SUPERGRAVITY
Something Fundamental and Something New

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Part I. Something fundamental–

SUSY and SG are attractive to me because the component formalism makes close contact with the basic principles of QFT.

1. Relativistic treatment of spin via Dirac γ-matrices. Complete set

$$I,\ \gamma^{\mu},\ \gamma^{\mu\nu},\ \gamma^{\mu\nu\rho}\ \ldots,$$

and their algebra, e.g. $\gamma^{\mu}\gamma^{\nu\rho} = \gamma^{\mu\nu\rho} + \eta^{\mu\nu}\gamma^{\rho} - \eta^{\mu\rho}\gamma^{\nu}$.
2. Basic eqtns. of geometry of gauge fields and Riemannian geometry, e.g. gauge field Bianchi identity

$$\partial_{\mu}F_{\nu\rho} + \partial_{\nu}F_{\rho\mu} + \partial_{\rho}F_{\mu\nu} = 0.$$

3. Spin and statistics: $\{\psi_{\alpha}(x), \psi_{\beta}(y)\} = 0$ even at the level of classical manipulations.

i. First example– Free Maxwell-Dirac theory:

$$S = -\int d^D x [\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\gamma^\mu \partial_\mu \psi]$$

Invariant under SUSY trf. rules in any spacetime dimension D.:

$$\delta A_\mu = -\bar{\epsilon}\gamma_\mu \psi + \bar{\psi}\gamma_\mu \epsilon, \qquad \delta\psi = \frac{1}{2}\gamma^{\nu\rho} F_{\nu\rho}\epsilon.$$

Proof:

$$\begin{aligned} \delta S_1 &= -\int d^D x F^{\mu\nu} \partial_\mu \delta A_\nu = +\int d^D x \, \partial_\mu F^{\mu\nu} \delta A_\nu \\ &\quad \rightarrow_{SUSY} \int d^D x \, \partial_\mu F^{\mu\nu} \, (\bar{\psi}\gamma_\nu \epsilon) \\ \delta S_{1/2} &= -\frac{1}{2}\int d^D x \, (\bar{\psi}\gamma^\mu \gamma^{\nu\rho}\epsilon) \, \partial_\mu F_{\nu\rho} \end{aligned}$$

We need some γ-matrix algebra: $\gamma^\mu \gamma^{\nu\rho} = \gamma^{\mu\nu\rho} + \eta^{\mu\nu}\gamma^\rho - \eta^{\mu\rho}\gamma^\nu$.

$$\delta S_1 = \int d^D x \, \partial_\mu F^{\mu\nu} \, \bar{\psi}\gamma_\nu \epsilon$$

$$dS_{1/2} = -\frac{1}{2}\int d^D x \, (\bar{\psi}[\gamma^{\mu\nu\rho} + \eta^{\mu\nu}\gamma^\rho - \eta^{\mu\rho}\gamma^\nu]\epsilon) \, \partial_\mu F_{\nu\rho}$$

Last two terms cancel δS_1, so first term must cancel by itself.

WHY does $\gamma^{\mu\nu\rho}\partial_\mu F_{\nu\rho}$ vanish?

$$\delta S_1 = \int d^D x \, \partial_\mu F^{\mu\nu} \, \bar{\psi}\gamma_\nu \epsilon$$
$$dS_{1/2} = -\frac{1}{2}\int d^D x \, (\bar{\psi}[\gamma^{\mu\nu\rho} + \eta^{\mu\nu}\gamma^\rho - \eta^{\mu\rho}\gamma^\nu]\epsilon) \, \partial_\mu F_{\nu\rho}$$

Last two terms cancel δS_1, so first term must cancel by itself.
WHY? BIANCHI IDENTITY:

$$\gamma^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = \frac{1}{3}\gamma^{\mu\nu\rho}(\partial_\mu F_{\nu\rho} + \partial_\mu F_{\nu\rho} + \partial_\mu F_{\nu\rho}) = 0$$

Spin + Statistics enters in a big way at interacting level, e.g. Fierz rearrangement.
At free level for Majorana spinor: if $[\lambda_\alpha(x), \lambda_\beta(y)] = 0$

$$S_{\mathrm{M}ajorana} = \tfrac{1}{2}\int d^D x \bar{\lambda}\gamma^\mu \partial_\mu \lambda = \tfrac{1}{4}\int d^D x \partial_\mu(\bar{\lambda}\gamma^\mu\lambda) = 0$$

ii. Second example – Universal part of Supergravity

$$
\begin{aligned}
S_2 &= \frac{1}{2}\int d^D x\, e\, e^{a\mu} e^{b\nu} R_{\mu\nu ab} = \frac{1}{2}\int d^D x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu} \\
S_{3/2} &= -\int d^D x\, e\, \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho
\end{aligned}
$$

$$D_\nu \psi_\rho \equiv (\partial_\nu + \frac{1}{4}\omega_{\nu ab}\gamma^{ab})\psi_\rho$$

To show: $\delta(S_2 + S_{3/2}) = 0$ to *lowest order* in ψ (for all dimensions D and for any type of fermion) under variations:

$\delta\psi_\rho = D_\rho \epsilon(x)$ (resembles $\delta A_\rho = \partial_\rho \theta(x)$ in gauge theory.)

$\delta e^a_\mu = -\frac{1}{2}\bar{\psi}_\mu \gamma^a \epsilon + c.c. \implies \delta g^{\mu\nu} = \frac{1}{2}(\bar{\psi}^\mu \gamma^\nu + \bar{\psi}^\nu \gamma^\mu)\epsilon + c.c.$

For the graviton:

$$\begin{aligned}\delta S_2 &= \frac{1}{2}\int d^D x\sqrt{-g}\,\delta g^{\mu\nu}\,[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R] \\ &= \frac{1}{2}\int d^D x\sqrt{-g}\,(\bar{\psi}^{\mu}\gamma^{\nu}\epsilon)\,[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R]\end{aligned}$$

For the gravitino:

$$\begin{aligned}\delta S_{3/2} &= -\frac{1}{2}\int d^D x\sqrt{-g}\bar{\psi}_{\rho}\gamma^{\rho\mu\nu}[D_{\mu}, D_{\nu}]\epsilon \\ &= -\frac{1}{8}\int d^D x\sqrt{-g}\bar{\psi}_{\rho}\gamma^{\rho\mu\nu}\gamma^{ab}R_{\mu\nu ab}\epsilon.\end{aligned}$$

$\gamma-$algebra again (but more difficult):

$$\begin{aligned}\gamma^{\rho\mu\nu}\gamma^{ab}R_{\mu\nu ab} &= [\gamma^{(5)} + \gamma^{(3)} + \gamma^{(1)}]R_{\dots} \\ &= \underset{(1)}{\gamma^{\rho\mu\nu ab}R_{\mu\nu ab}} + \underset{(2)}{2\gamma^{\mu\nu b}R_{\mu\nu\rho b}} + \underset{(3)}{4\gamma^{\nu\rho b}R_{\mu\nu}{}^{\mu}{}_{b}} \\ &+ 4\gamma^{\mu}R_{\mu\nu}{}^{\rho\nu} - 2\gamma^{\rho}R_{\mu\nu}{}^{\mu\nu}\,.\end{aligned}$$

Note: (1) and (2) =0 by Bianchi ident. and (3) = 0 by symmetry clash.

Last line gives:

$$\delta S_{3/2} = -\frac{1}{2}\int d^D x\sqrt{-g}(\bar{\psi}^\mu \gamma^\rho \epsilon)[R_{\mu\rho} - \frac{1}{2}g_{\mu\rho}R] = -\delta S_2\,.$$

Quod Erat Demonstrandum!!

There are higher order terms from $\delta\gamma^{\rho\mu\nu}$ which give
$\delta S_{3/2} = \int ..(\psi^3\epsilon)$, more difficult to cancel in $\mathcal{N}=1,\ D=4$ SG.
In higher dimension, this procedure leads to the limit $D \leq 11$.

Part II. The AdS/CFT correspondence:

AdS/CFT is a conjecture made by Juan Maldacena in late 1997. His paper has 11,984 citations !!!.

The conjecture asserts that the observables in two very different types of field theories should be equal, although the methods of computation are very different.

On the CFT side, $\exists$ a conformal invariant gauge theory in d spacetime dimensions and NO gravity. The observables are correlation functions of gauge invariant composite operators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle$. x, y, z are points in flat spacetime. It is difficult to calculate these correlators because strong coupling methods in field theory are often needed and are usually very crude. An exception occurs in a few SCFTs where the method of supersymmetric localization can be used.

The AdS side is a gravity or SG theory in $d + 1$-dimensional spacetime. That theory must have a special solution called anti-de Sitter space. AdS is a spacetime with a boundary. AdS/CFT asserts that the boundary limits of the bulk fields act as sources for the field theory operators.

It is remarkable that computations of correlators from the gravity theory involve much simpler classical techniques. One must solve PDE's and do some integrals.

These techniques were developed by Gubser, Klebanov, Polyakov and by Witten. Their papers have 6789 and 7857 citations.

These numbers prove that AdS/CFT permeates fundamental theoretical physics!

AdS/CFT correspondence applied to duality between

$\mathcal{N}=8,\ d=3$ A$BJM\ CFT_3 \leftrightarrow \mathcal{N}=8,\ D=4\ SG$

Based on work with S. Pufu, K. Pilch and N. Warner

A. i. An acute puzzle: ABJM contains $\Delta=1$ scalar operators $\mathcal{O}_{IJ}(x)$ in 35_v of SO(8).

$\langle\mathcal{O}_{IJ}(x)\mathcal{O}_{KL}(y)\mathcal{O}_{MN}(z)\rangle \neq 0$. Can be calculated exactly in the CFT because $\mathcal{O}_{IJ}(x)$ is in a short multiplet whose top component is $T_{\mu\nu}$.

ii. Many 3-pt correlators have been calculated in the gravity dual by evaluation of Witten diagram containing a cubic coupling from bulk Lagrangian. Gauged $\mathcal{N}=8,\ D=4\ SG$ contains 35 fields A^{IJ} dual to the $\mathcal{O}_{IJ}$, but there is no cubic A^3 coupling! Something new must be found to produce $\langle OOO\rangle$ from bulk SG!

iii. Resolution- SUSY requires that renormalized on-shell bulk action contains a *c*ubic BOUNDARY term in addition to standard bdy. terms from holographic renormalization.

New bdy. term is

$$S_3 = \frac{1}{8\pi G_4}\frac{1}{6}\int d^3x\sqrt{-h}\,A^{IJ}A^{JK}A^{KI}$$

This bdy term does produce $\langle \mathcal{O}_{IJ}(x)\mathcal{O}_{KL}(y)\mathcal{O}_{MN}(z)\rangle$ that matches the CFT result.

In $\mathcal{N}=4$ SYM, the 3-point correlators of chiral primary operators are protected, i.e. independent of $\lambda = g^2_{YM}N$.

But in ABJM, $\langle \mathcal{O}_{IJ}(x)\mathcal{O}_{KL}(y)\mathcal{O}_{MN}(z)\rangle$ contains strong coupling effects calculated using supersymmetric localization. So the agreement between the gravity and gauge theory results is a *precision test of holography.*

We will prove that: (no sum on I, J, K)

$$\begin{aligned}\langle \mathcal{O}_{IJ}(x)\mathcal{O}_{JK}(y)\mathcal{O}_{KI}(z)\rangle &= \frac{\sqrt{2}N^{3/2}}{6\pi^4}\frac{1}{|x-y||y-z||z-x|} \\ &= \frac{L^2}{4\pi^4 G_4}\frac{1}{|x-y||y-z||z-x|}\,.\end{aligned}$$

Coefficients are related by AdS/CFT dictionary. This is a general relation between the parameters of gauge and gravity theories which originates in the properties of M2-branes in 11-dim SG.

We will calculate the RHS using the modified Witten diagram containing the boundary term above.

B. First evidence for new S_{bdy} from consistent truncation of $\mathcal{N} = 8$ gauged SG to $\mathcal{N} = 1$. [DZF + S. Pufu, 1302.7310]

Truncation contains 3 chiral multiplets $z^\alpha = A^\alpha + iB^\alpha,\ \chi^\alpha$. The bosonic action is

$$
\begin{aligned}
S &= \frac{1}{8\pi G_4}\int d^4x\sqrt{-g}\Big[\frac{1}{2}R - \sum_{\alpha=1}^{3}\frac{|\partial_\mu z^\alpha|^2}{(1-|z^\alpha|^2)^2} \\
&+ \frac{1}{L^2}\left(-3 + \sum_{\alpha=1}^{3}\frac{2}{1-|z^\alpha|^2}\right)\Big].
\end{aligned}
$$

Simple Kähler metric– 3 decoupled copies of Poincaré disc.
Potential– Cos. const. + 3 decoupled terms.
NO CUBIC TERMS!

Find holomorphic superpotential $W(z^\alpha)$ such that V takes standard form in $\mathcal{N} = 1$ SG:

$$V = e^K \left(\nabla_\alpha W \, K^{\alpha\bar{\beta}} \nabla_{\bar{\beta}} \bar{W} - 3 W \bar{W} \right) \qquad \nabla_\alpha W \equiv (\partial_\alpha + K_\alpha) W .$$

Result: $W = (1 + z^1 z^2 z^3)/L$. An algebraic miracle that a highly coupled $W(z)$ produces an uncoupled $V(z, \bar{z})$! The cubic term in W is what we need in 2016, but how do we move it into the action?

In 2013, our goal was to calculate the Free Energy in a mass deformation of $\mathcal{N} = 8$ SG and match it to the calculation using localization by [Jafferis, 1012.3210] in a mass deformed version of ABJM

Not a straightforward procedure because .....

i. SUSY requires that the 3 scalars A^α are quantized with alternate quantization and the 3 pseudoscalars B^α by standard quantization.

This allows the 3 A^α to source $\Delta = 1$ scalar ops., $\mathrm{tr}(\bar{X}^\alpha X^\alpha)$, and the 3 pseudoscalars B^α to source the $\Delta = 2$ operators $\mathrm{tr}(\bar{\Psi}^\alpha \Psi^\alpha)$.

AdS/CFT mass formula $\Delta = \frac{d}{2} \pm \frac{1}{2}\sqrt{d^2 + 4m^2} \to \frac{3}{2} \pm \frac{1}{2}\sqrt{9-8}$

ii. We computed S_{ren} including all infinite CT's from holog. ren. procedure and the Legendre transform needed to implement alternate quantization but *still failed* to match the gauge theory.

Resolution: the cutoff procedure used in holog. ren. is not always compatible with SUSY. It may be necessary to add a finite local CT to restore SUSY. One way to test this is to construct a Bogomolny factorization argument.
Extension of Skenderis and Townsend 9909070 to general $K(z, \bar{z})$ and $W(z)$.

iii. Bogomolny argument:

a. Insert planar domain wall ansatz into $\mathcal{N} = 1$ bosonic action.

$$ds^2 = dr^2 + e^{2A(r)}\eta_{ij}dx^i dx^j \qquad z^\alpha = z^\alpha(r) \qquad \bar{z}^{\bar{\beta}} = \bar{z}^{\bar{\beta}}(r).$$

b. Manipulate by partial integration and grouping of terms to obtain factored form: (r_0 is radial cutoff.)

$$\begin{aligned} S &= \int^{r_0} d^3x dr \Big[e^{3A}(\partial_r A - e^{K/2}|W|)^2 \\ &\qquad -K_{\alpha\bar{\beta}}(\partial_r z^\alpha + \sqrt{\frac{W}{\bar{W}}} K^{\alpha\bar{\gamma}}\partial_{\bar{\gamma}}\bar{W})(c.c.)^{\bar{\beta}} \\ &\qquad \frac{\partial}{\partial r}(2e^3 A e^{K/2}|W|) \Big] \end{aligned}$$

The quadratic factors give the BPS eqtns for $A(r),\ z^\alpha(r),\ \bar{z}^{\bar{\beta}}(r)$. The action then vanishes except for the boundary term.

The boundary term must be cancelled by an equal and opposite CT. Otherwise the vacuum energy of the BPS domain wall will not vanish, violating SUSY. Thus we must add to the action:

$$S_3 \quad = \quad -\frac{1}{4\pi G_4} \int d^3x \, e^{3r_0/L} e^{K/2} |W|.$$

For Kähler potential and superpotential:

$K = z^\alpha \bar{z}^{\bar{\alpha}} + \ldots, \qquad W = (1 + z^1 z^2 z^3 + \ldots)/L,$

$$S_3 \to \frac{-1}{4\pi G_4 L} \int d^3x \, e^{3r_0/L} \Big[(1 + z^\alpha \bar{z}^{\bar{\alpha}}/2)[1 + \frac{1}{2}(z^1 z^2 z^3 + c.c.)] + \ldots \Big].$$

AdS/CFT asymptotic behavior $z \sim e^{-r/L}$ as $r \to \infty$. Thus we have

i. cubic + linear divergences that match CT's of holog. ren.

ii. finite cubic CT with the right coefficient to obtain $\langle OOO \rangle$.

iii. ... indicates terms which vanish faster than $e^{-3r/L}$ and so can be dropped.

c. **Find CT's by extension of local SUSY to the boundary.**

i. In usual proofs of invariance in SG, one is happy to achieve invariance up to total derivatives, i.e. $\delta S = \int d^4x \partial_\mu[\sqrt{-g}\bar{\epsilon}(x)X^\mu]$

ii. Now, however, we collect these bdy terms and write

$$\int d^4x \partial_\mu[\sqrt{-g}\,\bar{\epsilon}(x)X^\mu] = \int_{r=r_0} d^3x\sqrt{-h}\,\bar{\epsilon}\,X^r \equiv \delta S_{bdy}.$$

iii. Find set of CTs: $S_{CT} = \int d^3x\sqrt{-h}\,\mathcal{L}_{CT}$, such that $\delta_{SUSY} S_{CT} = -\delta S_{bdy}$.

iv. Similarly, require consistency of Euler-Lagrange variational principle at the bdy. Find bdy. conditions on the bulk fields.

d. First work out bdy. terms and CT's in global limit of any $\mathcal{N} = 1$ SG model. This is a limit in which the back reaction of the matter fields is consistently suppressed, so the gravitino can be dropped. Result is an action that has global SUSY on AdS_4.
Similar to construction of Festuccia and Seiberg, 1105.0689

i. In this global limit, the SUSY parameters are AdS Killing spinors. Killing spinors satisfy

$$(D_\mu + \frac{1}{2L}\gamma_\mu)\epsilon(r,x) = 0$$

They can be found explicitly for the AdS_4 metric $ds^2 = dr^2 + e^{2r/L}\eta_{ij}dx^i dx^j$. Their leading components grow at the bdy. as $\epsilon(r,x) \sim e^{r/2L}$.

ii. This limiting procedure works for any Kähler metric and any supot. of the form $W_{SG} = (1 + W(z^\alpha))$ with cubic $W(z^\alpha)$. This guarantees that the SG model has an AdS stationary point with cos. const. $\Lambda = 3/L^2$, the SUSY value.

iii. Further simplifications: info on CT's that we need is captured by case of one chiral multiplet $z,\ \chi$ with a flat Kähler potential $K = z\bar{z}$ and cubic $W = z^3/3$ or $z^1z^2z^3$.

iv. Result is a simple (off-shell) action.

$$
\begin{aligned}
S &= S_{kin} + S_F + S_{\bar{F}} \\
S_{kin} &= \int d^4x\sqrt{-g}\Big[-\partial_\mu z\partial^\mu \bar{z} - \frac{1}{2}\bar{\chi}\gamma^\mu D_\mu\chi \\
&\qquad +(F + z/L)(\bar{F} + \bar{z}/L) + 2z\bar{z}/L^2\Big] \\
S_F &= \int d^4x\sqrt{-g}[FW' - \frac{1}{2}W''\bar{\chi}P_L\chi + 3W/L] \\
S_{\bar{F}} &= (S_F)^*\,.
\end{aligned}
$$

The 3 terms $S_{kin},\ S_F,\ S_{\bar{F}}$ are *separately* invariant under:

$$\delta z = \bar{\epsilon}P_L\chi \qquad \delta P_L\chi = P_L(\gamma^\mu\partial_\mu z + F)\epsilon \qquad \delta F = \bar{\epsilon}(\gamma^\mu D_\mu - 1/L)P_L\chi.$$

S_F is very simple and so is its SUSY variation. It vanishes in flat spacetime, and the remaining AdS terms give

$$\delta S_F = \int d^4x\sqrt{-g}[\nabla_\mu(\bar{\epsilon}\gamma^\mu W' P_L \chi) - \bar{\epsilon}(\overleftarrow{D}_\mu \gamma^\mu - 2/L)W' P_L \chi].$$

Last term vanishes by adjoint of Killing spinor eqtn.
First term is the bdy term we are looking for! It is cancelled by CT

$$S_{cubic} = -\int d^3x\sqrt{-g}[W(z) + \bar{W}(\bar{z})].$$

With addition of bdy term from δS_{kin} and with change to previous normalization, one reproduces the CT S_3 from Bogomolny argument.

Total CT for $\mathcal{N}=1$ truncation of $\mathcal{N}=8$ SG

$$S_{CT} = -\frac{1}{4\pi G_4 L}\int d^3x\, e^{3r_0/L} e^{K/2}[1+\frac{1}{2}(z^1z^2z^3 + c.c.)]$$

IMPORTANT CLAIM– Results in the $\mathcal{N}=1$ truncation extend to $\mathcal{N}=8$ SG, which is a MUCH more complicated theory. Please see our paper for an $\mathcal{N}=8$ Bogomolny argument and explicit analysis in the $\mathcal{N}=8$ SG theory (not the global limit).

e. (Last topic) How to calculate

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle = \frac{L^2}{4\pi^4 G_4}\frac{1}{|x-y||y-z||z-x|}$$

from S_{CT} above.

Ingredients:

i. Use (Euclidean) AdS metric $ds^2 = \frac{L^2}{w_0^2}[dw_0^2 + dw_i dw_i]$ with $w_i = z^i$ and $w_0 = Le^{-r/L}$.

ii. Using $z^\alpha = (A^\alpha + iB^\alpha)/\sqrt{2}$, transform A^3 term in S_{CT} to new coordinates:

$$S_3 = -\frac{1}{8\pi G_4 L}\frac{L^3}{\sqrt{2}}\int \frac{d^3w}{w_0^3} A^1(w_0, w)A^2(w_0, w)A^3(w_0, w).$$

iii. In alt. quant., A^α is field dual to $\Delta = 1,\ \mathcal{O}_\alpha$ operator and has bulk-bdy propagator

$$K_1(w_0, w - x) = -\frac{1}{2\pi^2}\frac{w_0}{w_0^2 + (w - x)^2}$$

$$\implies A^\alpha(w_0, w) = \int d^3x\, K_1(w_0, w - x)\mathcal{A}^\alpha(x).$$

where $\mathcal{A}(x)$ is the boundary value of the bulk field $A^\alpha(w_0, w)$.

iv. The 3-point correlator we need is $\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle$

It commonly occurs in AdS/CFT that the canonically normalized bulk field differs from the actual source of the dual CFT operator by a constant called c. By studying the details of the fit to the free energy in 1302.7310, one finds $c = 2\sqrt{2}$.

Putting all the pieces together, including factor c^3, one gets

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle = \frac{L^3}{8\pi G_4}\frac{2}{\pi^6}\int \frac{d^3w}{w_0^3} \times \left[\frac{w_0}{w_0^2+(w-x)^2}\frac{w_0}{w_0^2+(w-y)^2}\frac{w_0}{w_0^2+(w-z)^2}\right].$$

This is the correlator evaluated at the cutoff w_0 near the AdS boundary. There are some cases, notably 2-point functions, in which this cutoff is essential, but in this case, the limit $w_0 \to 0$ is smooth, and we get

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle = \frac{L^2}{4\pi^7 G_4} I(x,y,z)$$

where $I(x,y,z)$ is the integral

$$I(x,y,z) = \int \frac{d^3w}{(w-x)^2(w-y)^2(w-z)^2} = \frac{\pi^3}{|x-y||y-z||z-x|}$$

Integral easily done using Feynman parameters and/or conformal inversion. The final result

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle = \frac{L^2}{4\pi^4 G_4}\frac{1}{|x-y||y-z||z-x|}$$

Perfect match to field theory result in the $\mathcal{N}=1$ truncation and can be extended to the full $\mathcal{N}=8$ theory!