Hydrodynamics

- Collective dynamics observed at RHIC
- Exact, analytic solutions: important to determine initial and final state
- Famous 1+1D solutions: Landau, Hwa, Bjorken
- Many new 1+1D solutions, few 1+3D, with spherical/axis/ellipsoidal symmetry
- Energy-mom. tensor in perfect fluid:
  \[ T^\mu_\nu = (\varepsilon + p)u^\mu u^\nu - pg^\mu_\nu \]
- Continuity & e-m. conservation:
  \[ \partial^\mu T^\mu_\nu = 0 \]
  \[ \partial_\mu T^\mu = 0 \]
- EoS: \( \varepsilon = \rho f, \kappa = \frac{c_s^2}{f} \) if const.
- Temperature equation via \( p = nT \)
  \[ T \partial_\mu u^\mu + \kappa u^\mu \partial^\mu T = 0 \]
- Without a conserved charge: use entropy density
  \( \varepsilon + p = T \sigma = T d\sigma + d\mu = \sigma d\tau \)
- Continuity equation for \( \sigma \):
  \[ \partial_\mu (\sigma u^\mu) = 0 \]
- If \( k = \text{const.} \): exactly the same eqs.
- Solutions valid for \( \{ u^\mu, n, T \} \) and/or \( \{ u^\mu, \sigma, T \} \)

Known ellipsoidal solution

- First 3D relativistic solution
  \[ u^\mu = \frac{\eta^\mu}{\tau}, \tau = \sqrt{\eta^2 x^2}, n = n_T \sqrt{b^2(s)} \]
  \[ T = T_f \left( \frac{\eta}{\tau} \right)^2 \left( 1 - \frac{1}{n(s)} \right) \]
  \[ s = \frac{\gamma^2}{2} + \frac{\gamma^2}{2} + \frac{\gamma^2}{2} \]
- \( n(s) \) arbitrary function of scaling variable \( s \)
- \( X, Y, Z \): axes of expanding ellipsoid
- Non-accelerating, i.e. \( u^\mu \partial^\mu = 0 \)
- Hadron, \( \gamma \), lepton source calculable
- Spectra, flow anisotropy, correlations calculable

Describes hadron data
  Even compatible with thermal dileptons
  Máté Csanád, András Szabó, Sándor Lőkös (Eötvös University, Budapest, Hungary)

Describes photon data

Higher order anisotropies?

- Finite number of nucleons \( \rightarrow \) anisotropy!
- Successfully utilized in numerical calculations
- Exact solutions handling this?

Generalization of elliptical symmetry

- **How to generalize** the elliptical scaling variable of
  \[ s = \frac{x^2}{X^2} + \frac{y^2}{Y^2} + \frac{z^2}{Z^2} \]
- Using
  \[ 1 = \frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2} \]
  \[ \varepsilon = \frac{X^2 + Y^2}{X^2 - Y^2} \]
  rewrite the transverse part of \( s \) to
  \[ s = \frac{R^2}{R^2}(1 + \varepsilon \cos(2\phi)) \]
- **Generalize** \( N \)-pole symm. in transverse plane
  \[ s = \frac{R^2}{R^2}(1 + \varepsilon \cos(N\phi)) \]
  \[ \varepsilon_2 = 0.8 \]
  \[ \varepsilon_3 = 0.5 \]
  \[ \varepsilon_4 = 0.4 \]

Multipole symmetries combined

- **Multipole symmetries** can be combined:
  \[ s = \sum N \frac{R^2}{R^2}(1 + \varepsilon \cos(N\phi - N\psi)) \]
- Aligned by \( N \)-th order reaction planes \( \psi_N \)
  \[ \varepsilon_2 = 0.8, \varepsilon_3 = 0, \varepsilon_4 = 0 \]
  \[ \varepsilon_2 = 0.8, \varepsilon_3 = 0.5, \varepsilon_4 = 0 \]
  \[ \varepsilon_2 = 0.8, \varepsilon_3 = 0.5, \varepsilon_4 = 0.4 \]
- Basically any shape can be described!
- In three dimensions: add \( \sum N \frac{R^2}{R^2}(1 + \varepsilon \cos(N\phi - N\psi)) \)
- No change in the longitudinal direction
- More general scaling variables possible, higher order asymmetries in 3D

New solutions

- **New solutions** with multipole symmetries
  \[ s = \sum N \frac{R^2}{R^2}(1 + \varepsilon \cos(N\phi - N\psi)) + \frac{Z^N}{Z^N} \]
  \[ u^\mu = \gamma \left( 1, \frac{R}{R} \cos \phi, \frac{R}{R}, \frac{R}{R} \right) \]
  \[ T = T_f \left( \frac{\eta}{\tau} \right)^3/2 \]
  \[ n = n_T \left( \frac{\eta}{\tau} \right)^3 n(s) \]
- Higher order harmonics: via hadronic source

Multipole velocity field?

- **Buda-Lund model**: hydro final state parametr.
- Add multipole densities (just as above)
- **Add multipole flow!**
  \[ u^\mu = (\gamma, \partial_\mu \Phi, \partial_\Phi, \partial_\Phi, \partial_\Phi) \]
  \[ \Phi = \sum N \frac{R^2}{R^2}(1 + \varepsilon \cos(N\phi)) + \frac{Z^N}{Z^N} \]
  \[ H \text{ is Hubble-like effect (cf. } N = 1) \]
  \[ \chi_N \text{ characterizes flow anisotropy} \]

Flow- and density anisotropies mix in \( \varepsilon_N \)

Observables

- **Hadronic source**: Maxwell-Boltzmann distribution:
  \[ S(x, p)d^3x = n(x) \exp \left( \frac{p_\mu u_\mu(x)}{T(x)} - (\tau - \tau) p_\mu u_\mu(x) \right) \]
- Transverse momentum spectrum and flow:
  \[ N(p) = \int S(x, p)d^3x \]
  \[ N(p_\perp) = \int_{|p_\perp|}^{2\pi} N(p)|_{p_\perp=0} d\alpha \]
  \[ \varepsilon_\perp(p_\perp) = \frac{2\pi}{2\pi} N(p)|_{p_\perp=0} \cos(n\alpha)da = \langle \cos(n\alpha) \rangle \]
- Choose Gaussian temperature profile, \( n(s) = e^{bs} \)