Conserved Quantities in Lemaître-Tolman-Bondi Cosmology

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Overview

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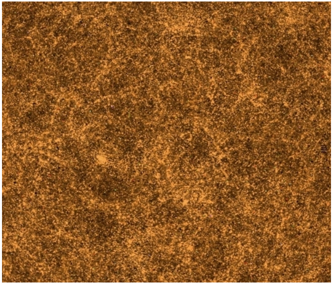

Image: SDSSIII

Why Perturb LTB Cosmology?

Why Perturb LTB Cosmology?

- Recent observations (e.g. SN1a) suggest late time accelerated expansion - driven by Dark Energy
- Inhomogeneous Cosmologies explain observations through inhomogeneous expansion not acceleration - no Dark Energy
- Many possible inhomogeneous cosmologies; LTB type models still actively researched (simplest model, toy model)
- Ongoing work interpreting observations (galaxy surveys, large scale structure surveys, CMB) and other redshift dependent observations in context of LTB background

Flat FRW vs LTB

Flat FRW vs LTB

- FRW: Maximally symmetric spatial section - expansion time dependent only

$$ds^2 = -dt^2 + a^2(t)dr^2 + a^2(t)r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- LTB: Spherically symmetric spatial section - expansion time and r coordinate dependant (not θ, ϕ)[a]

$$ds^2 = -dt^2 + X^2(r,t)dr^2 + Y^2(r,t)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

[a] Bondi 1947

The Standard Model of Cosmology - Flat FRW

The Standard Model of Cosmology - Flat FRW

- Background metric:

$$ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j$$

- Perturbed metric:

$$ds^2 = -(1 + 2\Phi)dt^2 + 2aB_i dx^i dt + a^2(\delta_{ij} + 2C_{ij})dx^i dx^j$$

with scalar, vector and tensor perturbations[a]

[a] e.g. Bardeen 1980

The Standard Model of Cosmology - Flat FRW

The Standard Model of Cosmology - Flat FRW

- Further decomposition of 3-spatial perturbations gives curvature perturbation ψ, identified with the intrinsic scalar curvature:

$$C_{ij} = E_{,ij} - \psi\delta_{ij} + vector + tensor \quad quantities^*$$

* On 3-spatial hypersurfaces

The Standard Model of Cosmology - Flat FRW

Constructing Gauge Invariant Quantities

- Splitting quantities into background + perturbation: no longer covariant - gauge dependent; construct gauge invariant quantities
- General gauge transformations:

$$\widetilde{\delta \mathbf{T}} = \delta \mathbf{T} + \pounds_{\delta x^\mu} \bar{\mathbf{T}}$$

- Tilde denotes new coordinates

$$\widetilde{x^\mu} = x^\mu + \delta x^\mu$$

bar denotes background.

- Key quantities gauge transformations:

$$\begin{aligned} \widetilde{\psi_{\rm FRW}} &= \psi_{\rm FRW} + \frac{\dot{a}}{a}\delta t \\ \widetilde{\delta\rho_{\rm FRW}} &= \delta\rho_{\rm FRW} + \dot{\bar{\rho}}\delta t \end{aligned}$$

The Standard Model of Cosmology - Flat FRW

Constructing Gauge Invariant Quantities

- Gauge choice: uniform density hypersurfaces, $\widetilde{\delta\rho_{\mathrm{FRW}}} = 0$

$$\delta t = -\frac{\delta\rho_{\mathrm{FRW}}}{\dot{\bar{\rho}}}$$

Get gauge invariant curvature perturbation

$$-\zeta \equiv \psi_{\mathrm{FRW}} + \frac{\dot{a}/a}{\dot{\bar{\rho}}}\delta\rho$$

- Evolution equations for ζ from time derivative, $\delta\rho$ from energy conservation $\nabla_\mu T^{\mu\nu} = 0$...

  ζ conserved in large scale limit - conserved perturbed quantities allow easily relate early to late times (e.g. curvature/physics early time relates to density/observables late time)

Conserved Quantities in Perturbed LTB

Perturbed LTB

- Background metric:

$$ds^2 = -dt^2 + X^2(r,t)dr^2 + Y^2(r,t)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Metric Perturbations:

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\Phi & XB_r & YB_\theta & Y\sin\theta B_\phi \\ XB_r & 2X^2C_{rr} & XYC_{r\theta} & XY\sin\theta C_{r\phi} \\ YB_\theta & XYC_{r\theta} & 2Y^2C_{\theta\theta} & Y^2\sin\theta C_{\theta\phi} \\ Y\sin\theta B_\phi & XY\sin\theta C_{r\phi} & Y^2\sin\theta C_{\theta\phi} & 2Y^2\sin^2\theta C_{\phi\phi} \end{pmatrix}$$

Conserved Quantities in Perturbed LTB

Perturbed LTB

- We have performed 1+3 decomposition into time and spatial sections of metric
- Decomposition of perturbations not completely straightforward without use of methods like spherical harmonic decomposition but...[a]
- Our undecomposed perturbations give simpler expressions, easing constructing conserved quantities.

[a] e.g. Clarkson, Clifton, February 2009

Conserved Quantities in Perturbed LTB

Perturbed LTB

- Background Energy Conservation:

$$\dot{\rho} + \rho(H_X + 2H_Y) = 0\,, \qquad H_X = \frac{\dot{X}}{X}\,, \qquad H_Y = \frac{\dot{Y}}{Y}$$

- Perturbed Energy Conservation:

$$\begin{aligned} \delta\dot{\rho} &+ (\delta\rho + \delta P)\left(H_X + 2H_Y\right) + \bar{\rho}' v^r \\ &+ \bar{\rho}\Big(\dot{\boldsymbol{C}}_{\boldsymbol{rr}} + \dot{\boldsymbol{C}}_{\boldsymbol{\theta\theta}} + \dot{\boldsymbol{C}}_{\boldsymbol{\phi\phi}} + \partial_r v^r + \partial_\theta v^\theta + \partial_\phi v^\phi \\ &+ \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right] v^r + \cot\theta v^\theta\Big) = 0 \end{aligned}$$

- Convenient to define spatial metric perturbation:

$$3\psi = \frac{1}{2}\delta g^k_k = \boldsymbol{C}_{\boldsymbol{rr}} + \boldsymbol{C}_{\boldsymbol{\theta\theta}} + \boldsymbol{C}_{\boldsymbol{\phi\phi}}$$

Conserved Quantities in Perturbed LTB

Constructing Gauge Invariant Quantities

- ψ transformation behaviour:

$$3\tilde{\psi} = 3\psi + \left[\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y}\right]\delta t + \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right]\delta r + \partial_i \delta x^i + \delta\theta\cot\theta\,,$$

- Gauge choices; uniform density:

$$\delta t\Big|_{\delta\tilde{\rho}=0} = -\frac{1}{\dot{\bar{\rho}}}\left[\delta\rho + \bar{\rho}'\delta r\right]\,.$$

comoving:

$$\delta x^i = \int v^i dt\,.$$

Conserved Quantities in Perturbed LTB

Constructing Gauge Invariant Quantities

- Gives gauge invariant **Spatial Metric Trace Perturbation (SMTP)** on comoving, uniform density hypersurfaces:

$$
\begin{aligned}
-\zeta_{\mathbf{SMTP}} \quad = \quad & \psi + \frac{\delta\rho}{3\bar{\rho}} + \frac{1}{3}\Bigg\{ \left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}}\right)\int v^r dt \\
+ \quad & \partial_r \int v^r dt + \partial_\theta \int v^\theta dt + \partial_\phi \int v^\phi dt + \cot\theta \int v^\theta dt \Bigg\}
\end{aligned}
$$

Conserved Quantities in Perturbed LTB

Constructing Gauge Invariant Quantities

- Get gauge invariant density perturbation on uniform curvature hypersurfaces

$$\begin{aligned} \delta\tilde{\rho}\Big|_{\psi=0} &= \delta\rho + \bar{\rho}\Bigg\{ 3\psi + \left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}}\right) \int v^r dt \\ &+ \partial_r \int v^r dt + \partial_\theta \int v^\theta dt + \partial_\phi \int v^\phi dt + \cot\theta \int v^\theta dt \Bigg\} \end{aligned}$$

- May be related to ζ_{SMTP} as

$$\delta\tilde{\rho}\Big|_{\psi=0} = -3\bar{\rho}\zeta_{\mathrm{SMTP}}$$

Conserved Quantities in Perturbed LTB

Conserved Quantities in Perturbed LTB

- $\zeta_{\rm SMTP}$ Evolution Equation:

$$\dot{\zeta}_{\mathbf{SMTP}} = \frac{H_X + 2H_Y}{3\bar{\rho}}\delta P_{\mathbf{nad}}$$

- Valid on all scales.
- For barotropic fluids $\dot{\zeta}_{\mathbf{SMTP}} = 0$

Conserved Quantities in Perturbed LTB

Conclusion and Further Research

- $$\dot{\zeta}_{\mathbf{SMTP}} = \frac{H_X + 2H_Y}{3\bar{\rho}}\delta P_{\mathrm{nad}}$$
- Research already extended to other spacetimes.
  i.e.$\dot{\zeta}_{\mathbf{SMTP}}$ already extended to Lemaitre and FRW
- Potential wider use of $\zeta_{\mathbf{SMTP}}$ in inhomogeneous spacetimes generally versus standard FRW model.

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