Infrared dynamics of the massive ϕ^4 theory on de Sitter space.

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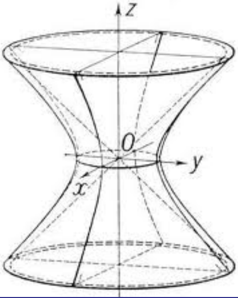
De Sitter space.

D-dimensional de Sitter (dS) space-time is the hyperboloid,

$$X_0^2 - X_i^2 = -1, i = 1, \ldots, D$$

inside $(D+1)$-dimensional Minkowski space-time,
$ds^2 = dX_0^2 - dX_i^2$.



Poincare Patch.

We consider the half of de Sitter space half

$$X_0 - X_D \geq 0$$

which is referred to as expanding Poincare patch (PP):

$$ds^2 = \frac{1}{\eta^2}(d\eta^2 - d\vec{x}^2)$$

where $\eta = e^{-t}$, $\eta \in (+\infty, 0)$.

Free scalar field

Let's consider theory of free real massive scalar field with action

$$S = \int d^D x \sqrt{-g} \left(\frac{1}{2} g^{ik} \partial_i \phi \partial_k \phi - m^2 \phi^2 \right)$$

Varying the action, one can get Klein-Gordon equation.

$$\left(\Box + m^2 \right) \phi(x, \eta) = 0$$

where d'Alamber operator is

$$\Box \phi = \frac{1}{\sqrt{-g}} \partial_\mu \left(g^{\mu\nu} \sqrt{-g} \partial_\nu \phi \right)$$

Harmonics in free theory.

Solving equation of motion, one can get

$$\phi(x,\eta) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \eta^{(D-1)/2} \left(a h(k\eta) e^{-i\vec{k}\vec{x}} + a^* h^*(k\eta) e^{i\vec{k}\vec{x}} \right)$$

Where a is a complex constant, h - solution of Bessel equation with index

$$\nu = \sqrt{\left(\frac{D-1}{2}\right)^2 - m^2}$$

Quantizing the field

To quantize the field, let's consider the operators

$$\hat{\phi}(x,\eta) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left(e^{-ikx} g_k(\eta)\hat{a_k} + e^{ikx} g_k^*(\eta)\hat{a}_k^+\right) \qquad (1)$$

and

$$\hat{\pi}(x,\eta) = \eta^2 \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left(\hat{a}_k e^{-ikx} g_k'(\eta) + \hat{a}_k^+ e^{ikx} {g'}_k^*(\eta)\right) \qquad (2)$$

where $g_k(\eta) = \eta^{(D-1)/2} h(k\eta)$, $\hat{a_k}$ and $\hat{a}_k^+$ are annihilation and creation operators correspondingly.

Theory with self-interaction ϕ^4

Let's add the interaction into the lagrangian of the theory

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

where

$$\mathcal{L}_{int} = \frac{\lambda}{4!}\phi^4$$

Then hamiltonian of interaction is

$$H_{int} = \frac{\lambda}{4!}\int \phi^4\sqrt{-g}d^{D-1}x$$

Keldysh propagator

We would like to calculate Keldysh propagator
$G_K =< \{\phi_H(x)\phi_H(y)\} >$
It's Fourier image has the form

$$D^K(\eta_1, \eta_2, p) = (\eta_1\eta_2)^{\frac{D-1}{2}} d^K(p\eta_1, p\eta_2) \qquad (3)$$

where $p = |\vec{p}|$ and

$$d^K(p\eta_1, p\eta_2) = h(p\eta_1)h^*(p\eta_2)\left(\frac{1}{2} + n_p\right) + h(p\eta_1)h(p\eta_2)k_p + \text{c.c.} \qquad (4)$$

Here $n_p = \langle a_p^+ \, a_p\rangle$ and $\kappa_p = \langle a_p \, a_{-p}\rangle$

Two-loop.

$$n_p(\eta) \approx -\frac{\lambda^2}{3\,(2\pi)^{2(D-1)}} \int d^{D-1}q_1\, d^{D-1}q_2\, d^{D-1}q_3$$
$$\int_\infty^\eta d\eta_3 \int_\infty^\eta d\eta_4\, (\eta_3\eta_4)^{D-2}\, \delta^{(D-1)}\left(\vec{p} + \vec{q}_1 + \vec{q}_2 + \vec{q}_3\right)$$
$$h(p\eta_3)h(q_1\eta_3)h(q_2\eta_3)h(q_3\eta_3)h^*(p\eta_4)h^*(q_1\eta_4)h^*(q_2\eta_4)h^*(q_3\eta_4),$$
$$k_p(\eta) \approx \frac{2\,\lambda^2}{3\,(2\pi)^{2(D-1)}} \int d^{D-1}q_1\, d^{D-1}q_2\, d^{D-1}q_3$$
$$\int_\infty^\eta d\eta_3 \int_\infty^{\eta_3} d\eta_4 (\eta_3\eta_4)^{D-2} \delta^{(D-1)}\left(\vec{p} + \vec{q}_1 + \vec{q}_2 + \vec{q}_3\right)$$
$$h^*(p\eta_3)h(q_1\eta_3)h(q_2\eta_3)h(q_3\eta_3)h^*(p\eta_4)h^*(q_1\eta_4)h^*(q_2\eta_4)h^*(q_3\eta_4). \quad (5)$$

Heavy and light particles

Heavy field

$$m > \frac{D-1}{2} \Longrightarrow \nu \in i\mathbb{R} \Longrightarrow D_K \sim \log(p\eta) \quad (6)$$

has logarithmic behavior with respect to η, hence, linear wrt t
And light one

$$m < \frac{D-1}{2} \Longrightarrow \nu \in \mathbb{R} \Longrightarrow D_K \sim (p\eta)^{-\nu} \quad (7)$$

has power-like behavior with respect to η , hence, exponential with respect to t

Choosing vacuum

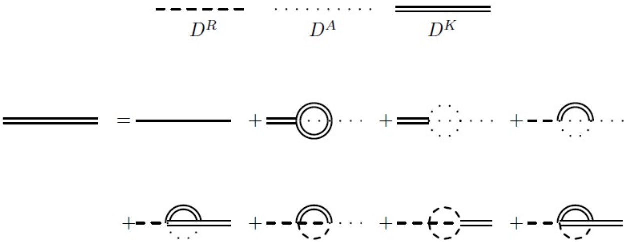
The behavior of functions n and k depends on chose of the initial vacuum, which corresponds to chose of different harmonics. We can go from one vacuum to another by Bogolubov rotation.

For Bunch-Davis (in) vacuum harmonics are Hankel functions, n and k both logarithmic.

For out-Jost vacuum harmonics are pure Bessel function, n is logarithmic, while k is constant.

So, using this fact, we will further neglect k in comparison with n.

Keldysh diagram technique



Dyson-Scwinger equation

$$
\begin{aligned}
D^K(\eta_1,\eta_2,p) = D_0^K(\eta_1,\eta_2,p) - \\
- \frac{\lambda^2}{6\,(2\pi)^{2(D-1)}} \int d^{D-1}q_1\, d^{D-1}q_2\, d^{D-1}q_3 \iint_\infty^0 \frac{d\eta_3\, d\eta_4}{(\eta_3\eta_4)^D}\, \delta^{(D-1)}\left(\vec{p}-\vec{q}_1-\vec{q}_2-\vec{q}_3\right) \\
\Big[3\, D_0^K(\eta_1,\eta_3,p) D^K(\eta_3,\eta_4,q_1) D^K(\eta_3,\eta_4,q_2) D_0^A(\eta_3,\eta_4,q_3) D_0^A(\eta_4,\eta_2,p) \\
- \frac{1}{4} D_0^K(\eta_1,\eta_3,p) D_0^A(\eta_3,\eta_4,q_1) D_0^A(\eta_3,\eta_4,q_2) D_0^A(\eta_3,\eta_4,q_3) D_0^A(\eta_4,\eta_2,p) \\
- \frac{3}{4} D_0^R(\eta_1,\eta_3,p) D^K(\eta_3,\eta_4,q_1) D_0^A(\eta_3,\eta_4,q_2) D_0^A(\eta_3,\eta_4,q_3) D_0^A(\eta_4,\eta_2,p) \\
+ D_0^R(\eta_1,\eta_3,p) D^K(\eta_3,\eta_4,q_1) D^K(\eta_3,\eta_4,q_2) D^K(\eta_3,\eta_4,q_3) D_0^A(\eta_4,\eta_2,p) \\
- \frac{3}{4} D_0^R(\eta_1,\eta_3,p) D^K(\eta_3,\eta_4,q_1) D_0^R(\eta_3,\eta_4,q_2) D_0^R(\eta_3,\eta_4,q_3) D_0^A(\eta_4,\eta_2,p) \\
- \frac{1}{4} D_0^R(\eta_1,\eta_3,p) D_0^R(\eta_3,\eta_4,q_1) D_0^R(\eta_3,\eta_4,q_2) D_0^R(\eta_3,\eta_4,q_3) D^K(p\eta_4,p\eta_2) \\
+ 3\, D_0^R(\eta_1,\eta_3,p) D_0^R(\eta_3,\eta_4,q_1) D^K(\eta_3,\eta_4,q_2) D^K(\eta_3,\eta_4,q_3) D^K(\eta_4,\eta_2,p) \Big]
\end{aligned}
$$

Anzatz

$$D^{K,R,A}(\eta_1,\eta_2,p)=\int d^{D-1}x\, e^{i\vec{p}\vec{x}}\, G^{K,R,A}(\eta_1,\vec{x},\eta_2,0)\,,$$

$$D^{K}(\eta_1,\eta_2,p)=(\eta_1\eta_2)^{\frac{D-1}{2}}d^{K}(p\eta_1,p\eta_2),$$

$$D^{A}_{R}(\eta_1,\eta_2,p)=\mp\theta(\pm\Delta\eta)(\eta_1\eta_2)^{\frac{D-1}{2}}d^{-}(p\eta_1,p\eta_2),$$

where $\Delta\eta=\eta_1-\eta_2$, $p=|\vec{p}|$ and

$$d^{-}(p\eta_1,p\eta_2)=2\text{Im}\left[h(p\eta_1)h^{*}(p\eta_2)\right],$$

$$d^{K}(p\eta_1,p\eta_2)=h(p\eta_1)h^{*}(p\eta_2)\left(\frac{1}{2}+n_p\right)+h(p\eta_1)h(p\eta_2)k_p+\text{c.c.}\,.$$

Kinetic equation

$$
\begin{aligned}
\frac{n_p(\eta) - n_p(\eta_*)}{\log(\eta) - \log(\eta_*)} \to \frac{dn_{p\eta}}{d\log(p\eta)} = -\frac{\lambda^2\,|A|^2}{6} \int \frac{d^{D-1} l_1}{(2\pi)^{D-1}} \frac{d^{D-1} l_2}{(2\pi)^{D-1}} \int_\infty^0 dv\, v^{D-2} \\
\Big\{ 3\mathbb{R} \left[v^{i\mu} \quad h^*(l_1) h^*(l_2) h\left(\left|\vec{l}_1 + \vec{l}_2\right|\right) h(l_1 v) h(l_2 v) h^*\left(\left|\vec{l}_1 + \vec{l}_2\right| v\right)\right] \times \\
\times \left[(1+n_{p\eta}) n_{l_1} n_{l_2} (1 + n_{|\vec{l}_1+\vec{l}_2|}) \; - \; n_{p\eta}(1+n_{l_1})(1+n_{l_2}) n_{|\vec{l}_1+\vec{l}_2|}\right] \\
+ 3\mathbb{R} \left[v^{i\mu} \quad h^*(l_1) h(l_2) h\left(\left|\vec{l}_1 - \vec{l}_2\right|\right) h(l_1 v) h^*(l_2 v) h^*\left(\left|\vec{l}_1 - \vec{l}_2\right| v\right)\right] \times \\
\times \left[(1+n_{p\eta}) n_{l_1} (1+n_{l_2}) (1 + n_{|\vec{l}_1-\vec{l}_2|}) \; - \; n_{p\eta}(1+n_{l_1}) n_{l_2} n_{|\vec{l}_1-\vec{l}_2|}\right] \\
+ \mathbb{R} \left[v^{i\mu} \quad h^*(l_1) h^*(l_2) h^*\left(\left|\vec{l}_1 + \vec{l}_2\right|\right) h(l_1 v) h(l_2 v) h\left(\left|\vec{l}_1 + \vec{l}_2\right| v\right)\right] \times \\
\times \left[(1+n_{p\eta}) n_{l_1} n_{l_2} n_{|\vec{l}_1+\vec{l}_2|} \; - \; n_{p\eta}(1+n_{l_1})(1+n_{l_2})(1 + n_{|\vec{l}_1+\vec{l}_2|})\right] \\
+ \mathbb{R} \left[v^{i\mu} \quad h(l_1) h(l_2) h\left(\left|\vec{l}_1 + \vec{l}_2\right|\right) h^*(l_1 v) h^*(l_2 v) h^*\left(\left|\vec{l}_1 + \vec{l}_2\right| v\right)\right] \times \\
\times \left[(1+n_{p\eta})(1+n_{l_1})(1+n_{l_2})(1 + n_{|\vec{l}_1+\vec{l}_2|}) \; - \; n_{p\eta} n_{l_1} n_{l_2} n_{|\vec{l}_1+\vec{l}_2|}\right] \Big\}.
\end{aligned}
$$

Solution $n \ll 1$

$$\frac{dn_{p\eta}}{d\log(p\eta)} \approx \Gamma_1 \, n_{p\eta} - \Gamma_2 \tag{8}$$

Here Γ_1 and Γ_2 are the particle decay and production rates, correspondingly.
We should remember, that $\eta = e^{-t}$ and goes in the back direction.
So, the concentration goes to stable stationary point

$$n_{p\eta} = \Gamma_2/\Gamma_1$$

Solution $n \gg 1$

$$\frac{dn_{p\eta}}{d\log(p\eta)} \approx -\bar{\Gamma}\, n_{p\eta}^3, \quad \text{where}$$

Note that $\bar{\Gamma}$ is independent of p. This equation has the obvious solution:

$$n_{p\eta} \approx \frac{1}{\sqrt{2\,\bar{\Gamma}\,\log\left(\eta/\eta_\star\right)}},$$

where $\eta_\star = \frac{\mu}{p}\, e^{-\frac{C}{2\bar{\Gamma}}}$ and C is the integration constant.

Conclusion

- We received kinetic equation for theory ϕ^4 in IR limit.
- We found exploding solution
- It motivates us to consider the feedback of de Sitter space on pair production
- Because of pair production curvature of de Sitter space should decrease
- It may be a key for understanding a problem of cosmological constant

$$\Lambda_{experimental} \sim 10^{-120} \Lambda_{theoretical}$$

Thanks for Your attention