Conserved Quantities in Lemaître-Tolman-Bondi Cosmology

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From arXiv:1403.7661 AL, Karim A. Malik

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Flat FRW (Homogeneous) versus LTB (Inhomogeneous)

Standard Flat FRW cosmology vs this research; LTB comology

Flat FRW

FRW: Maximally symmetric spatial section - expansion time dependent only

Background and Perturbed Metric - Flat FRW

Background only: $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$
Perturbed (e.g. Bardeen 1980):

$ds^2 = -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2(\delta_{ij} + 2C_{ij})dx^i dx^j$

with scalar, vector and tensor perturbations

Decompostion of Perturbations - Flat FRW

Further decomposition of 3-spatial perturbations gives curvature perturbation ψ, identified with the intrinsic scalar curvature:

$$C_{ij} = E_{,ij} - \psi\delta_{ij} + vector + tensor \quad quantities^*$$

* On 3-spatial hypersurfaces

Constructing Gauge Invariant Quantities

Splitting quantities into background + perturbation - gauge dependent; construct gauge invariant quantities General gauge transformations: $\widetilde{\delta\mathbf{T}} = \delta\mathbf{T} + \pounds_{\delta x^\mu}\bar{\mathbf{T}}$, tilde denotes new coordinates; $\widetilde{x^\mu} = x^\mu + \delta x^\mu$, bar denotes background. Key quantities gauge transformations (e.g. Malik and Wands 2009):

$$\widetilde{\psi_{\rm FRW}} = \psi_{\rm FRW} + \frac{\dot{a}}{a}\delta t$$

$$\widetilde{\delta\rho_{\rm FRW}} = \delta\rho_{\rm FRW} + \dot{\bar{\rho}}\delta t$$

Constructing Conserved Gauge Invariant Quantities

Gauge choice: uniform density hypersurfaces, $\widetilde{\delta\rho_{\rm FRW}} = 0$ gives $\delta t = -\frac{\delta\rho_{\rm FRW}}{\dot{\bar{\rho}}}$ Get gauge invariant curvature perturbation $-\zeta \equiv \psi_{\rm FRW} + \frac{\dot{a}/a}{\dot{\bar{\rho}}}\delta\rho$ Evolution equations for ζ from time derivative, $\delta\rho$ from energy conservation $\nabla_\mu T^{\mu\nu} = 0$... ζ conserved in large scale limit - conserved perturbed quantities allow easily relate early to late times (e.g. curvature/physics early time relates to density/observables late time)

LTB

LTB: Spherically symmetric spatial section - expansion time and r coordinate dependant (not θ, ϕ)

Background and Perturbed LTB

Background metric (Bondi 1947):

$$ds^2 = -dt^2 + X^2(r,t)dr^2 + Y^2(r,t)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Metric Perturbations (AL and Malik 2014): $\delta g_{\mu\nu} =$

$$\begin{pmatrix} -2\Phi & XB_r & YB_\theta & Y\sin\theta B_\phi \\ XB_r & 2X^2C_{rr} & XYC_{r\theta} & XY\sin\theta C_{r\phi} \\ YB_\theta & XYC_{r\theta} & 2Y^2C_{\theta\theta} & Y^2\sin\theta C_{\theta\phi} \\ Y\sin\theta B_\phi & XY\sin\theta C_{r\phi} & Y^2\sin\theta C_{\theta\phi} & 2Y^2\sin^2\theta C_{\phi\phi} \end{pmatrix}$$

Φ, Bs, Csarbitrary functions of coordinates.

Perturbed LTB

1+3 decomposition into time and spatial sections of metric. Decomposition of perturbations not completely straightforward without use of methods like spherical harmonic decomposition (e.g. Clarkson, Clifton, February 2009) but our undecomposed perturbations ease constructing conserved quantities.

Perturbed LTB

Perturbed Energy Conservation (AL and Malik 2014):

$$\begin{aligned} \delta\dot{\rho} &+ (\delta\rho + \delta P)(H_X + 2H_Y) + \bar{\rho}' v^r \\ &+ \bar{\rho}(\dot{C}_{rr} + \dot{C}_{\theta\theta} + \dot{C}_{\phi\phi} + \partial_r v^r + \partial_\theta v^\theta + \partial_\phi v^\phi \\ &+ \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right] v^r + \cot\theta v^\theta) = 0 \end{aligned}$$

Convenient to define spatial metric perturbation:

$$3\psi = \frac{1}{2}\delta g^k_k = C_{rr} + C_{\theta\theta} + C_{\phi\phi}$$

Gauge Invariant Quantities LTB

Gauge invariant Spatial Metric Trace Perturbation (SMTP) on comoving, uniform density hypersurfaces:

$$\begin{aligned} -\zeta_{\rm SMTP} &= \psi + \frac{\delta\rho}{3\bar{\rho}} + \frac{1}{3}\Bigg\{\left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}}\right)\int v^r dt \\ &+ \partial_r\int v^r dt + \partial_\theta\int v^\theta dt + \partial_\phi\int v^\phi dt + \cot\theta\int v^\theta dt\Bigg\} \end{aligned}$$

Gauge invariant density perturbation on uniform curvature hypersurfaces:

$$\begin{aligned} \delta\tilde{\rho}\Big|_{\psi=0} &= \delta\rho + \bar{\rho}\Bigg\{3\psi + \left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}}\right)\int v^r dt \\ &+ \partial_r\int v^r dt + \partial_\theta\int v^\theta dt + \partial_\phi\int v^\phi dt + \cot\theta\int v^\theta dt\Bigg\} \end{aligned}$$

May be related to $\zeta_{\rm SMTP}$ as $\delta\tilde{\rho}\Big|_{\psi=0} = -3\bar{\rho}\zeta_{\rm SMTP}$

Conclusion/Result: Conserved GI Quantity in Perturbed LTB

$\zeta_{\rm SMTP}$ Evolution Equation: $\dot{\zeta}_{\rm SMTP} = \frac{H_X + 2H_Y}{3\bar{\rho}}\delta P_{\rm nad}$ - Valid on all scales - For barotropic fluids $\dot{\zeta}_{\rm SMTP} = 0$

Research - $\dot{\zeta}_{\rm SMTP}$ - already extended to Lemaitre and FRW cosmologies