Overlap Quark Propagator in Coulomb Gauge QCD

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Outline

- The Overlap quark propagator is explored for the first time in Coulomb gauge. Results for other lattice fermion discretizations can be found in Ref. [1].
- Coulomb gauge is well suited for studying the interrelation between confinement and chiral symmetry breaking.
- Continuum methods give values for the chiral condensate and constituent quark mass, which are too low, due to truncation effects.
- The dressing functions and the dynamical quark mass are evaluated.
- The effect of artificial chiral symmetry restoration by removing low-lying modes of the Dirac operator is explored.

Quark Propagator and Energy Dispersion

The inverse quark propagator in Coulomb gauge is decomposed as

$$S^{-1}(\boldsymbol{p}, p_4) = i\gamma_i p_i A_{\mathsf{S}}(\boldsymbol{p}) + i\gamma_4 p_4 A_{\mathsf{T}}(\boldsymbol{p}) + \gamma_4 p_4 \gamma_i p_i A_{\mathsf{D}}(\boldsymbol{p}) + \mathbb{1} B(\boldsymbol{p}) ,$$

with $A_{\mathsf{S}}, A_{\mathsf{T}}, A_{\mathsf{D}}, B$ referring to *spatial*, *temporal*, *mixed* and *scalar* dressing functions, respectively. The ratio $M(\boldsymbol{p}) = B(\boldsymbol{p})/A_{\mathsf{S}}(\boldsymbol{p})$ is denoted as dynamical quark mass. Since the dressing functions are independent of p_4, the static propagator can be evaluated by integrating over the component p_4

$$S(\boldsymbol{p}) = \frac{B(\boldsymbol{p}) - i\boldsymbol{\gamma}\cdot\boldsymbol{p} A_{\mathsf{S}}(\boldsymbol{p})}{2\omega(\boldsymbol{p})},$$

yielding the dispersion relation

$$\omega(\boldsymbol{p}) = A_{\mathsf{T}}(\boldsymbol{p}) A_{\mathsf{S}}(\boldsymbol{p}) \sqrt{\boldsymbol{p}^2 + M^2(\boldsymbol{p})} .$$

In a continuum Coulomb gauge model it has been shown that the energy dispersion and the dressing functions A_{S}, B diverge, signaling the confinement of quarks, Ref. [2]. Clearly, the lattice discretization introduces an infrared regulator, so the dressing functions are finite in the IR.

Unbreaking Chiral Symmetry

Here an interesting question arises: if removing the chiral condensate from the quark propagator, is the energy dispersion $\omega(\boldsymbol{p})$ still infrared divergent? The low-modes of the quark propagator are removed via the prescription

$$S^k_{\mathsf{TRUNC}} = S_{\mathsf{FULL}} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |v_i\rangle\langle v_i| .$$

Overlap Fermions

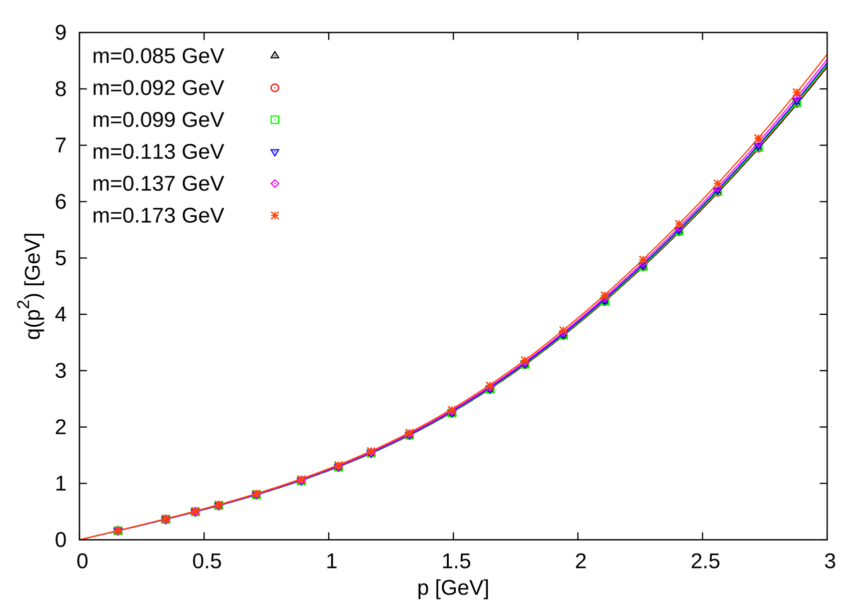
The Overlap Dirac operator is given as

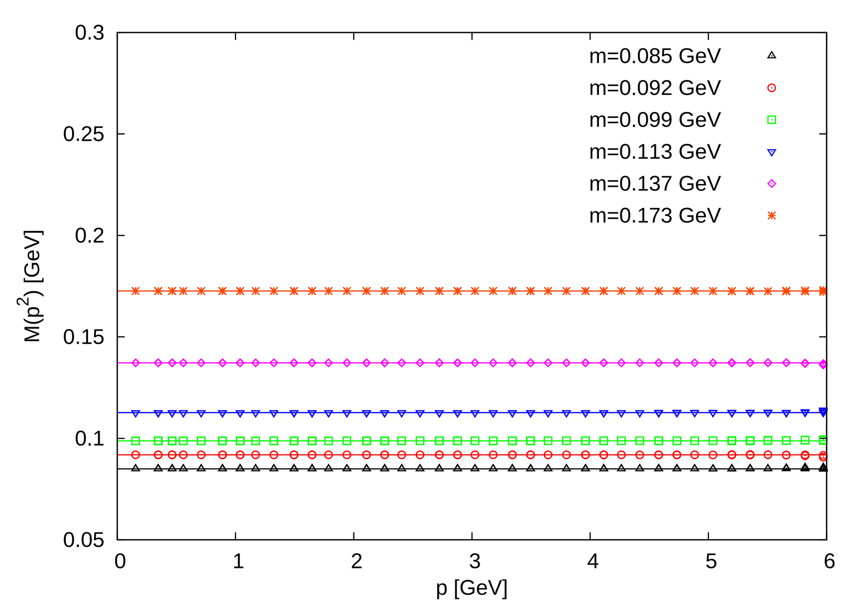
$$D(m_0) = \left(1 - \frac{m_0}{2\rho}\right) D(0) + m_0 , \quad D(0) = \rho\left(\mathbb{1} + \gamma_5 \mathsf{sign}\left[H_{\mathsf{W}}(-\rho)\right]\right)$$

with H_{W} the Hermitian Wilson-Dirac kernel, $\rho = 1.6$. Redefining the massless Overlap quark propagator $\widetilde{S} = S - \frac{1}{2\rho}$, via

$$\left(\widetilde{S}^{(0)}\right)^{-1}(p) = i\gamma_\mu q_\mu + \mathbb{1} m ,$$

we identify the Overlap lattice momenta q_μ and current quark mass m, Ref. [3]. The current quark mass is here indeed a constant and no further corrections have to be performed, which could spoil the result. This is a great advantage of using Overlap fermions.



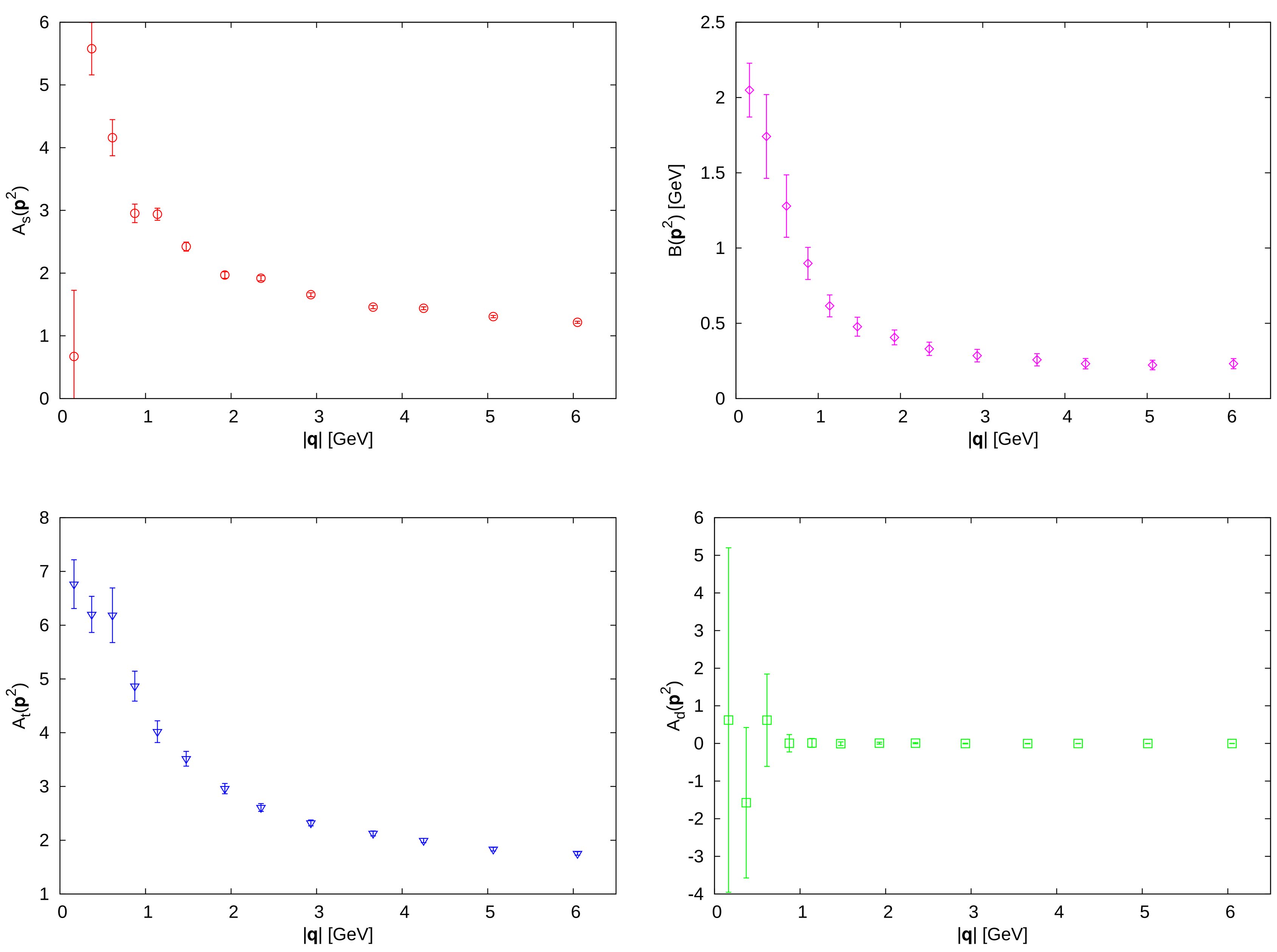


Setup of the Calculation

We use quenched gauge field configurations generated with the Lüscher-Weisz gauge action. The size of the lattice is 20^4 and the lattice spacing is $a = 0.2$ fm. The configurations are fixed to Coulomb gauge. The residual gauge invariance with respect to space independent gauge transformations is fixed to the integrated Polyakov gauge. Up to now we have evaluated 63 propagators, which enter our simulation.

Preliminary Results

- The spatial and scalar dressing functions, $A_{\mathsf{S}}(\boldsymbol{p})$ and $B(\boldsymbol{p})$, approach their tree-level behavior for large momenta, i.e. $A_{\mathsf{S}} = 1, B = m$. For intermediate momenta both dressing functions start to grow. For small momenta the values get much larger, which is consistent with dynamical mass generation. Note, that $A_{\mathsf{S}}(\boldsymbol{p})/|\boldsymbol{p}|$ is diverging, since it is a dressing function of the static quark propagator.
- The temporal dressing function $A_{\mathsf{T}}(\boldsymbol{p})$ is only well-defined when the gauge invariance w.r.t. space independent gauge transformations is fixed. It reaches the tree-level result for large momenta and starts to build up for intermediate momenta. It is an interesting question, if an IR-divergent behavior is possible in the continuum limit.
- The mixed component $A_{\mathsf{D}}(\boldsymbol{p})$ seems to vanish for all momenta. At tree-level it is zero, but it stays zero also in the intermediate momentum regime. For low momenta more statistics is needed to make a concrete statement.



Artificial Chiral Symmetry Restoration

- Preliminary results show that $M(\boldsymbol{p})$ goes to the current quark mass for all momenta, as enough modes are removed from the quark propagator. This is an expected result from chiral symmetry considerations.
- The spatial dressing function A_{S}, relevant for the IR-diverging dispersion relation, remains unchanged. This is a crucial signal that confinement stays intact after chiral symmetry restoration.
- Due to the explicit quark mass, the scalar dressing function $B(\boldsymbol{p})$ does not vanish, but it goes to smaller values in the IR. The bare quarks still interact with gluons.

References

[1] G. Burgio, M. Schröck, H. Reinhardt, and M. Quandt, *Phys.Rev.* **D86** (2012) 014506.
[2] S. L. Adler and A. Davis, *Nucl.Phys.* **B244** (1984) 469.
[3] CSSM Lattice collaboration, *Phys.Rev.* **D65** (2002) 114503.

More

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