Plane Waves as Tractor Beams

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Outline

Introduction
- What is negative radiation pressure?
- NRP on kinks
- Other approaches

NRP in two channel scattering
- Some simple examples in 1d
- Force in 2d scattering

Applications
- Scattering on cosmic strings
- Neutron scattering on XY model vortices

Conclusions

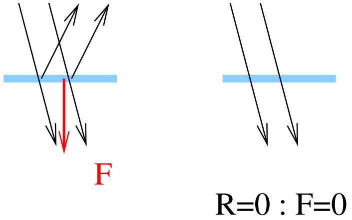
Radiation pressure

Scattering of light: the pressure acting on a platelet

$$P = (1 + R^2 - T^2)W\,,$$

$$W = \epsilon_0 E_0^2/2$$

R: reflection, T: transmission

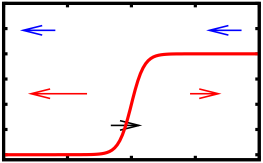


Can the pressure be negative?
What objects can be pulled towards the radiation source?
Unitarity (conservation of energy): $R^2 + T^2 = 1$, $P = 2WR^2 > 0$.
How can P be negative?

NRP on kinks

Kinks in the ϕ^4 theory

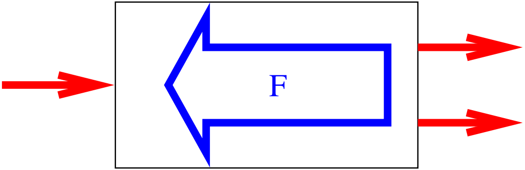
- linearized perturbations: scattering reflectionless
- nonlinearities generate higher harmonics



- More momentum in the forward direction $\rightarrow$ NRP (Romańczukiewicz 2004, Forgács, Lukács and Romańczukiewicz 2007.)
- It should be possible with 2 channels, different momenta

Other approaches

Gain media (Mizraki and Fainman, 2010)



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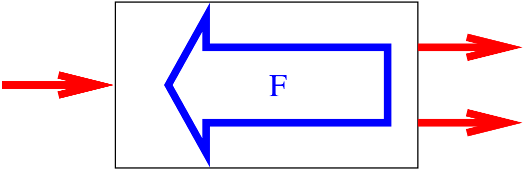
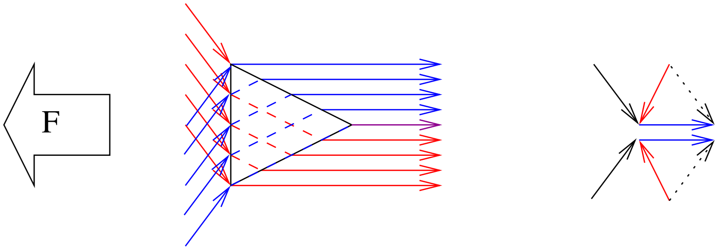


Structured beams (Sukhov and Dogariu, 2011)



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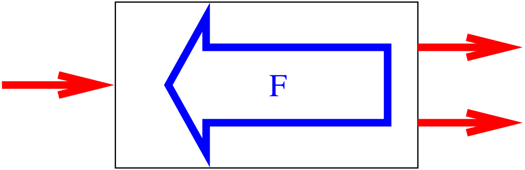

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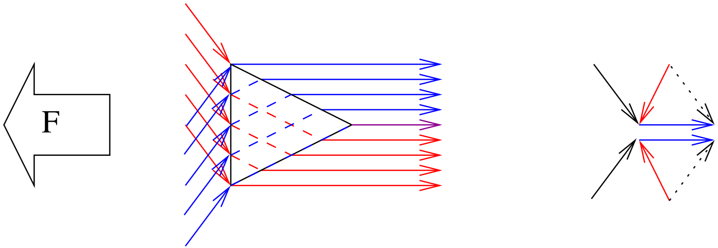

Negative ε, μ (Veselago 1967)

Birefringent media

Anisotropic dielectric tensor

$$\varepsilon = \mathrm{diag}(\varepsilon_x, \varepsilon_y, \varepsilon_z)$$

Two types of wave: polarizations
Wave propagation in the z direction

- Two modes: the x and y polarizations, assume $n_x < n_y$
- Momentum flux of plane waves $P_i = \varepsilon_i/2$
- Energy fluxes $S_i = \sqrt{\varepsilon_i/\mu}/2$

Pressure

$$\frac{p_z}{|E_{x0}|^2} = P_x(1 + |R_{xx}|^2 - |T_{xx}|^2) + P_y(|R_{yx}|^2 - |T_{yx}|^2)\,,$$

where T_{ij} transmission, R_{ij} reflection, channel $j \to i$
Energy conservation: $\sum_i S_i(|R_{ij}|^2 + |T_{ij}|^2) = S_j$.

A macroscopic example

Dielectric

$$n_x = 3, \quad n_y = 6$$

Incoming wave: 1 kW/cm^2 at $\omega = 1$GHz

Pressure

Averaged over thickness $P_z = -0.053$ Pa

Radiation pressure at total reflection: ~ 0.6 Pa

This is a macroscopic effect.

An optical example

Dielectric

Liquid crystal 5CB, at 25 °C, $\lambda = 5893$ Å : $n_x = 1.53$, $n_y = 1.72$

Scatterer: rotated birefringent platelet
E.g., $L = 0.1$ mm, $\theta = \pi/4$

Pressure

- x polarization $p_z = -5.52 \times 10^{-13}$Pa (m/V)$^2|E_0|^2$
- y polarization $p_z = 7.48 \times 10^{-13}$Pa (m/V)$^2|E_0|^2$

Accuracy: -4.95×10^{-13}Pa (m/V)$^2|E_0|^2$ and
8.14×10^{-13}Pa (m/V)$^2|E_0|^2$
A few percents of radiation pressure on a totally reflecting mirror!

Force in 2d scattering

Scattering in 2d, rotational invariance

- Partial wave expansion (Fourier trf. in ϑ)
- S-matrix elements S_ℓ: $n \times n$ matrix (n: channels)
- S_ℓ unitary (conservation of energy)

Momentum balance: force master formula

$$F = F_x + iF_y = -4 \sum_\ell \left\{ A^\dagger S_{\ell+1}^\dagger K S_\ell A - A^\dagger K A \right\} ,$$

$A = (A_1, \ldots, A_n)^T$ amplitude, $K = \mathrm{diag}(k_1, \ldots, k_n)$ wave numbers

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A consequence of unitarity:

$$\mathrm{Re}\, |A_a|^2 k_a (1 - S^*_{aa,\ell+1} S_{aa\ell}) > 0$$

one channel radiation pressure positive

NRP: $k_b > k_a$ necessary

Scattering off cosmic strings

Inside a cosmic string: GUT Higgs zero, flux of broken gauge field
Cosmic string catalyzed baryon number violation:

$$B + \text{string} \rightarrow \ell + \text{string}$$

A simplified description:

- 2 channel Aharonov–Bohm scattering
- Large cross section:
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A simplified description:

- 2 channel Aharonov–Bohm scattering
- Large cross section:
  cosmic string catalyzed baryon number violation
- Force: string friction (moving in a plasma)
  scattering energy $m_i/\sqrt{1-v^2}$
  - Decoupled approximation

    $$F_i = -4 n_i v (1 - \exp(2\pi i \nu_i \Phi))$$

    n_i density, ν_i coupling to the X-boson
  - only valid for light modes, heavy modes give negative contribution

Scattering off cosmic strings – 2ch approximation

- Two types of wave (particle species)
  - Neglect spin degrees of freedom
  - 1 heavy baryon, 1 light lepton, mass ratio 1.5 : 2.
- Neglect vortex core
  $r \to \infty$ asymptotic form of A
  $\to$ a two channel Aharonov-Bohm scattering problem

$$\left(\nabla + i\mathbf{A}\frac{\sigma_2}{2}\right)^2 \rho - K^2 \rho = 0\,, \quad \sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}\,,$$

  where
  - $\mathbf{A} = \mathbf{e}_\vartheta / r$
  - $\rho = (u, d)$, u: heavy and d light mode
  - fermionic boundary cond. $\rho(r, \vartheta + 2\pi) = -\rho(r, \vartheta)$

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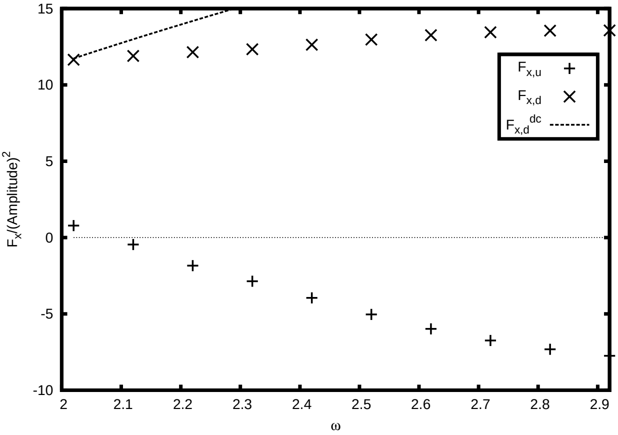
Main properties of the scattering: similar to AB

- NRP for large range of parameters
- Cross section $\gg$ geometric, $1/k$
- $1/\sin\vartheta/2$ in scattering amplitude
- Large cross section from one channel to another

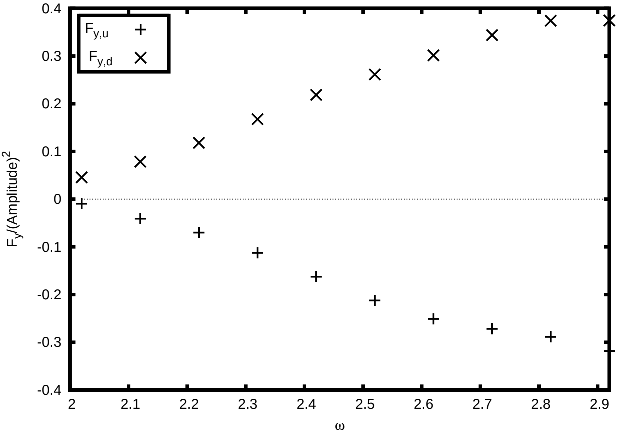
Force: x

x component



Force: y

y component



Neutron scattering: XY model vortices I

XY model

rotators (spins) in a plane, with nearest neighbor interaction

Magnetic vortex: singularity of magnetization **M**

gM energy difference between parallel and antiparallel spin neutrons

Diagonalize Hamiltonian locally

- 2 modes, $\hbar^2/2/m(k_d^2 - k_u^2) = gM$
- small momentum transfer

Measurable manifestation of the same phenomenon as NRP

- Large cross sections, $1/k$ (A-B)
- Large spin-flip cross section
  $E = 4.1 \times 10^{-5}$ eV (45 Å neutrons), $\sigma_{du} = 1.19 \times 10^{-4}$ m
- Can be calculated perturbatively

Conclusions

- Many approaches to tractor beams (e.g., structured beams)
- Multi-channel scattering, $k_i \neq k_j$ fairly general
- One-dimensional examples:
  - polarizations of EM waves
  - higher harmonics (kink)
- Two dimensions
  - Cosmic strings: baryon decay
  - Magnetic XY-vortex

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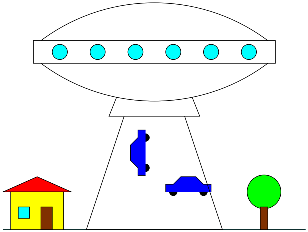

THANK YOU FOR YOUR ATTENTION!

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Cosmic strings

- Classical field theoretical solution
- Thin, elongated object
- String core: a zero of a Higgs field
- Energy density localized in the core

Important parameter: string tension

$$\mu = E/L$$

- Electroweak string: $G\mu \approx 10^{-32}$ (μ: 10 mg/Solar diam)
- GUT string: $G\mu \approx 10^{-6}$ (μ: Solar mass/Solar diam)

Cosmic strings are high energy localized objects that provide a link between astrophysics and particle physics.

Physics of cosmic strings

- Formation: during phase transitions
- Evolution of a string network
  - Friction dominated era: scattering of particles
  - Scaling $v \sim 0.65$
  - collisions, interlinking
  - radiation (e.g. at cusps formed in collisions)
  - string tension contracts loops
  - expansion: the network becomes more diluted
- Signatures of cosmic strings
  - Scattering of material off strings: structure formation: galaxies, voids, filaments (fractal dimension)
  - Contribution to CMB anisotropy: best fit with GUT strings $G\mu = (2.04 \pm 0.13) \times 10^{-6}$, Contribution to multipole $\ell = 10$:$f_{10} = 0.11 \pm 0.05$ (Hindmarsh et al., 2007, 2008)
  - Gravitational lensing
  - Gravitational radiation