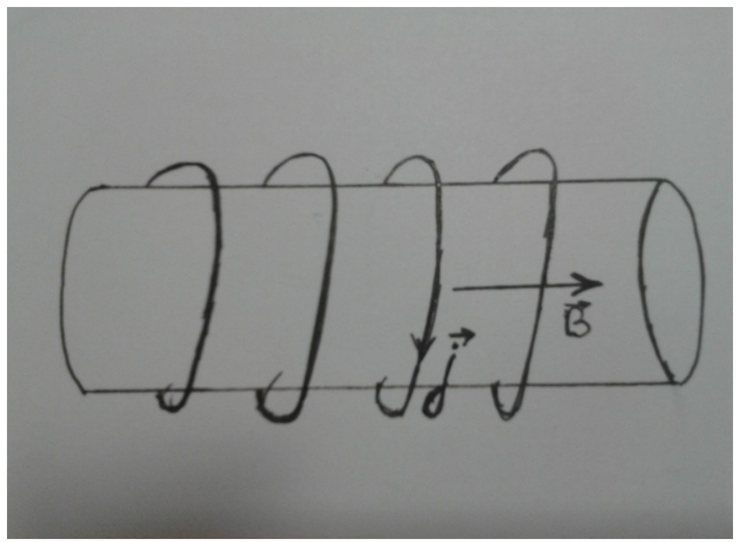
Induced vacuum current and magnetic field in the background of a cosmic string modeled by an impenetrable magnetic-flux-carrying tube

Bogolyubov Institute for Theoretical Physics
of the National Academy of Sciences of Ukraine

Iryna Ivanchenko



«ETTORE MAJORANA» FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE
INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS
Erice, 24 June – 3 July 2014

Vacuum polarization effects in a magnetic cosmic string background are considered. Cosmic string is modeled by finite radius magnetic-flux-carrying tube that is impenetrable for quantum matter. The vacuum polarization depends on the choice of a boundary condition at the edge of the tube and on a magnetic flux of string.

Impenetrable tube with magnetic flux can match objects such as:

Abrikosov vortices in condensed matter physics

artificial solenoids (Bohm-Aharonov effect)

Cosmic strings that may have formed in the Universe as a result of phase transitions

Objects of cosmological scale:

Topological strings:

- *Infinitely long, straight*
- *Closed rings*

Superstrings:

- *F –strings*
- *D– strings*

Cosmic strings are objects that are theoretically expected in many modifications of the Standard Model. They are close to the experimental observations.

Possibilities of detecting cosmic strings *:*

In fluctuations of microwave radiation
- *In the emission of gravitational waves*
- *Gravitational lensing by cosmic strings*
- *Nonthermal radiation from the cusp to the superconducting string*
- *"Millisecond" radio-flare "spark“*

We already have two ways of detecting cosmic string :

- *Gravitational lensing by cosmic string:*
In 2004 the analysis of fluctuations of light gravitationally lensing quasar Q0957+561.

- *“Millisecond” radio-flare “spark”:*
In 2007 a “millisecond” radio-flare “spark” was observed.
And in 2008 “spark” was explained as a flash from the cusp to the superconducting string.

Nielsen-Olesen model

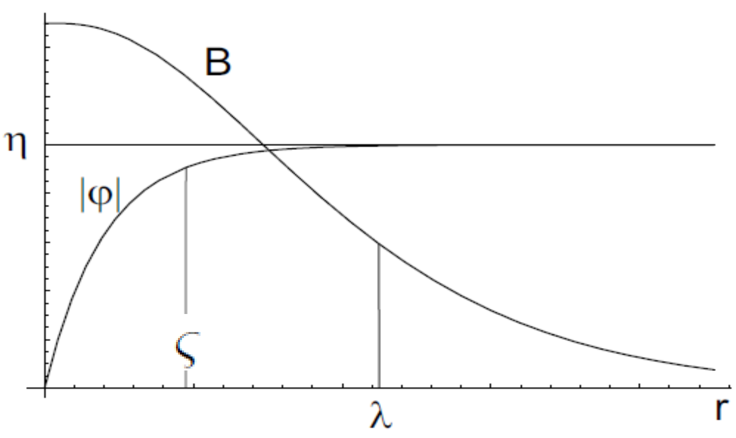
Lagrangian of the theory with spontaneously broken symmetry:

$$L = D_\mu \phi^\dagger D^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + c_2 |\phi|^2 - c_4 |\phi|^4$$

$$D_\mu = \partial_\mu - igA_\mu \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The minimum of the energy is located at $\phi = \eta e^{i\theta} \ (\eta = \sqrt{c_2/(2c_4)})$

The amplitude of the magnetic and scalar fields near the cosmic strings:



The magnetic field close to the strings exponentially decreases with the distant from it. The characteristic length :

$$\lambda = 1/(|g|\eta)$$

The scalar field also varies according to the exponential law with a characteristic length

$$\varsigma = 1/\sqrt{2c_2}$$

The case of a singular string:

The operator of a second-quantized scalar field can be represented in the form:

$$\Psi(\mathbf{x},t) = \sum\!\!\!\!\!\!\int_{\lambda} \frac{1}{\sqrt{2E_\lambda}} \left[e^{-iE_\lambda t}\, \psi_\lambda(\mathbf{x})\, a_\lambda + e^{iE_\lambda t}\, \psi_{-\lambda}(\mathbf{x})\, b_\lambda^{+}\right]$$

The time-independent Klein-Gordon equation:

$$\left(-\boldsymbol{\nabla}^2 + m^2\right)\psi_\lambda(\mathbf{x}) = E_\lambda{}^2\psi_\lambda(\mathbf{x})$$ where: $$\boldsymbol{\nabla} = \boldsymbol{\partial} - i\mathbf{V}(\mathbf{x})$$

The complete set of solutions to Klein-Gordon equation in the field of the vortex that satisfy the regularity condition:

$$\psi_{kn\mathbf{p}}(\mathbf{x}) = (2\pi)^{\frac{1-d}{2}} J_{|n-\Phi|}(kr) e^{in\varphi} e^{i\mathbf{p}\mathbf{x}_{d-2}}$$

where $0 < k < \infty, \quad n \in \mathbb{Z}, \quad -\infty < p_\nu < \infty, \quad \nu = \overline{3, d}$

They are normalized to a delta function:

$$\int d^d x\ \psi^*_{kn\mathbf{p}}(\mathbf{x})\psi_{k'n'\mathbf{p}'}(\mathbf{x}) = \frac{\delta(k-k')}{k}\delta_{nn'}\delta(\mathbf{p}-\mathbf{p}')$$

We only will have φ –component of the current, the remaining components vanish identically

The vacuum current:

$$\mathrm{j}(\mathrm{x}) = \frac{1}{4\mathrm{i}} \left\langle \mathrm{vac} \left| \left\{ \left[\Psi^\dagger(t,\, \mathrm{x}),\, \nabla\Psi(t,\, \mathrm{x}) \right]_+ - \left[\nabla\Psi^\dagger(t,\, \mathrm{x}),\, \Psi(t,\, \mathrm{x}) \right]_+ \right\} \right| \mathrm{vac} \right\rangle$$

$$\mathbf{j}(\mathbf{x}) = (2i)^{-1} \sum\!\!\!\!\!\!\int_\lambda E_\lambda{}^{-1} \left\{ \psi_\lambda^*(\mathbf{x}) \left[\nabla\psi_\lambda(\mathbf{x}) \right] - \left[\nabla\psi_\lambda(\mathbf{x}) \right]^* \psi_\lambda(\mathbf{x}) \right\}$$

The vector potentials of electromagnetic field:

$$V_1(\mathbf{x}) = -\Phi \frac{x^2}{(x^1)^2 + (x^2)^2}, \quad V_2(\mathbf{x}) = \Phi \frac{x^1}{(x^1)^2 + (x^2)^2}, \quad V_\nu(\mathbf{x}) = 0, \quad \nu = \overline{3, d} \ ,$$

$$B^{3\cdots d}(\mathbf{x}) = 2\pi\, \Phi\, \delta(x^1)\, \delta(x^2),$$

φ –component of the current:

$$j_\varphi(\mathbf{x}) \equiv r^{-1} \left[x^1 j_2(\mathbf{x}) - x^2 j_1(\mathbf{x}) \right] =$$

$$= (2\pi)^{1-d}\, r^{-1} \int d^{d-2}p \int_0^\infty dk k \left(p^2 + k^2 + m^2 \right)^{-\frac{1}{2}} \sum_{n\in\mathbb{Z}} (n - \Phi) J^2_{|n-\Phi|}(kr)$$

To obtain the final expressions , need to regularize:

$$j_{\varphi,reg}(x, \mathrm{F}) = j_\varphi(x, F) - j_\varphi(x, F = 0)$$

The vacuum current is a periodic function of the flux which vanishes at F=0, 1/2, 1. It is a negative function on the interval 0<F<1/2 , and is a positive function on the interval 1/2<F<1.

After performing the integration in the case of space dimension d = 2 we get:

$$j_\varphi(\mathbf{x}) = \frac{\sin(F\pi)}{4\pi^2 r^2}\left(F-\frac{1}{2}\right)\left\{-4\left[\left(F-\frac{1}{2}\right)^2+m^2r^2\right]\int_{2mr}^{\infty}\frac{du}{u}K_{2F-1}(u)+\right.$$
$$\left.+mr\left[K_{2F}(2mr)+K_{2(1-F)}(2mr)\right]\right\},\quad d=2,$$

The asymptotic expressions for the vacuum current at small and large distances from the vortex are given by :

$$j_\varphi(\mathbf{x}) = \frac{4\sin(F\pi)}{(4\pi)^{\frac{d}{2}+1}}\left(F-\frac{1}{2}\right)\frac{\Gamma\left(\frac{d-1}{2}+F\right)\Gamma\left(\frac{d-1}{2}+1-F\right)}{\Gamma\left(\frac{d}{2}+1\right)}r^{-d}\left\{1+O\left[(mr)^2\right]\right\},$$
$$mr \ll 1,$$

$$j_\varphi(\mathbf{x}) = \frac{2\sin(F\pi)}{(4\pi)^{\frac{d+1}{2}}}\left(F-\frac{1}{2}\right)e^{-2mr}m^{\frac{d-3}{2}}r^{-\frac{d+3}{2}}\left\{1+O\left[(mr)^{-1}\right]\right\},\ mr \gg 1.$$

The magnetic field of strength B is induced in the vacuum:

$$\partial_r B^{3\ldots d}_{(I)}(\mathbf{x}) = -e^2 j_\varphi(\mathbf{x}) \qquad B^{3\ldots d}_{(I)}(\mathbf{x}) = e^2 \int_r^\infty dr\, j_\varphi(\mathbf{x})$$

we obtain:

$$B^{3\ldots d}_{(I)}(\mathbf{x}) = \frac{16\, e^2\, \sin(F\pi)}{(4\pi)^{\frac{d+3}{2}}} \frac{r^{1-d}}{\Gamma(\frac{d+1}{2})} \int_{mr}^\infty dw\, (w^2 - m^2 r^2)^{\frac{d-1}{2}} \times$$
$$\times \left\{ w\left[K^2_{1-F}(w) - K^2_F(w)\right] + \left(2F-1\right) K_F(w) K_{1-F}(w) \right\}.$$

The total flux of the vacuum magnetic field (in 2π units)

$$\Phi^{(I)} = \int_0^\infty dr\, r B^{3\ldots d}_{(I)}(\mathbf{x})$$

Specifically, we have: $$\Phi^{(I)} = \frac{2\, e^2\, m^{d-3}}{3(4\pi)^{\frac{d+1}{2}}} \Gamma\left(\frac{3-d}{2}\right) F\,(1-F)\left(F - \frac{1}{2}\right)$$

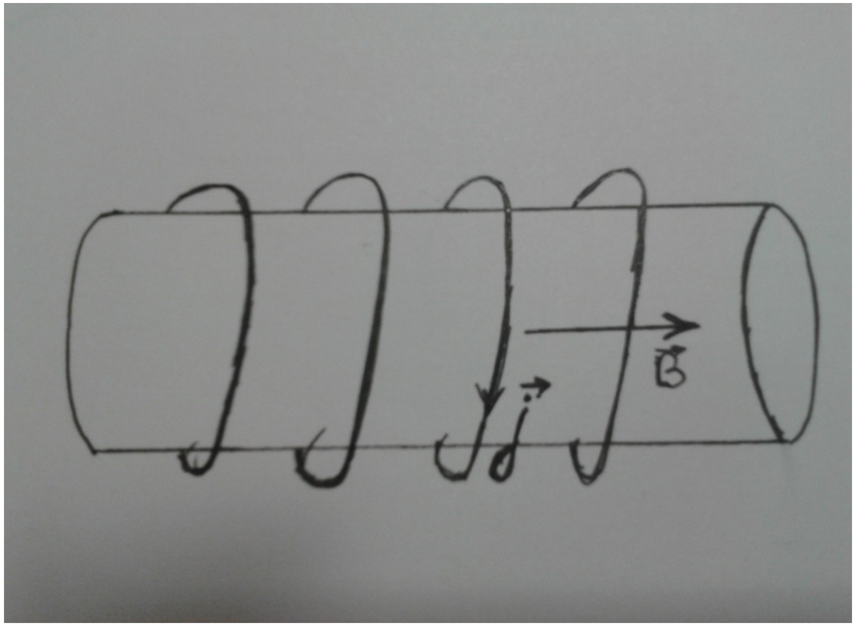
Thus, the current density near the string behaves as r^{-d} and goes to ∞ !
Induced magnetic flux is infinite for d > 2

String with finite radius

Physically interesting case is vacuum polarization by the magnetic cosmic string of finite radius.

- ***The cosmic string is modeled by an impenetrable magnetic tube with some boundary conditions.***

Problem:



At the boundary of the tube, the scalar field requires imposition of boundary conditions, which lead to vacuum polarization. We consider partial case of the Neumann boundary conditions:

$$\nabla_r \psi_\lambda |_{r=r_0} = 0$$

The solutions of Klein-Gordon equation:

$$\psi_{kn}(\mathbf{x}) = (2\pi)^{-1/2} e^{in\varphi} \Upsilon_{|n-e\Phi/2\pi|}(kr, kr_0)$$

$$\Upsilon_\nu(v,u) = \frac{Y_\nu'(u) J_\nu(v) - J_\nu'(u) Y_\nu(v)}{[Y_\nu^2(u) + J_\nu^2(u)]^{1/2}}$$

$$\int d^2\mathbf{x}\, \psi_{kn}^* \psi_{k'n'} = \langle \lambda' | \lambda \rangle = \frac{\delta(k-k')}{k} \delta_{n,n'}$$

$$j_\varphi(r) = \frac{1}{2\pi r} \int_0^\infty dk\, k(k^2+m^2)^{-1/2} S(kr, kr_0)$$

$$S(v,u) = \sum_{n=0}^{\infty} \left[(n+1-F)\, \Upsilon_{n+1-F}^2(v,u) - (n+F)\, \Upsilon_{n+F}^2(v,u) \right]$$

$$j_{\varphi,reg}(x,\mathrm{F}) = j_\varphi(x,F) - j_\varphi(x,F=0)$$

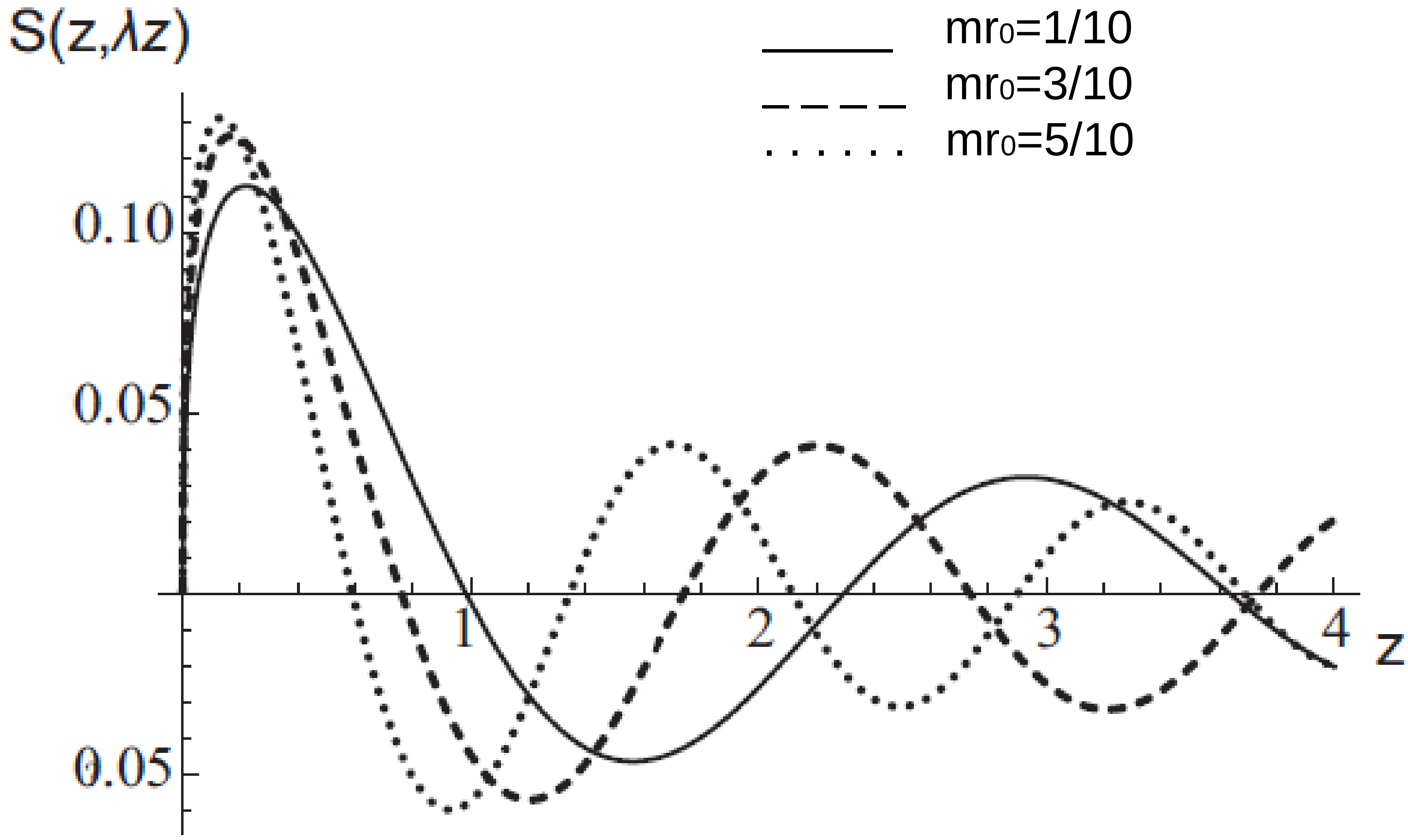
Our attempts at an analytical solution failed and hence we have to adopt a numerical approach

$$r^2 j_\varphi = \frac{1}{2\pi} \int_0^\infty dz\, z \left(z^2 + \left(\frac{mr_0}{\lambda} \right)^2 \right)^{-1/2} [S_F(z, \lambda z) - S_{F=0}(z, \lambda z)]$$

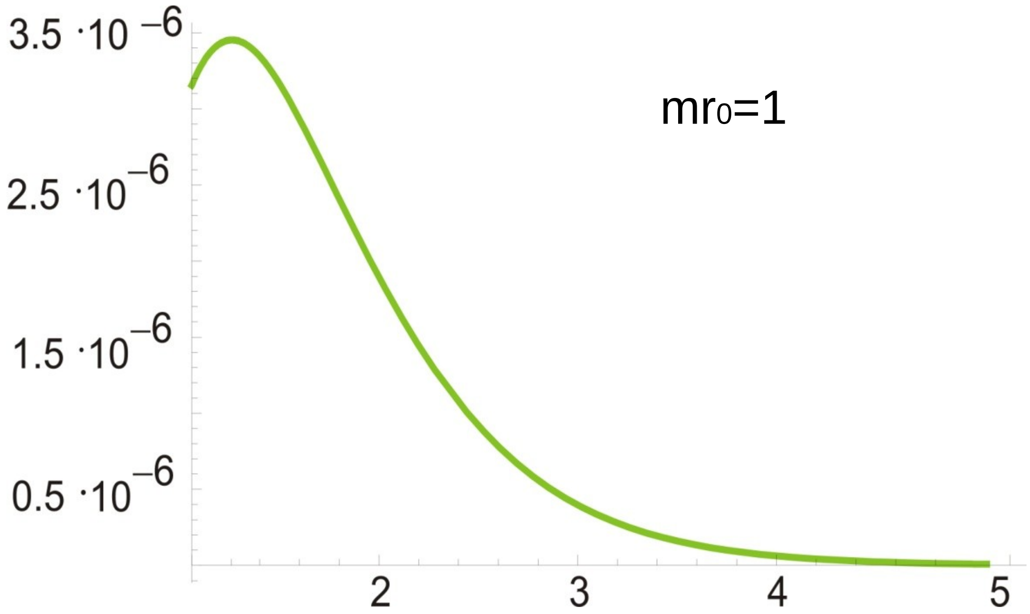
where $z = kr,\ z \in (0,\infty)$ and $\lambda = r_0/r,\ \lambda \in [0,1]$

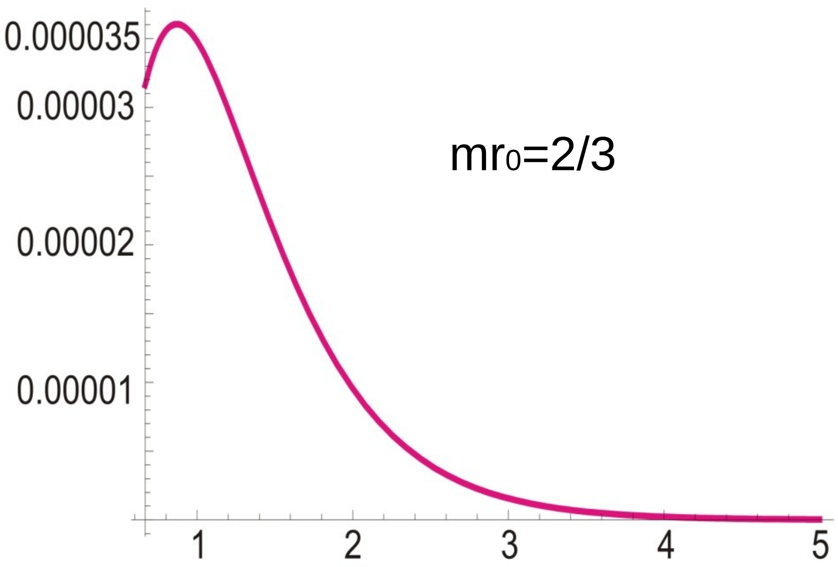
Behavior of S(z, λz) at different values of λ

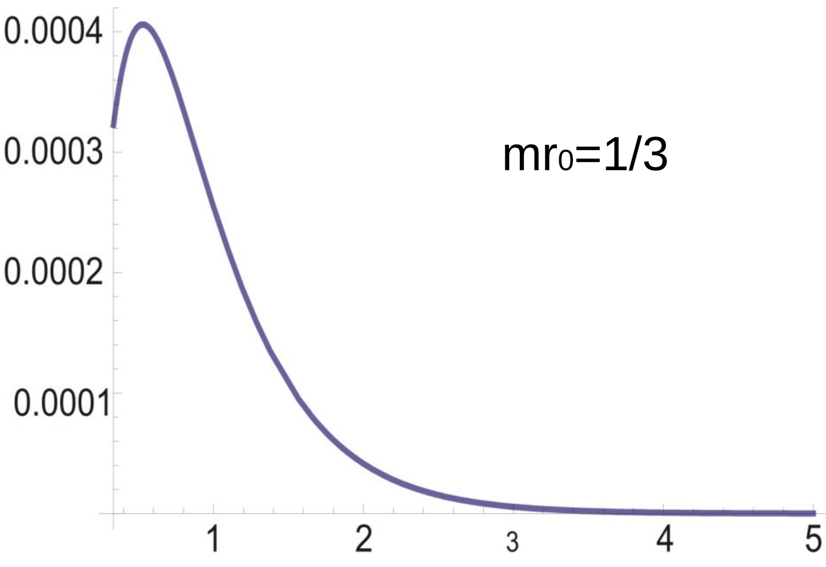
S(z, λz) is an oscillation function with amplitude that quite slowly decreases at large z.



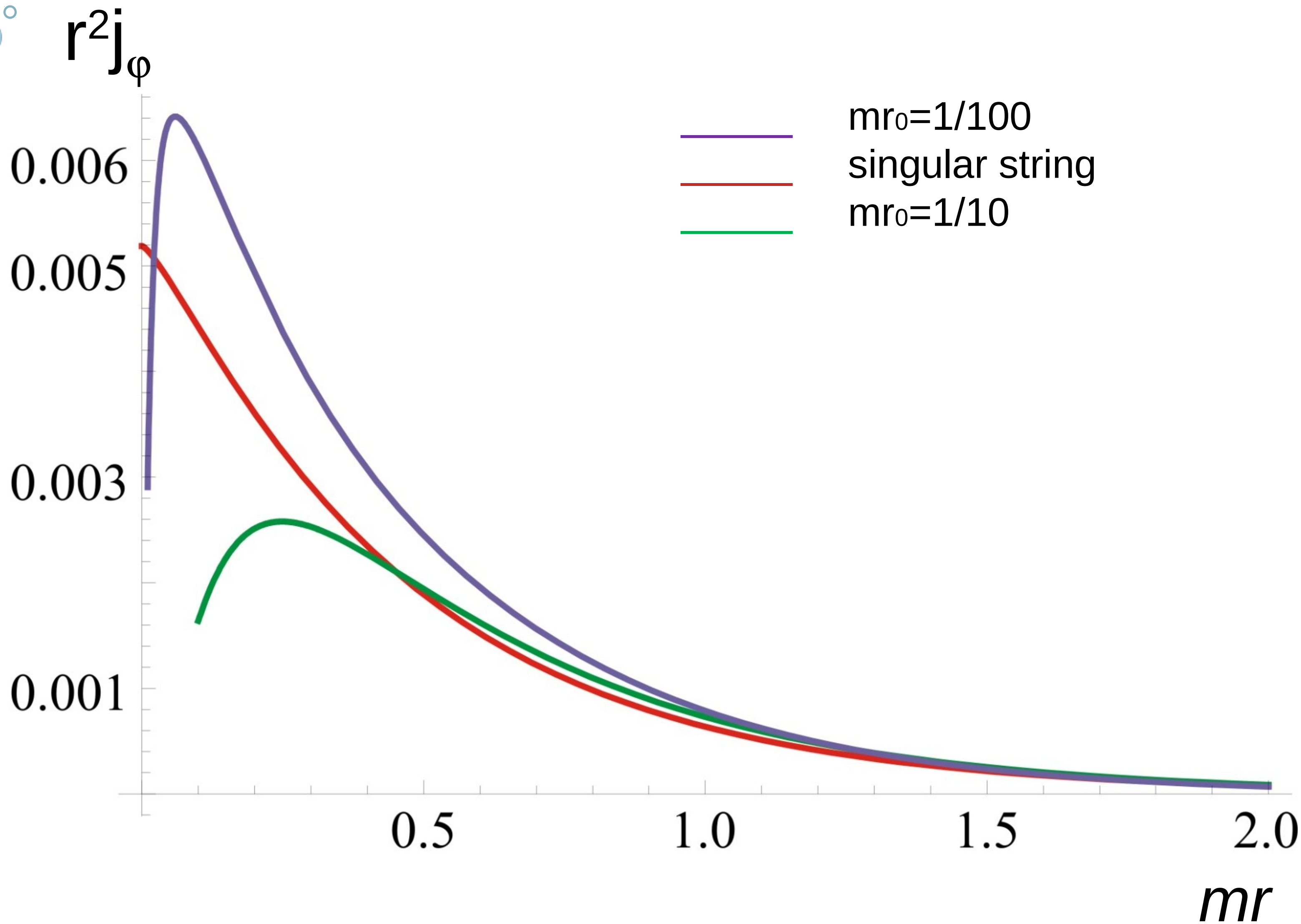
The vacuum current ($r^2 j_\varphi$) for different value of mr_0





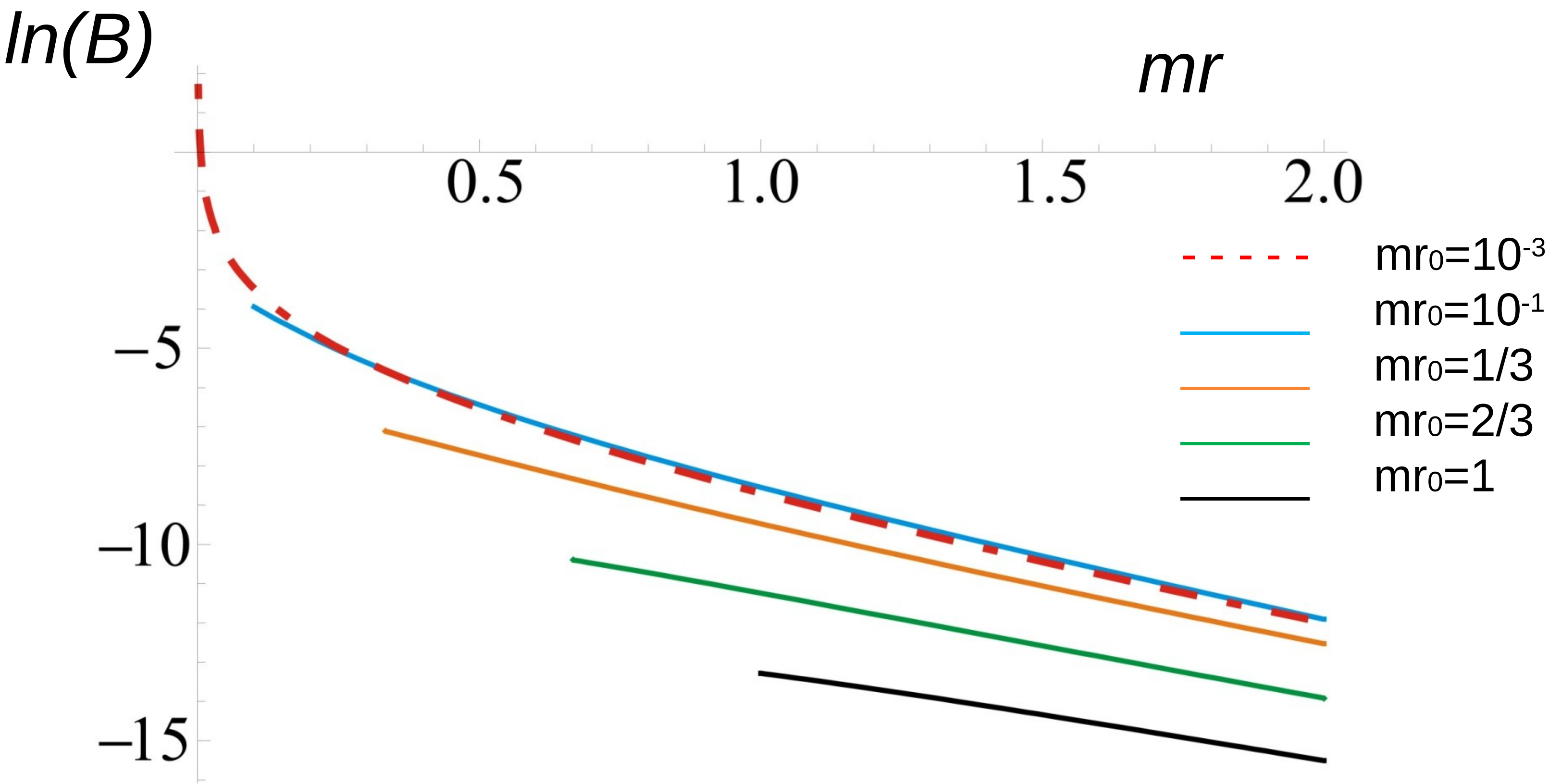


The vacuum current density can be bigger for a string with finite small radius than for a singular string



Magnetic field for different mr_0

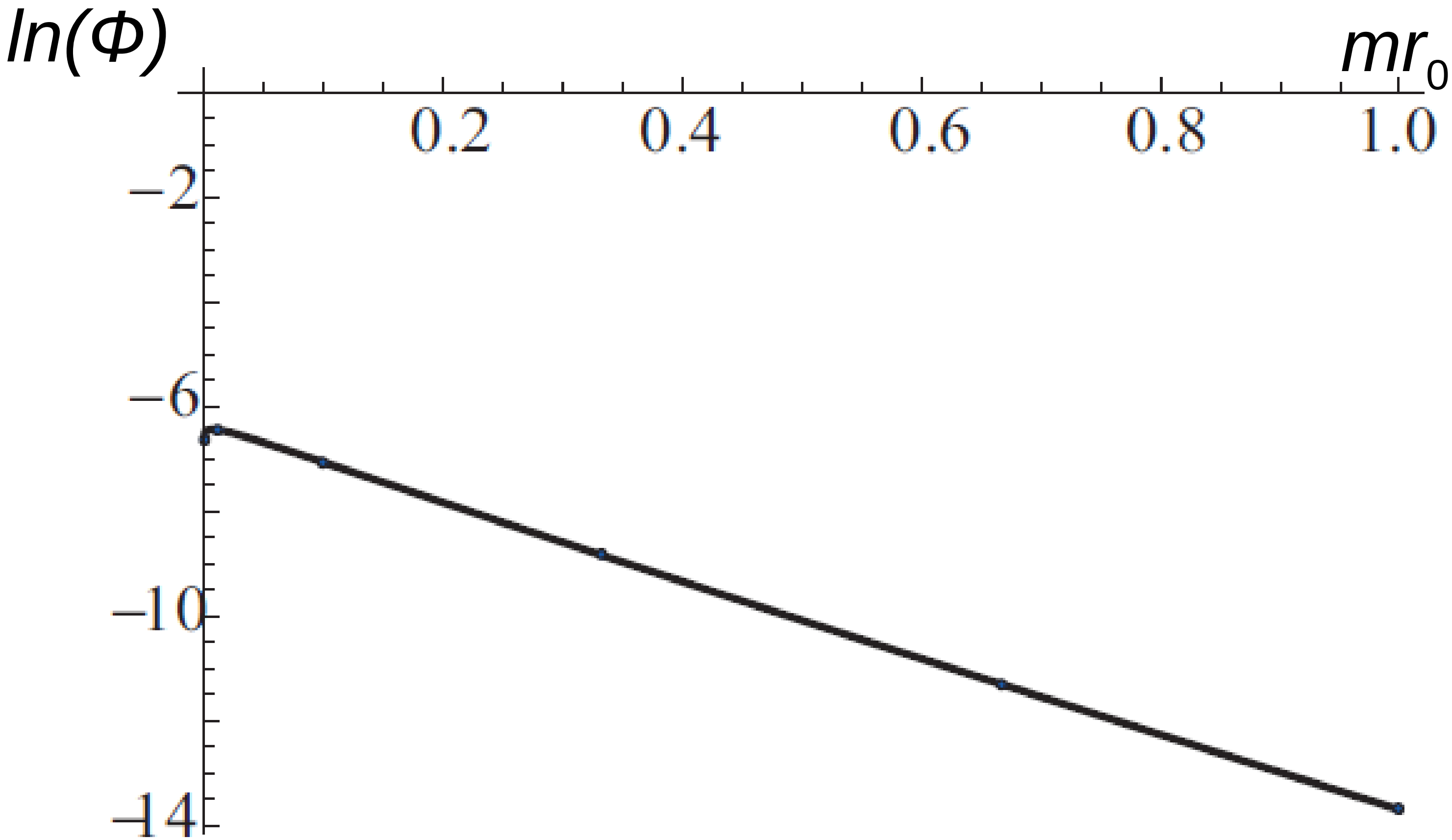
We not have a limit transition to singular string if $mr_0 = 0$



Magnetic flux

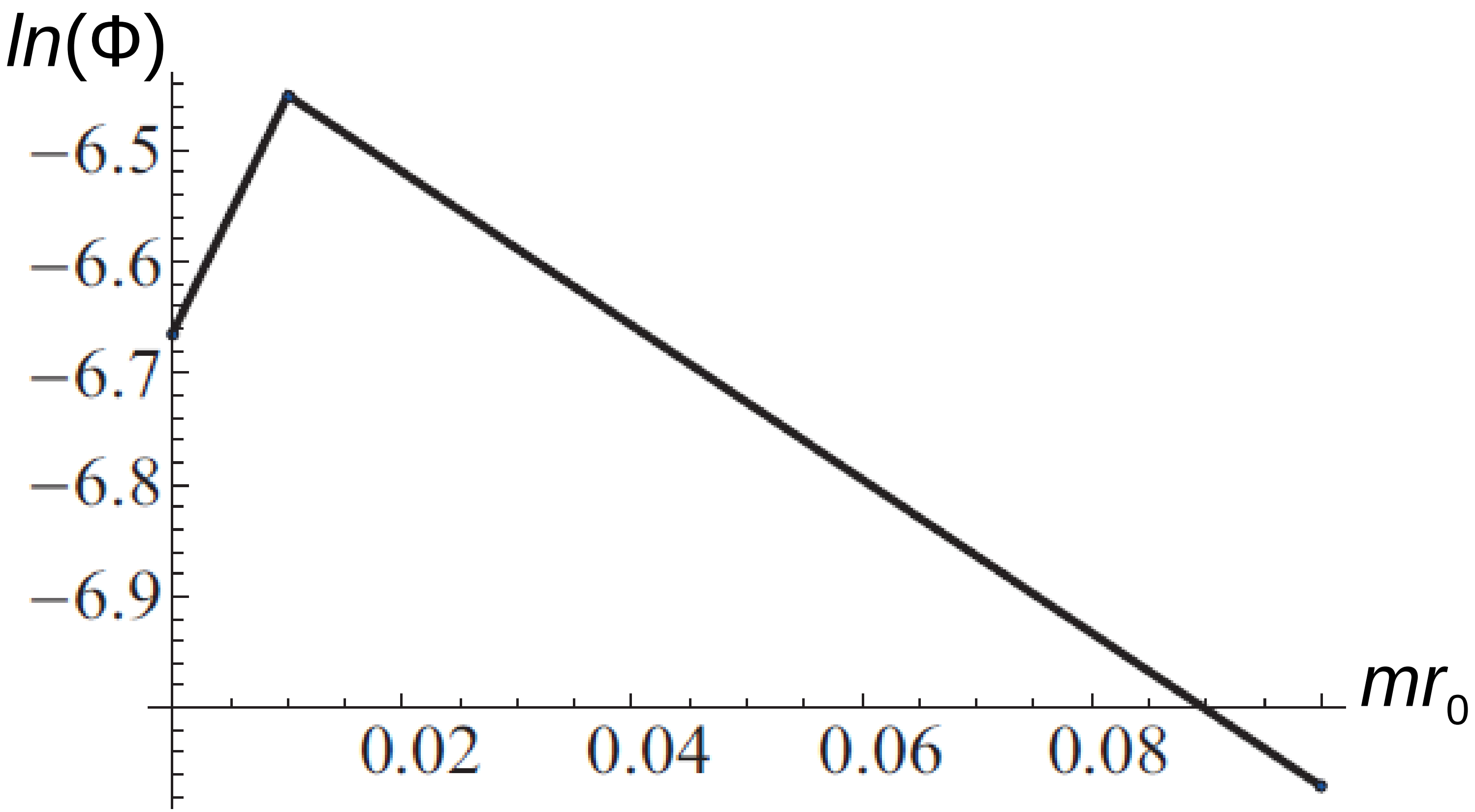
All point of string with finite radius create a line.

Magnetic flux: $\Phi = Ae^{+\alpha(mr_0)}$, where A=1/584,841, α=-7,347



Magnetic flux

There is no limit transition to singular string at $mr_0 \to 0$



Conclusions:

1. *The advantages of **singular string**- analytical solution. The advantages of **the model of the string as a tube with finite radius** - solves the problem of divergent induced quantities.*

2. ***We obtained for induced vacuum current:***
   - only φ-component of the current isn't equal to 0;
   - the limiting transition $r \to 0$ isn't transfer the model of the string as an impenetrable magnetic-flux-carrying tube with Neumann boundary conditions to the singular magnetic string.

3. ***The density of vacuum current and induced magnetic field for tubes with different thicknesses:***
   - the vacuum effects increase with decreasing of tube radius;
   - in the some area of space the vacuum effects can be bigger for string with finite small radius $mr_0 \ll 1$ than for singular string.

Thank you for attention!