Some Features of Levy Stability Related to Intermittency in ^{32}S-Em Collisions at 200A GeV/c

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Abstract: An attempt is made to study the behaviour of higher order and second order scaled facrorial moments of relativistic shower particles produced in ^{32}S-Em Collisions at 200 AGeV/c in η-phase space for different N_s- intervals. It has been observed that the two ratios of higher order and second order anomalous fractal dimensions (d_q/d_2) expressed in terms of Levy stable law gives an evidence of self-similar cascade mechanism responsible for multiparticle production. The experimental results give an agreement for the requirement of the Levy stable region ($0 \leq \mu \leq 2$).

Introduction:

The experimental observation of large rapidity fluctuations [1] has provided interest and excitement about their nature and origin. Bialas and Peschanski [2] have proposed the most suitable method known as scaled factorial moments (SFMs) to study the non-statistical fluctuations in the distributions of relativistic shower particles produced in high-energy collisions. They suggested a power law scaling behaviour of SFMs ($< F_q> \propto M$) on the bin size and described the phenomenon as "intermittency".

The SFMs method can-not only predict the existence of large non-statistical fluctuations but it could also investigate the pattern of fluctuations and their origin. There exists abundant evidence of power law behaviour in experimental data of e^+e^- annihilation , muon-hadron, ν-nucleus ,hadron-hadron , hadron-nucleus and nucleus-nucleus collisions .

No one has claimed the existence of formation of quark matter in such experiments. Thus, intermittency seems to be a general property of multiparticle production and this effect is not fully explained by any single model proposed, so further experimental information is needed for necessary improvements.

According to the predictions of a simple scale-invariant cascade model [3], the higher order scale factorial moments are related to the second order scaled factorial moments by a modified power law, which may provide some vital information about the underlying dynamics. It has been found that the slopes of the power law between higher order and second order SFMs are independent of the phase space size and phase space dimension.

Analysis of modified power law has been used to investigate hadronic collisions [3] and nuclear collisions data [4,5]. The dependence of ratios of higher order anomalous fractal dimension on the order of moments can help to search for an intermittent type of fluctuations in the multiparticle production process.

The Levy stable law [3,6] has been used to study such dependences, where multiplicity fluctuation is described quite successfully. The study of variations of these ratios on the order of moments has suggested the existence of self-similar cascade processes and a second order phase transition. If the underlying mechanism is a self-similar cascade mechanism, then it leads to intermittent fluctuations and this type of behaviour is characterized by multifractals, whereas, if it is a second order phase transition, e. g., quark gluon plasma, the behaviour is characterized by monofractals.

Experimental Technique:

In this experiment two stacks of Ilford G5 nuclear emulsion plates exposed horizontally to a ^{32}S-beam at 200 AGeV from Supper Proton Synchrotron, SPS at CERN have been utilized for data collection. The scanning of the plates is performed with the help of Leica DM2500M microscope with a 10X objective and 10X ocular lens provided with semi-automatic scanning stages. The method of line scanning was used to collect the inelastic ^{32}S-Em interactions. A sample of 330 primary interactions was collected for performing angular measurement.

For present data η_{min} is –4.03473 and η_{max} is 5.46032.

N_s – intervals	Order of moments	ϕ_2 (η)
$25 \leq N_s \leq 60$	3	0.0833 ± 0.0220
	4	0.0927 ± 0.0295
	5	0.0969 ± 0.0304
	6	0.1062 ± 0.0470
$61 \leq N_s \leq 100$	3	0.0521 ± 0.0160
	4	0.0641 ± 0.0171
	5	0.0806 ± 0.0127
	6	0.0947 ± 0.0147
$N_s \geq 101$	3	0.0603 ± 0.0121
	4	0.0736 ± 0.0173
	5	0.0996 ± 0.0131
	6	0.1047 ± 0.0230

Mathematical Formalism and Experimental Results:

Bialas and Gazdzicki [7] proposed a new scaled variable X(η) which drastically reduces the distortion produced in the study of multiplicity fluctuations due to non-uniformity of single charged particle distribution. We have used this new-scaled variable X related to the single particle density distribution ρ(η) as:

$$X(\eta) = \frac{\int_{\eta_1}^{\eta} \rho(\eta')d(\eta')}{\int_{\eta_1}^{\eta_2} \rho(\eta')d(\eta')} \quad .... (1)$$

where ρ(η) = (1/N)dn/dη is the single particle pseudorapidity density of the shower particles and and are the minimum and maximum values of the pseudorapidity distribution for a given value of the pseudorapidity falling in the interval of an individual shower tracks in an events.

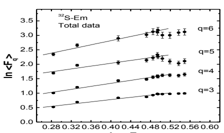

Fig. 1. Variation of ln $\langle F_q \rangle$ as a function of ln$\langle F_2 \rangle$ in the interaction of ^{32}S-Em at 200A GeV. Solid lines represent the linear fits to the data.

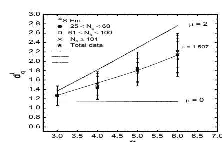

Fig. 3. Variation of d^{l}_{q} as a function of q for different N_s - intervals. The dotted lines represented the two boundaries of Levy stable regions corresponding to μ = 0 and μ = 2, respectively. The experimental results are shown by different symbols.

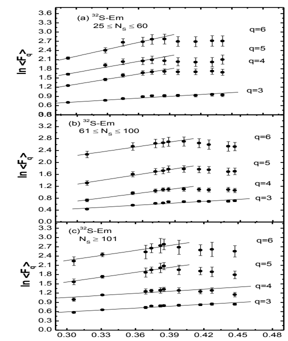

Fig. 2. Variation of ln $\langle F_q \rangle$ as a function of ln$\langle F_2 \rangle$ in the interaction of ^{32}S-Em at 200A GeV for different N_s-intervals. Solid lines represent the linear fits to the data.

Intermittency:

An effective method to study the nature of the multiplicity fluctuations in high-energy collisions is to examine the dependence of the scaled factorial moments (SFMs) on the bin width, which is given as:

$$< F_q >_H = \frac{M^{q-1}}{N_{ev}} \sum_{i=1}^{N_{ev}} \sum_{m=1}^{M} \frac{n_{m,i}(n_{m,i}-1)..........(n_{m,i}-q+1)}{< N >^q} \quad ......(2)$$

where M is the number of bins, $n_{m,i}$ is the number of relativistic charged particles in the m^{th} bin (m = 1, 2, 3…….M) of i^{th} event and < N > represents the mean multiplicity of the hadrons. The pseudorapidity interval Δη is divided into M bins of uniform width δη= Δη/M.

For the presence of non-statistical self-similar fluctuations of many different sizes in rapidity space follows the power law behaviour of $< F_q>$ on M given as:

$<F_q(M)> \propto (M)^{\phi q}$, $(M \rightarrow \infty)$ ……..(3) Where ϕq is called intermittency index.

Also $\ln < F_q> = \beta_q \ln < F_2> + C_q$ ….(4) with $\beta_q = \phi_q/\phi_2$. The slopes, β_q, can be obtained by plotting ln$< F_q>$ as a function of ln$< F_2(M)>$. C_q is constant.

It is noted from the figures above that the values of the scaled factorial moments are larger for the $25 \leq N_s \leq 60$ sample and smaller for the $N_s \geq 101$ sample. Observation of such a power law may indicate a self-similar cascade mechanism in multiparticle production process. Although the data points for q = 3 and 4 in Figs. 1 and 2 respectively exhibit linear behaviour, the data points for q = 5 and 6 show a saturation behaviour for large values of ln $<F_2>$ in case of different N_s-intervals. A similar deviation is also observed for the total data in Fig. 1. The deviation from linear region in different N_s-intervals is more pronounced than the total data due to small statistics of events in N_s-intervals. We have therefore excluded the last three points from the linear fits of the graph shown in Figs. 1 and 2 for all values of q as these points deviated systematically from the linear behaviour.

The values of slopes, ϕ_2(η) for different N_s-intervals for the best fitted lines are listed in Table below:

Levy Stable Law and Anomalous Fractal Dimension

The anomalous fractal dimension, d_q, in term of intermittency index, ϕ_q, is represented by the following relation: Which measure the fractality of a system.

$$d_q = \phi_q / (q-1)$$

The linear increasing trend of the fractal dimension, d_q, on the order of the moments, q, supports multifractality and hadrons in the final state are produced as a result of self-cascade mechanism, while on the other hand, a constant d_q for different q suggests monofractality and does not favour the origin of any exotic phenomenon.

The ratio of the anomalous fractal dimension, d_q/d_2, is given: $d_q/d_2 = 1/(q-1) [(q^\mu - q)/(2^\mu - 2)]$

Levy stable laws are considered to be useful in describing the intermittency in multiparticle production at high-energy collisions. The Levy index, μ, helps in classifying the intermittency regimes due to different kinds of phase transitions during the cascade process. If $\mu < 1$, then it corresponds to a thermal phase transition whereas, its value for $\mu > 1$ is due to non- thermal phase transition during the cascading process.

According to the Levy stable theory, the value of μ is confined to the interval $0 \leq \mu \leq 2$,

The dotted lines in the figure represent the two boundaries of Levy stable regions corresponding to μ = 0 and μ = 2, respectively.

The value of the Levy index, μ, for our data, obtained from the fits in Fig. 4 is 1.507 ± 0.076, which is consistent with the Levy stable theory and multifractals corresponds to "wild" singularities. Thus, it is observed that the Levy index, μ, obtained gives a clear evidence of self-similar cascade mechanism responsible for multiparticle production in the present interactions and rules out the possibility of formation of a quark-gluon plasma [8].

References:

[1] JACEE Collaboration, T. H. Bermett, et al., Phys. Rev. Lett. 50 (1983) 2062. [2] A. Bialas, R. Peschanski, Nucl. Phys. B 273 (1986) 703. [3] W. Ochs, Phys. Lett. B 247 (1990) 101; W. Ochs, Z. Phys. C 50 (1991) 339.[4] KLM Collaboraion,R. Holynski et al., Phys. Rev. Lett. 62 (1989) 733; [5] EMU01 Collaboration, M. I. Adamovich et al., Phys. Rev. Lett. 65 (1990) 412; [6] Ph. Brax, R. Peschanski, Phys. Lett. B 253 (1991) 225. [7] A. Bialas, M. Gazdzicki, Phys. Lett. B 252 (1990) 1476.