall $\chi^2$ in the fitted region is not improved.

$$\chi^2/n_{dof} = 287.6/252$$ compared to $$\chi^2/n_{dof} = 259.4/252$$ without DGLAP valence contribution.
### HERAPDF1.0 → HERAPDF1.5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>IIM fit</th>
<th>IIM+DGLAP(_{\text{valence}}) fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Uncertainty</td>
</tr>
<tr>
<td>(R_{IIM}) (fm)</td>
<td>0.593</td>
<td>0.004</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0.258</td>
<td>0.002</td>
</tr>
<tr>
<td>(x_0)</td>
<td>(0.59 \times 10^{-4})</td>
<td>(0.03 \times 10^{-4})</td>
</tr>
</tbody>
</table>

**Table:** Parameters and total uncertainties of the IIM dipole and IIM+DGLAP\(_{\text{valence}}\) fits performed for \(Q^2 \geq 3.5\) GeV\(^2\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>B-SAT fit</th>
<th>B-SAT+DGLAP(_{\text{valence}}) fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Uncertainty</td>
</tr>
<tr>
<td>(A_g)</td>
<td>2.35</td>
<td>0.04</td>
</tr>
<tr>
<td>(\lambda_g)</td>
<td>0.072</td>
<td>0.006</td>
</tr>
<tr>
<td>(Q_0^2) (GeV(^2))</td>
<td>2.02</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Table:** Parameters and total uncertainties of the B-SAT dipole and B-SAT+DGLAP\(_{\text{valence}}\) fits performed for \(Q^2 \geq 3.5\) GeV\(^2\).
### Fit Conditions

<table>
<thead>
<tr>
<th>Fit Conditions</th>
<th>GBW</th>
<th>IIM</th>
<th>$\chi^2/n_{dof}$</th>
<th>ACOT</th>
<th>RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal fit</td>
<td>718.8/352</td>
<td>397.6/352</td>
<td>424.9/352</td>
<td>715.2/781</td>
<td>764.5/781</td>
</tr>
<tr>
<td>$Q^2 \geq 3.5$ GeV$^2$</td>
<td>559.7/252</td>
<td>259.4/252</td>
<td>261.7/252</td>
<td>248.3/249</td>
<td>288.8/249</td>
</tr>
<tr>
<td>DGLAP$_{valence}$</td>
<td>739.5/252</td>
<td>287.6/252</td>
<td>371.4/252</td>
<td>248.3/249</td>
<td>288.8/249</td>
</tr>
</tbody>
</table>

**Table:** Quality of fits in terms of $\chi^2/n_{dof}$ for GBW, IIM and B-SAT dipole model as well as ACOT and RT DGLAP schemes for various fit conditions described in the H1 paper.

- ACOT and RT DGLAP fits are discussed in the H1 paper.
- "Nominal fits" for the dipole models are performed for $x < 0.01$ while for the DGLAP fits at $Q^2_{min} \geq 3.5$ GeV$^2$.
- DGLAP$_{valence}$ fits are performed in a common phase space for the Dipole and DGLAP models.
Input Data into HERAPDF fits

Input data:

- **HERAPDF1.0** (NLO)
  - combined **CC/NC HERA-I inclusive** data [JHEP01(2010) 109]

- **HERAPDF1.5** (NLO, NNLO)
  - the currently recommended set, available at LHAPDF
    - + combined **CC/NC High Q^2 HERA-II inclusive** data [prelim]
      
      _increase of statistics, sensitive to valence quarks_

- **HERAPDF1.6** (NLO)
    
    _determination of \( \alpha_s \)_

- **HERAPDF1.7** (NLO)
  - + combined **charm \( F_2 \)** data [prelim]
    
    _constrains on charm mass_
  - + combined **low energy** run data [prelim]
    
    _sensitive to \( F_L \)_