# Consistent Pauli reduction on group manifolds

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ISSP, June 2016

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# Motivations

- Search for a unified theory: reconciliate matter and gravity in one framework.

- Important historical example: Kaluza-Klein theory.
  electromagnetism and gravity unification by embedding in a five dimensional space-time. Provide mechanism for dimensional reduction.

- Idea still influencial today (string theory):
  $\rightarrow$ *consistent* Kaluza-Klein truncation.
  Powerful tool: any solution of the lower-dimensional theory can be uplifted to a solution of the higher-dimensional theory

# Kaluza - Klein theory

$$\{x^\mu, y\}\,, \quad \mu = 0, \ldots, 3 \qquad S = \int \mathrm{d}^4x\,\mathrm{d}y\,\sqrt{|det(\tilde{g})|}\,\tilde{R}$$

$$\tilde{g}_{5\times 5} = \begin{array}{c} \begin{array}{cc} 4d & 1d \end{array} \\ \left(\begin{array}{c|c} g_{\mu\nu} + A_\mu A_\nu & A_\mu \\ \hline A_\nu & 1 \end{array}\right) \end{array} \begin{array}{c} \\ 4d \\ 1d \end{array}$$

Dimensional reduction hypothesis: $\partial_y g_{\mu\nu} = 0\,, \quad \partial_y A_\mu = 0$

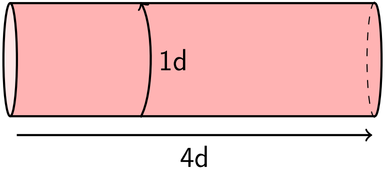
$$\longrightarrow \int \mathrm{d}^4x\,\sqrt{|det(g)|}\big(\,R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\big)\int \mathrm{d}y$$

Photon as geometry !

# Compactification

$$S = \int \mathrm{d}^4x \sqrt{|det(g)|}( R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}) \int \mathrm{d}y$$

$\rightarrow$ *Compactify* fifth dimension, $\int \mathrm{d}y = 2\pi r$




r « very small », fifth dimension imperceivable.

# A more general toy model

Free massless scalar field in (D+1) dimensional space satisfy

$$\hat{\Box}\hat{\Phi}(x, y) = 0$$

Now, suppose we compactify on a circle of ‘small’ radius R

$$\hat{\Phi}(x, y) = \sum \phi_n(x) e^{iny/R}, \quad \rightarrow \quad \Box\phi_n(x) - \frac{n^2}{R^2}\phi_n(x) = 0$$

In this case, truncating to the massless sector (n=0) is *consistent*.

But in general, could have
$\Box\phi_1 + \frac{\phi_1}{R^2} = \phi_0, \rightarrow \phi_1 = 0$ *inconsistent* .

Q: Are the fields I keep sources for the fields I discard ?

# A simple group theoretical argument

$\{e^{iny/R}, n \in \mathbb{Z}\}$ = Representations of U(1) = Sphere harmonics of $S^1$

- n=0 mode: singlet, "uncharged" under U(1)
- n≠0 modes: doublets, "charged" under U(1)

Here, truncating to the massless sector = keep only the singlets modes
$\longrightarrow$ *consistency guaranteed* !

Why ?

Doublets cannot act as sources for singlets.
$\rightarrow$ safe to throw them out.

Same group theoretical argument behind the "DeWitt" reduction.

# DeWitt reduction vs Pauli reduction

Take internal space compact group manifold G.

$$\hat{\Phi}(x,y) = \sum_{\alpha} \varphi_\alpha(x) H_\alpha(y)$$

$H_\alpha$ : harmonics. $\alpha$ related to rep. of the isometry group $G_L \times G_R$.

Massless spectrum includes Yang-Mills gauge bosons in $G_L \times G_R$.

- DeWitt (1963): Keep set of fields singlet under $G_L$ (or $G_R$) $\longrightarrow$ automatically consistent but gauge group $G_L$ (or $G_R$).
- Pauli reduction: retain all the $G_L \times G_R$ gauge bosons $\longrightarrow$ consistency not guaranteed.

**Conjecture by Duff, Nilsson, Pope (1986)**

$\exists$ consistent Pauli reduction of the bosonic string.

# DFT reformulated

$$\text{Coordinates } (x^\mu, Y^M) \quad \mu = 0..n-1, \quad M = 1..2d$$

$$D \; = \; n \; + \; d$$

$\downarrow$ enlarged internal space

$$n \; + \; 2d$$

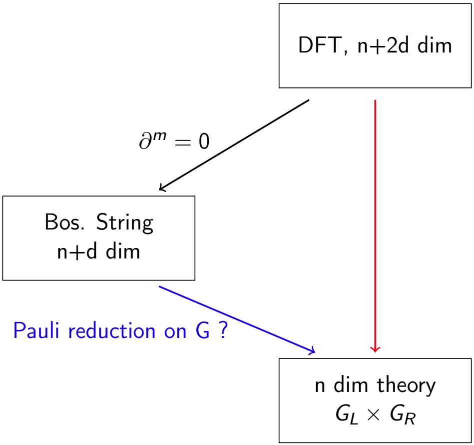
Then, project d unphysical coordinates with the section constraints

$$\eta^{MN}\partial_M\partial_N A = 0, \qquad \eta^{MN}\partial_M A \partial_N B = 0$$

for any fields or gauge parameters A,B.

$$\eta_{MN} = \begin{pmatrix} 0 & \delta_m{}^n \\ \delta_m{}^n & 0 \end{pmatrix}, \text{ SO(d,d) invariant tensor}$$

# What do we gain from DFT ?




Generalised Scherk-Schwarz

- twist matrix $U \in SO(d,d)$
- scale factor $\rho$

n+2d          $n$

$$\mathcal{H}_{MN}(x,y) = U_M{}^A(y)\, U_N{}^B(y)\, M_{AB}(x)\ ,$$

$$g_{\mu\nu}(x,y) = e^{-4\gamma\varphi(x)}\, \mathrm{g}_{\mu\nu}(x)\ ,$$

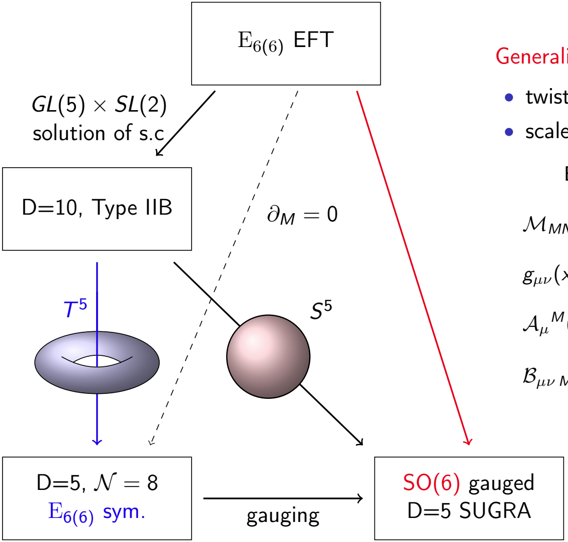
$$\mathcal{A}_\mu{}^M(x,y) = (U^{-1})_A{}^M(y) A_\mu{}^A(x)\ ,$$

$$\mathcal{B}_{\mu\nu}(x,y) = B_{\mu\nu}(x)\ ,$$

$$e^{\Phi} = \rho^{\frac{n-2}{2}}(y) e^{\varphi(x)}\ .$$

Physically interesting example: G=SO(4)

# DFT embedded in EFT




Generalised Scherk-Schwarz:

- twist matrix $U \in SO(d, d)$
- scale factor $\rho$

EFT 5$D$

$$\mathcal{M}_{MN}(x, Y) = U_M{}^{\underline{K}}(Y)\, U_N{}^{\underline{L}}(Y)\, M_{\underline{KL}}(x)$$

$$g_{\mu\nu}(x, Y) = \rho^{-2}(Y)\, \mathrm{g}_{\mu\nu}(x)$$

$$\mathcal{A}_\mu{}^M(x, Y) = \rho^{-1}(Y)(U^{-1})_{\underline{N}}{}^M(Y) A_\mu{}^{\underline{N}}(x)$$

$$\mathcal{B}_{\mu\nu\, M}(x, Y) = \rho^{-2}(Y) U_M{}^{\underline{N}}(Y)\, B_{\mu\nu\, \underline{N}}(x)$$

# Results

Explicit reduction ansatz for all fields of the reduced theory

$$
\begin{aligned}
e^{4\beta\phi} &= \Delta^{2\gamma}(x,y)\, e^{4\gamma\varphi(x)} \\
C_{mn} &= \widetilde{C}_{mn}(y) + \Delta^{2\gamma}(x,y)\, \kappa_A{}^D \mathcal{K}_{D\,m} \mathcal{K}_B{}^p\, G_{pn}(x,y)\, e^{4\gamma\varphi(x)} M^{AB}(x)\ , \\
C_{\mu\,m} &= \kappa_A{}^D \mathcal{K}_{D\,m}\, A_\mu{}^A(x) \\
&\quad + \Delta^{2\gamma}(x,y)\, \kappa_C{}^E \mathcal{K}_A{}^n \mathcal{K}_{E\,n} \mathcal{K}_D{}^p\, G_{pm}(x,y)\, e^{4\gamma\varphi(x)} M^{CD}(x) A_\mu{}^A(x)\ , \\
C_{\mu\nu} &= B_{\mu\nu}(x) - \kappa_B{}^C \mathcal{K}_A{}^m\, \mathcal{K}_{C\,m}\, A_{[\mu}{}^A(x) A_{\nu]}{}^B(x) \\
&\quad - \Delta^{2\gamma}(x,y)\, \kappa_C{}^E\, \mathcal{K}_B{}^n \mathcal{K}_{E\,n}\, \mathcal{K}_D{}^p\, \mathcal{K}_A{}^m\, G_{pm}(x,y) \\
&\quad \times e^{4\gamma\varphi(x)} M^{CD}(x)\, A_{[\mu}{}^A(x) A_{\nu]}{}^B(x)\ .
\end{aligned}
$$

# Conclusion

- DFT manifestly covariant under SO(d,d), T-duality group
  $\rightarrow$ embedded in EFT, covariant under exceptional groups
- New tools for non-toroidal compactifications !
  (generalised Scherk-Schwarz ansatz)
- Full consistency proof of the Pauli reduction of the bosonic string:
  explicit expressions for all fields of the reduced theory