Nonstationary QFT

Linerally growing loop corrections in strong electric field

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Introduction

We study loop corrections to sQED in strong electric field by using Schwinger-Keldysh diagrammatic technique instead of Feynman one.

Why should we study sQED in this way?

- It is similar to the situation in curved space-time such as Hawking radiation, de-Sitter instabilities and so on, when $|\text{out}\rangle \neq |\text{in}\rangle$, but is more simpler.
- Feynman diagrammatic technique calculates the non-diagonal matrix elements $\langle\text{out}|A|\text{in}\rangle$, whereas Keldysh one gives $\langle\text{in}|A|\text{in}\rangle$.
- One can study dependence of different quantities on time and it has a clear physical meaning, as a solution of corresponding kinetic equation.

Formulation of the problem

We consider scalar QED in $(3+1)$ dimensions.

$$S = \int d^4x \left[|D_\mu \phi|^2 - m^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu}^2 - j_\mu^{cl} A^\mu \right]$$

External source creates background classical electromagnetic field, that is a solution of Maxwell's equations

$$\partial^\mu F_{\mu\nu}^{cl} = j_\nu^{cl}$$

Full gauge potential can be divided into two pieces $A_\mu = A_\mu^{cl} + a_\mu$ - classical and quantum fields. And photons will be the quanta of the field a_μ.

We consider eternal electric field $\vec{E} = \text{const}$ in gauge $A_\mu^{cl} = (0, -Et, 0, 0)$.

The problem

How do level populations of photons $n_{\mu\nu} = \langle a_\mu^\dagger a_\nu \rangle$ and scalar bosons $n_\pm = \langle a_\pm^\dagger a_\pm \rangle$ depend on time in such system?

Free theory for photons

After simple manipulation on initial action we get

$$S = \int d^4x \left[|D^{cl}_\mu \phi|^2 - m^2|\phi|^2 - \frac{1}{4} f^2_{\mu\nu} + e^2 a^2_\mu |\phi|^2 + a_\mu e (i\phi D^{cl\,\mu} \phi^* - i\phi^* D^{cl\,\mu} \phi) \right]$$

Where $D^{cl}_\mu = \partial_\mu + ieA^{cl}_\mu$, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$.

Free theory for field a_μ is the same as for usual EM field and we can use the following decomposition.

$$a_\mu(x) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{\sqrt{2E_q}} \left(a_{\mu\,p} e^{-ipx} + \text{h.c.} \right), \left[a_{q,\mu}, a^\dagger_{p,\nu} \right] = -g_{\mu\nu} (2\pi)^3 \delta^{(3)}(q-p)$$

Wave equation on scalar field operators $\left[D_{cl}^2 + m^2\right]\phi = 0$

Field decomposition we choose due to spatial homogeneity

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3}\left[a_{\vec{p}}\, e^{i\,\vec{p}\,\vec{x}} f_p(t) + b_{\vec{p}}^{\dagger}\, e^{-i\,\vec{p}\,\vec{x}} f_{-p}^{*}(t)\right]$$
$$\left[a_p, a_q^{\dagger}\right] = \left[b_p, b_q^{\dagger}\right] = (2\pi)^3\delta^{(3)}(q-p)$$

And equation on harmonics $f_p(t)$ has the following form

$$\left[\partial_t^2 + \omega_p^2(t)\right]\, f_p(t) = 0,\ \omega_p(t) = \sqrt{m^2 + \left[\vec{p} + e\vec{A}(t)\right]^2}$$

For constant field it can be solved analytical, but we need only a obvious feature that $f_p(t) = f\left(p_1 + eEt, p_{\perp}\right)$.

In an interacting field theory $H = H_0 + V$ we get the following formula for Green function

$$G(x,y) = \langle T\phi_H(x)\phi_H(y)\rangle = \langle S^{-1}T\left[\phi_{H_0}(x)\phi_{H_0}(y)S\right]\rangle$$

$$S = T\exp\left(-i\int V_{H_0}(x)d^4x\right), S^{-1} = \tilde{T}\exp\left(i\int V_{H_0}(x)d^4x\right)$$

Where A_H denotes, that the operator A evolutes due to Hamiltonian H.

Usually physicists suppose that interaction is adiabatically turned off in a distinct future and past. Hence, S acts as $S\left|\text{in}\right\rangle = e^{i\psi}\left|\text{out}\right\rangle$ and we can't calculate diagonal elements $\langle\text{in}|\text{A}|\text{in}\rangle$.

If we don't throw S^{-1} away, we have to expand it also. But now we need 4 propagators to express all Wick's contractions.

$$G_0^{--} = \langle T\phi_{H_0}(x)\phi_{H_0}(y)\rangle \, G_0^{++} = \langle \tilde{T}\phi_{H_0}(x)\phi_{H_0}(y)\rangle$$
$$G_0^{-+} = \langle \phi_{H_0}(x)\phi_{H_0}(y)\rangle \, , G_0^{+-} = \langle \phi_{H_0}(y)\phi_{H_0}(x)\rangle$$

Instead of 4 linearly dependent propagators we rotate to another propagators.

- $D^K = \frac{1}{2}\left\langle \left\{\phi(x), \bar{\phi}(y)\right\}\right\rangle = \frac{1}{2}\left[G^{++} + G^{--}\right]$ tells us about density matrix of corresponding particles.
- $D^{R,A} = \mp\theta(\mp\Delta t)\left\langle \left[\phi(x), \bar{\phi}(y)\right]\right\rangle$ contain information about energy levels of particles.

For example we consider the following potential $V_{H_0} = \lambda \frac{\phi_{H_0}^4}{4!}$. In the order $\mathcal{O}(\lambda^2)$ we have

$$\Delta_1 G(x,y) =$$

$$-i\lambda \langle T\phi_{H_0}(x)\phi_{H_0}(y) \int \phi_{H_0}^4(z)dz\rangle + i\lambda \langle \tilde{T} \int \phi_{H_0}^4(z)dz \; T\phi_{H_0}(x)\phi_{H_0}(y))\rangle =$$

$$\frac{i}{2}\lambda \int dz \left[G^{-+}(x,z)G^{++}(z,z)G^{+-}(z,y) - G^{-+}(x,z)G^{++}(z,z)G^{+-}(z,y)\right]$$

So, we just put in usual Feynman diagram sign in each vertex and then sum over these signs with corresponding propagators.

Density matrix can be calculated by using the following formula.

$$G^K_{\mu\nu}(q,t_1,t_2) = \left[-\frac{g_{\mu\nu}}{2} + n_{\mu\nu}(t_1,t_2)\right] \frac{e^{-i|q|(t_1-t_2)}}{2|q|} + \kappa_{\mu\nu}(t_1,t_2)\, \frac{e^{-i|q|(t_1+t_2)}}{2|q|} + h.c.$$

$$n_{\mu\nu}(t_1,t_2) = \langle \mathrm{in}|a^\dagger_\mu(t_1)a_\nu(t_2)|\mathrm{in}\rangle \ , \ \kappa_{\mu\nu}(t_1,t_2) = \langle \mathrm{in}|a_\mu(t_1)a_\nu(t_2)|\mathrm{in}\rangle$$

Loop corrections, given by Keldysh diagram technique, can be expressed in the terms of density matrices and can be interpreted as solution of underlying Boltzmann kinetic equation.

Loop corrections

We consider the following limit $t_1 - t_2 = \text{const}, T = \frac{t_1+t_2}{2} \to \infty$. After that we get the leading contribution to photon's density matrix.

$$n_{\mu\nu}(q, t_1, t_2) =$$

$$e^2 \int_{t_0}^{t_1} dt_3 \int_{t_0}^{t_2} dt_4 \frac{e^{-i|q|(t_3-t_4)}}{2|q|} \int \frac{d^3k}{(2\pi)^3} \left[f_k(t_3) \overleftrightarrow{D}_\mu f_{k+q}(t_3) \right] \left[f_k^*(t_4) \overleftrightarrow{D}_\nu f_{k+q}^*(t_4) \right]$$

$$\kappa_{\mu\nu}(q, t_1, t_2) =$$

$$-2e^2 \int_{t_0}^{t_1} dt_3 \int_{t_0}^{t_3} dt_4 \frac{e^{i|q|(t_3+t_4)}}{2|q|} \int \frac{d^3k}{(2\pi)^3} \left[f_k(t_3) \overleftrightarrow{D}_\mu f_{k+q}(t_3) \right] \left[f_k^*(t_4) \overleftrightarrow{D}_\nu f_{k+q}^*(t_4) \right]$$

After changing of variables $T = \frac{1}{2}(t_3 + t_4), \tau = t_3 - t_4$ we get

$$n_{\mu\nu} \propto e^2 (T - t_0), \kappa_{\mu\nu} \approx \text{const}$$

This appears IR divergences because if $T \to \infty$, then an effective small parameter $e^2 T$ goes to infinity and perturbation theory breaks down.

The other propagators can be systematically studied in the similar way

- Loop corrections to the propagators of scalar bosons turn out to be suppressed

$$\frac{dn^{\pm}}{dT} \propto \frac{e^2}{T^2}, \; \frac{d\kappa^{\pm}}{dT} \propto \frac{e^2}{T^2}, \; T \to \infty$$

- Advanced and retarded propagators are suppressed for both photons and scalars bosons, because their correction have the following form

$$D_1^A(p, t_1, t_2) = \int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 D_0^A(p, t_1, t_3)\, \Sigma^A(p, t_3, t_4)\, D_0^A(p, t_4, t_2),$$

  And in the limit $t_1 - t_2 = \text{const}, t_1, t_2 \to \infty$ this corrections are suppressed.

This propagators are suppressed and in the following calculations we won't take into account loop corrections to them.

Because only photon's Keldysh propagator diverges we have to sum up all leading loop corrections to him. It can be done by using Dyson-Schwinger equation.

$$G^K_{\mu\nu}(p,t_1,t_2) = G^K_{0\,\mu\nu}(p,t_1,t_2) + \sum_{s_1,s_2,s_3=\pm} e^2 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 \int \frac{d^3k}{(2\pi)^3} \times$$
$$\times G^{s_1 s_2}_{0\,\mu\rho}(p,t_1,t_3) D^{s_2 s_3}(p-k,t_3,t_4) \overleftrightarrow{D}_\rho(t_3) \overleftrightarrow{D}_\sigma(t_4) D^{s_2 s_3}(k,t_3,t_4) G^{s_3 s_1}_{\sigma\nu}(p,t_4,t_2)$$

As it was mentioned, only photon density matrix gets the largest contribution. Therefore, extracting diverging parts from LHS and RHS of equation we get kinetic equation on density matrix.

$$\frac{dn^\gamma_{\mu\nu}}{dt} = -\Gamma^{\rho}_{1\,\mu}\left(-g_{\rho\nu} + n^\gamma_{\rho\nu}\right) + \Gamma^{\rho}_{2\,\mu} n^\gamma_{\rho\nu}$$

$$\Gamma_{1\,\mu\nu}(q) = e^2 \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} d\tau \frac{e^{-2i|q|\tau}}{|q|} \left[f_k(\tau) \overleftrightarrow{D}_\mu f_{k-q}(\tau)\right] \left[f^*_k(-\tau) \overleftrightarrow{D}_\nu f^*_{k-q}(-\tau)\right]$$

$$\Gamma_{2\,\mu\rho}(q) = e^2 \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} d\tau \frac{e^{-2i|q|\tau}}{|q|} \left[f^*_k(\tau) \overleftrightarrow{D}_\mu f^*_{k-q}(\tau)\right] \left[f_k(-\tau) \overleftrightarrow{D}_\rho f_{k-q}(-\tau)\right]$$

It can be checked that all $\Gamma_{1,2,\mu\nu}(q), n_{\mu\nu}(q)$ are transversal. Hence, all quantity has the following tensor structure

$$A_{\mu\nu}(q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) A$$

And kinetic equation can be written as

$$\frac{dn}{dt} = \Gamma_1 + (\Gamma_1 - \Gamma_2)\, n$$
$$n(t) = \frac{\Gamma_2}{\Gamma_2 - \Gamma_1} + C_1 e^{(\Gamma_1 - \Gamma_2)t}$$

System gets large IR contributions either distinct future or past. Hence, eternal electric field can be existed.

Conclusion

- Loop corrections to sQED in electromagnetic field were studied.
- Kinetic equation, describing dynamics of level populations, was obtained.
- Solutions of this equation show presence of IR divergences near either distinct future or past.

Thank you for your attention!