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A STUDY OF THE INTERACTION OF ULTRARELATIVISTIC NEUTRINOS WITH RELIC NEUTRINOS NEAR THE Z-BOSON PEAK
In experiment we observe particles with an energy about $10^{19}$ eV. The important question is how particles with such an energy reach the Earth. For example, electrons lose the energy during their motion. Hadrons have a great scattering cross section, so they lose the energy, having covered a distance about $L_{\text{cosm}} = 1$ Gpc.

So, perhaps in a process with an ultrarelativistic neutrino a particle with a high energy can be produced. The reason is that a neutrino has a very small scattering cross section, and therefore a very small probability of interaction with other particles. In this thesis we consider a process with an ultrarelativistic neutrino and calculate the cross section of this process.
If the energy is about $\sqrt{s} \approx m_e$, the scattering cross sections are equal to [1]

$$\sigma(\nu + \gamma \rightarrow \nu + \gamma) \approx 10^{-66} \text{ cm}^2$$

and

$$\sigma(\nu + \gamma \rightarrow \nu + \gamma + \gamma) \approx 10^{-52} \text{ cm}^2.$$ 

If the energy $\sqrt{s} > m_e$, we can observe a new process [2]

$$\sigma(\nu + \gamma \rightarrow \nu + e^+ + e^-) \approx 10^{-47} \text{ cm}^2.$$
INTERACTION OF AN ULTRARELATIVISTIC NEUTRINO WITH RELIC NEUTRINOS

Scattering cross section of the process with the $W^{\pm}$-boson production is [3]

$$\sigma_{\text{asym}}(\nu_i + \bar{\nu}_j \rightarrow l_i + \bar{l}_j) \approx \frac{\pi \alpha^2}{2 \sin^4 \theta_W M_{W}^2} \approx 10^{-34} \text{cm}^2.$$

For the process with the $Z$-boson production the scattering cross section is [4]

$$\langle \sigma(\nu_i + \bar{\nu}_j \rightarrow Z \rightarrow f_i + \bar{f}_j) \rangle \approx \frac{4 \pi G_F}{\sqrt{2}} = 4.2 \cdot 10^{-32} \text{ cm}^2.$$

The probability of the interaction of a neutrino with a relic neutrino at the distance $L_{\text{cosm}} = 1 \text{ Gpc} = 3 \cdot 10^{27} \text{ cm}$ [5]:

$$P_{\text{cosm}}(\nu + \bar{\nu}) = \max \{\sigma(\nu + \bar{\nu})\}(n_{\nu}^0 + n_{\bar{\nu}}^0)L_{\text{cosm}} = 3.3 \cdot 10^{-3}.$$
THE FEYNMAN DIAGRAM FOR THE PROCESS \( v_i + \bar{v}_i \rightarrow Z \rightarrow f_i + \bar{f}_i \)

\( \bar{u}(k_1) \) and \( u(k_2) \) are the wave functions of the final antifermion and fermion respectively.

\( u(p_1) \) and \( \bar{v}(p_2) \) are the wave functions of the initial antineutrino and neutrino respectively.
THE MATRIX ELEMENT OF THE PROCESS

\[ \nu_i + \bar{\nu}_i \rightarrow Z \rightarrow f_i + \bar{f}_i \]

To calculate the cross section, we write the matrix element

\[
M = i \frac{g^2}{8 \cos^2 \theta_W} \frac{1}{q^2 - M_Z^2 + i M_Z \Gamma_Z} \bar{u}(k_1) \gamma^\mu (g^f_V - g^f_A \gamma^5) v(k_2) \times \bar{v}(p_2) \gamma_\mu (1 - \gamma^5) u(p_1).
\]

The matrix element squared is

\[
|M|^2 = \frac{g^4}{4 \times 64 \cos^4 \theta_W} \frac{1}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} Sp_1 Sp_2,
\]

where

\[
Sp_1 = \sum \bar{v}(p_2) v(p_2) \gamma_\mu (1 - \gamma^5) u(p_1) \bar{u}(p_1) \gamma_\nu (1 - \gamma^5),
\]

\[
Sp_2 = \sum \bar{v}(k_2) v(k_2) \gamma^\mu (g^f_V - g^f_A \gamma^5) u(k_1) \bar{u}(k_1) \gamma^\nu (g^f_V - g^f_A \gamma^5).
\]
A CALCULATION OF THE NEUTRINO AND FERMION CURRENTS

For the neutrino current we have

\[ S_{p\_1} = Sp[\hat{p}_2 \gamma_\mu (1 - \gamma^5) \hat{p}_1 \gamma_\nu (1 - \gamma^5)], \]
\[ S_{p\_1} = 2Sp[\hat{p}_2 \gamma_\mu \hat{p}_1 \gamma_\nu] - 2Sp[\gamma^5 \hat{p}_1 \gamma_\nu \hat{p}_2 \gamma_\mu], \]
\[ S_{p\_1} = 8\{p_{2\mu} p_{1\nu} + p_{2\nu} p_{1\mu} - p_1 p_2 g_{\mu\nu} - i \varepsilon_{\rho\nu\sigma\mu} p_1^\rho p_2^\sigma\} \]

Calculations of the fermion current yield

\[ S_{p\_2} = Sp[\hat{k}_2 \gamma^\mu (g_V^f - g_A^f \gamma^5) \hat{k}_1 \gamma^\nu (g_V^f - g_A^f \gamma^5)], \]
\[ S_{p_2} = Sp[\hat{k}_2 \gamma^\mu (g_L^2 + 2g_V^f g_A^f) \hat{k}_1 \gamma^\nu] - 2g_V^f g_A^f Sp[\gamma^5 \hat{k}_1 \gamma^\nu \hat{k}_2 \gamma^\mu], \]
\[ S_{p\_2} = 4(g_L^2 + 2g_V^f g_A^f)\{k_2^\nu k_1^\mu + k_2^\mu k_1^\nu - k_1 k_2 g^{\mu\nu}\} + 8i \varepsilon^{\beta\nu\alpha\mu} k_1^\beta k_2^\alpha. \]

We use the notations \( g_R = g_V^f + g_A^f \) and \( g_L = g_V^f - g_A^f. \)
The product of the currents. The Mandelstam variables

Multiplying the fermion and neutrino traces, we get

\[ S_{p_1} \times S_{p_2} = 64\{g_L^2(p_1 k_2)(p_2 k_1) + g_R^2(p_1 k_1)(p_2 k_2)\} \].

We consider the process in the centre-of-inertia frame. It is convenient to use the Mandelstam variables

\[ s = 2p_1 p_2 = 2k_1 k_2; \]
\[ t = -2p_1 k_1 = -2p_2 k_2 = -\frac{s(1 - \cos \theta^*)}{2}; \]
\[ u = -2p_1 k_2 = -2p_2 k_1 = -\frac{s(1 + \cos \theta^*)}{2} \].
THE CROSS SECTION OF $\nu_i + \bar{\nu}_j \rightarrow Z \rightarrow f_i + f_j$

Taking into account the previous formulas we get

$$|M|^2 = \frac{g^4 s^2}{64 \cos^4 \theta_W} \times \frac{1}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \{g_L^2 (1 + \cos \theta^*)^2 + g_R^2 (1 - \cos \theta^*)^2\}.$$ 

To calculate the cross section, we use the following formula

$$\frac{d\sigma}{d\Omega} = \frac{|M|^2}{64 \pi^2 s^2}.$$ 

Then

$$\frac{d\sigma}{d\cos \theta^*} = \frac{G_F^2 M_Z^4 s}{64 \pi} \frac{1}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \{g_L^2 (1 + \cos \theta^*)^2 + g_R^2 (1 - \cos \theta^*)^2\},$$

where $g^2 = 4 \sqrt{2} G_F M_W^2$ and $M_W = M_Z \cos \theta_W.$
CONCLUSIONS

- We have considered some processes with ultrarelativistic neutrinos. For various neutrino energies we list some possible reactions and the corresponding cross sections and probabilities.

2. We have calculated the cross sections for scattering of an ultrarelativistic neutrino and a relic neutrino

\[ \nu_i + \bar{\nu}_j \rightarrow Z \rightarrow f_i + \bar{f}_j. \]
REFERENCES