“It is unfortunate to note that the situation with respect to the experimental checks of GR is not much better than it was a few years after the theory was discovered- say in the 1920s. It is a great challenge to ... try to improve this situation”

R. Dicke
“Here we have a case that allowed one to suggest that the relativists with their sophisticated work were not only magnificent cultural ornaments but might actually be useful to science!

“Everyone is pleased: the relativists who feel they are ... suddenly experts in a field they hardly knew existed; the astrophysicists for having enlarged ... their empire by the annexation of another subject: general relativity!

“What a shame it would be if we had to go and dismiss all the relativists again!”

Thomas Gold, Texas Symposium, (1963)
Now: Solar System and Pulsars

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bound</th>
<th>Effects</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma - 1$</td>
<td>$2.3 \times 10^{-5}$</td>
<td>Time delay, light deflection</td>
<td>Cassini tracking</td>
</tr>
<tr>
<td>$\beta - 1$</td>
<td>$2.3 \times 10^{-4}$</td>
<td>Nordtvedt effect, Perihelion shift</td>
<td>Nordtvedt effect</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.001</td>
<td>Earth tides</td>
<td>Gravimeter data</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>$10^{-4}$</td>
<td>Orbit polarization</td>
<td>Lunar laser ranging</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$4 \times 10^{-7}$</td>
<td>Spin precession</td>
<td>Solar alignment</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>$4 \times 10^{-20}$</td>
<td>Self-acceleration</td>
<td>Pulsar spin-down</td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>0.02</td>
<td>-</td>
<td>Combined PPN bounds</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>$4 \times 10^{-5}$</td>
<td>Binary pulsar acceleration</td>
<td>PSR 1913+16</td>
</tr>
<tr>
<td>$\zeta_3$</td>
<td>$10^{-8}$</td>
<td>Newton's 3rd law</td>
<td>Lunar acceleration</td>
</tr>
<tr>
<td>$\zeta_4$</td>
<td>0.006</td>
<td>-</td>
<td>Usually not independent</td>
</tr>
</tbody>
</table>
“... by pushing a theory to its extremes, we also find out where the cracks in its structure might be hiding.”

John Wheeler

Decadal Survey 2000
Regimes

Cosmological scales

Ferreira & Starkman, 2009

Adapted from Baker, Psaltis, Skordis 2009
The Large Scale Structure of the Universe

Planck

SDSS

Angular scale

Multipole moment, $\ell$

$D_V(\mu K^2)$

Planck 2013

Standard

CMASS DR9

best-fit model

$\chi^2=81.5 / 59$
“The elegant logic of general relativity theory, and its precision tests, recommend GR as the first choice for a working model for cosmology. But the Hubble length is fifteen orders of magnitude larger than the length scale of the precision tests, at the astronomical unit and smaller, a spectacular extrapolation.”

Jim Peebles, IAU 2000
Einstein Gravity

\[ \frac{1}{16\pi G} \int d^4 x \sqrt{-g} R(g) + \int d^4 x \sqrt{-g} \mathcal{L}(g, \text{matter}) \]

Metric of space time

Curvature

Lovelock’s theorem (1971): “The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant.”
Initial Conditions

Reionization ("EoR")

Dark ages

Recombination

Initial Conditions
Initial Conditions

Primordial Gravitational Waves

Primordial Tilt

Planck XXII

Planck XXII

Tuesday, 30 June 15
Initial Conditions

Primordial Gravitational Waves

Primordial Tilt

Only 2 numbers

Planck XXII

Tuesday, 30 June 15
Acceleration

Where strange things do happen...

Planck XVIII
Effective Field Theory

"Cutoff": \( m \quad a_i \sim \mathcal{O}(1) \)

\[- \frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = \lambda + \frac{M_p^2}{2} R + a_1 R_{\mu\nu} R^{\mu\nu} + a_2 R^2 + a_3 R_{\mu\nu}\lambda_\rho R^{\mu\nu}\lambda_\rho + a_4 \Box R + \frac{b_1}{m^2} R^3 + \frac{b_2}{m^2} R R_{\mu\nu} R^{\mu\nu} + \frac{b_3}{m^2} R_{\mu\nu} R^{\nu\lambda} R_{\lambda\mu} \]

\[ \frac{M_p^2}{2} R + a_2 R^2 \sim \frac{M_p^2}{2} R \left( 1 + 2a_2 \frac{R}{M_p} \right) \quad \text{but} \quad \frac{R}{M_p} \ll 1 \]

Deviations from GR unlikely in low \( R \) and late times ...
The Feynman/Weinberg “Theorem”

\[ g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \]  
Spin-2 field

\[
S = \frac{1}{16\pi G} \int d^4x \left[ -\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h + \partial_\lambda h \partial^\lambda h \right]
\]

\[ g_{\mu\nu}(y) = \frac{\partial x^\alpha}{\partial y^\mu} \frac{\partial x^\beta}{\partial y^\nu} g_{\alpha\beta}(x) \rightarrow h_{\alpha\beta} \rightarrow h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha \]

Couple to matter: \( S_M = \int d^4x h_{\alpha\beta} T^{\alpha\beta}_M \)

Self energy of the graviton: \( T^G_{\mu\nu} \sim (\partial h)(\partial h) \)

\[ \Box h_{\mu\nu} = 16\pi G(T^M_{\mu\nu} + T^G_{\mu\nu}) \]

Unique non-linear completion is GR...
Modified Gravity

- Higher dimensions
- Non-local
- Higher-order

New degrees of freedom

- Scalar
- Vector
- Tensor

Generalisations of $S_{EH}$

Einstein-Dilaton-Gauss-Bonnet

Cascading gravity

Randall-Sundrum I & II

Kaluza-Klein

Strings & Branes

DGP

2T gravity

Some degravitation scenarios

Scalar-tensor & Brans-Dicke

Ghost condensates

Galileons

the Fab Four

KGB

Coupled Quintessence

Horndeski theories

Scalar

$\mathcal{L} = \frac{1}{2} f(R) \mathcal{L}$

$\mathcal{L} = \frac{1}{2} f(G) \mathcal{L}$

$f(T)$

Einstein-Aether

Lorentz violation

Einstein-Cartan-Sciama-Kibble

Conformal gravity

Lorentz violation

Massive gravity

Bigravity

EBI

Bimetric MOND

arXiv:

1310.1086
1209.2117
1107.0491
1110.3830

Tessa Baker 2013
Example: Jordan-Brans-Dicke

\[ S = \int \sqrt{-g} d^4x \left[ \phi R - \frac{\omega}{\phi} (\nabla \phi)^2 \right] \]

\[ \Box \phi = \frac{1}{(2\omega + 3)} T_{\text{matter}} \]

\[ G = \frac{4 + 2\omega}{3 + 2\omega} \frac{1}{\phi_0} \]

Recall Dirac: \( \Box \) \( \frac{1}{G} \) \( \propto \) \( \rho \) \hspace{1cm} \text{GR:} \hspace{0.5cm} \omega \to \infty
The Process

Theory Space  Regime

Parametrization

Observables
The Universe: background cosmology

\[ ds^2 = a^2 \gamma_{\mu\nu} dx^\mu dx^\nu \]

FRW equations

\[ G_{\alpha\beta} = 8\pi G T_{\alpha\beta} \quad \Rightarrow \quad \mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho \]

Any theory (modified gravity or otherwise)

\[ G_{\alpha\beta} = 8\pi G T_{\alpha\beta} + U_{\alpha\beta} \quad \Rightarrow \quad \rho_X(\tau), P_X(\tau) \]
The Universe: background cosmology

\[ D_V(z) = \left[ (1 + z)^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3}. \]

BOSS, Anderson et al 2013.
The Universe: large scale structure
Linear Perturbation Theory \((10 - 10,000h^{-1} Mpc)\)

\[
ds^2 = a^2(\gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu
\]

Diffeomorphism invariance \(\rightarrow\) Gauge invariant
Newtonian potentials

\[
\rho \rightarrow \rho(\tau)[1 + \delta(\tau, \mathbf{r})]
\]

\[
\delta G_{\alpha\beta} = 8\pi G \delta T_{\alpha\beta}
\]

\[
\delta G^{(gi)}_{0i} : \quad 2\nabla^2 \hat{\Phi} - 6\mathcal{H}k \hat{\Gamma} = 8\pi Ga^2 \rho \delta^{(gi)}
\]

\[
\delta G^{(gi)}_{00} : 
\]

\[
\delta G^{(gi)}_{ij} : \quad 2k \hat{\Gamma} = 8\pi G (\rho + P) \theta^{(gi)}
\]

\[
\hat{\Phi} - \hat{\Psi} = 8\pi Ga^2 (\rho + P) \Sigma^{(gi)}
\]

\((+ \quad \delta G^{(gi)}_{ii} \quad \text{equation})\)
Extending Einstein’s equations

\[ \delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}^M + \delta U_{\mu\nu} \]

Linear in \( \hat{\Phi}, \hat{\Gamma}, \hat{\chi}, \hat{\dot{\chi}} \)

Baker, Ferreira, Skordis 2012

ArXiv:1209.2117
Extending Einstein’s equations

Key: Matter + Metric + New degree of freedom

\[-\alpha^2 \delta G^0_0(g_i) = \kappa \alpha^2 G \rho_M \delta_M^{(g_i)} + \alpha_0 k^2 \dot{\chi} + \alpha_1 k \ddot{\chi} + A_0 k^2 \dot{\Phi} + F_0 k^2 \dot{\Gamma}\]
Extending Einstein’s equations

Key: Matter + Metric + New degree of freedom

\[-a^2 \delta G^0_i (gi) = \kappa a^2 G \rho_M \delta^j_M(g) + \alpha_0 k^2 \dot{\chi} + \alpha_1 k \ddot{\chi} + A_0 k^2 \hat{\Phi} + F_0 k^2 \dot{\Gamma}\]

\[-a^2 \delta G^0_i (gi) = \nabla_i \left[ \kappa a^2 G \rho_M (1 + \omega_M) \theta^j_M (gi) + \beta_0 k \dot{\chi} + \beta_1 \dot{\chi} \right] + B_0 k^2 \hat{\Phi} + I_0 k \dot{\Gamma}\]

\[a^2 \delta G^i_i (gi) = 3 \kappa a^2 G \rho_M \Pi^j_M (gi) + \gamma_0 k^2 \dot{\chi} + \gamma_1 k \dot{\chi} + \gamma_2 \ddot{\chi} + C_0 k^2 \hat{\Phi} + C_1 k \dot{\Phi} + J_0 k^2 \dot{\Gamma} + J_1 k \dot{\Gamma}\]

\[a^2 \delta G^i_j (gi) = D^i_j \left[ \kappa a^2 G \rho_M (1 + \omega_M) \Sigma_M + \epsilon_0 \dot{\chi} + \frac{\epsilon_1}{k} \dot{\chi} + \frac{\epsilon_2}{k^2} \ddot{\chi} \right] + D_0 \hat{\Phi} + \frac{D_1}{k} \dot{\Phi} + K_0 \dot{\Gamma} + \frac{K_1}{k} \dot{\Gamma}\]
... but “Integrability condition” can help

\[ U_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S_U}{\delta g^{\alpha\beta}} \]

Use general principles to restrict \( S_U \)

\[ S_U = \int d^4x \sqrt{-g} L(N, N^i, h_{ij}, (^{(3)} R_{ij}, K_{ij}) \]

with general time dependent coefficients.

Gleyzes, Langlois, Vernizzi 1411.3712
... but “Integrability condition” can help

$$U_{\alpha\beta} = -\epsilon$$

Integrability

Use general principles to restrict

with general time dependent coefficients.

7 free functions of time

Gleyzes, Langlois, Vernizzi 1411.3712
More statistical power ...

\[ N(k) \propto k^3 \]
The non-linear regime

\[ \frac{P_{f_R} - P_{\Lambda CDM}}{P_{\Lambda CDM}} \]

- \( \Delta \) No cham. simulation
- \( \square \) Full \( f_R \) simulation
- \( \cdots \) Linear (HS)

\( |f_{R0}| = 10^{-4} \)

\( |f_{R0}| = 10^{-6} \)

k (h Mpc\(^{-1}\))

Courtesy of Hans Winther
What about the non-linear regime?

Baryon, feedback and bias
What we observe.

\[ \delta, \bar{\nu} \]

\[ \bar{\nu}, \Phi, \Psi \]
Large Scales: horizon scale effects

Alonso et al 2014 (2MASS)

Alonso et al 2015
Large Scales: horizon scale effects
Large Scales: the problem with cosmic variance

\[ G_{\text{eff}} = G_0(1 + \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 - \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 + 0.2 \Omega_\Lambda) \]

\[ G_{\text{eff}} = G_0(1 - 0.2 \Omega_\Lambda) \]

ISW- late time effects on large scales

\[ \propto \int (\dot{\Phi} + \dot{\Psi}) d\eta \]
Large Scales: the problem with cosmic variance

Systematic effect due to stellar densities

Ross et al (BOSS) 2012
Not so large scale: “quasi-static” regime

A preferred length scale - the horizon

\[ \mathcal{H}^{-1} \equiv \left( \frac{\dot{a}}{a} \right)^{-1} \propto \tau \sim 3000 h^{-1} \text{Mpc} \]

Focus on scales such that \( k \tau \gg 1 \)

Most surveys \( \leq 300 h^{-1} \text{Mpc} \)

\[ -k^2 \Phi = 4\pi G \mu a^2 \rho \Delta \]

\[ \gamma \Psi = \Phi \]

Note: not applicable to CMB!
Not so large scale: “quasi-static” regime

The “quasi-static” functions reduce to a simple form

\[ \mu = \mu_0(a) \left[ 1 + \left( \frac{M_1(a)}{k} \right)^2 \right] \]

\[ \gamma = \gamma_0(a) \left[ 1 + \left( \frac{M_2(a)}{k} \right)^2 \right] \]

which depends on locality, Lorentz invariance, extra degrees of freedom, screening, etc.

Goal: to use k and z dependent measurements of \((\gamma, \mu)\) to constrain PPF functions

DeFelice et al 2011
Baker et al 2012
Silvestri et al 2013
Growth of Structure

Growth rate

\[ f(k, a) = \frac{d \ln \delta_M(k, a)}{d \ln a} \]

\[ f \text{ satisfies a simple ODE} \]

\[ \frac{df}{d \ln a} + qf + f^2 = \frac{3}{2} \Omega_M \xi \]

with \[ q = \frac{1}{2}[1 - 3w(1 - \Omega_M)] \] and \[ \xi = \frac{\mu}{\gamma} \]
$$f(k, a) = \frac{d \ln \delta_M(k, a)}{d \ln a}$$
Weak Lensing
Weak Lensing of the CMB

\[ \frac{[L(L+1)]^2 W_L^2}{2\pi} \times 10^7 \]

Planck (2015)
Planck (2013)
SPT
ACT

Planck 2015
Galaxy Weak Lensing

Simpson et al 2012
(CFHTLens)
Cross correlating data sets

Reyes et al 2010
Example: Jordan-Brans-Dicke

\[ S = \int \sqrt{-g} d^4x \left[ \phi R - \frac{\omega}{\phi} (\nabla \phi)^2 \right] \]

\[ \omega > 692 \]

Avillez & Skordis 2014
Galaxy Weak Lensing

\[ (1 + \frac{1}{\gamma}) \mu - 2 \]

Simpson et al 2012
(CFHTLenS)
State of the art: Planck 2015

\[ \gamma - 1 \]

\[ \xi - 1 \]
## The Future is now

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Now</th>
<th>Soon</th>
<th>Future</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photo-z:LSS (weak lensing)</td>
<td>DES, RCS, KIDS</td>
<td>HSC</td>
<td>LSST, Euclid, SKA</td>
</tr>
<tr>
<td>Spectro-z (BAO, RSD, ...)</td>
<td>BOSS</td>
<td>MS-DESI, PFS, HETDEX, Weave</td>
<td>Euclid, SKA</td>
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<tr>
<td>SN Ia</td>
<td>HST, Pan-STARRS, SCP, SDSS, SNLS</td>
<td>DES, J-PAS</td>
<td>JWST, LSST</td>
</tr>
<tr>
<td>CMB/ISW</td>
<td>WMAP</td>
<td>Planck</td>
<td></td>
</tr>
<tr>
<td>sub-mm, small scale lensing, SZ</td>
<td>ACT, SPT</td>
<td>ACTPol, SPTPol, Planck, Spider, Vista</td>
<td>CCAT, SKA</td>
</tr>
<tr>
<td>X-Ray clusters</td>
<td>ROSAT, XMM, Chandra</td>
<td>XMM, XCS, eRosita</td>
<td></td>
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<tr>
<td>HI Tomography</td>
<td>GBT</td>
<td>Meerkat, Baobab, Chime, Kat 7</td>
<td>SKA</td>
</tr>
</tbody>
</table>
The Future: Redshift Space Distortions

Percival 2013

Tuesday, 30 June 15
DETF-IV (scale indep.) constraints

Growth (e.g. RSDs)

\[ \bar{\mu}_0 = \frac{\mu}{\gamma} \]

Lensing

\[ \Sigma_0 = (1 + \gamma) \frac{\mu}{\gamma} \]

Leonard et al 2015
Example: Jordan-Brans-Dicke

\[
S = \int \sqrt{-g} d^4x \left[ \phi R - \frac{\omega}{\phi} (\nabla \phi)^2 \right]
\]

Cosmology

Now: \( \frac{1}{\omega} < 6 \times 10^{-3} \) \hspace{1cm} Avillez & Skordis 2014

Euclid: \( \frac{1}{\omega} < 3 \times 10^{-4} \) \hspace{1cm} (RSDs only) Baker, Ferreira & Skordis, 2013

Solar System

Now: \( \frac{1}{\omega} < 1 \times 10^{-4} \) \hspace{1cm} Cassini
Summary

- The large scale structure of the Universe can be used to test gravity (different eras probe different scales).
- There is an immense landscape of gravitational theories (how credible or natural is open for debate).
- We need a unified framework to test gravity.
- Focus on linear scales at late times (for now).
- Non-linear scales can be incredibly powerful but much more complicated.
- Need new methods and observations to access the really large scales (is HI tomography the future?).
- There are a plethora of new experiments to look forward to.