Some properties of BPS skyrmions

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1. Introduction: Skyrme models

The Skyrme model is a topological model of nuclear matter. One modification of it, the BPS Skyrme model has nice mathematical properties and zero binding energy. We investigate here the dynamics of radial pulsations of BPS Skyrmions.

1.1 Motivation

• Low-energy effective theories: Φ-models
• Topological charge

\[ \mathcal{L}_2 = -\frac{\alpha r^4}{2} J_{\mu \nu} K_{\mu \nu}, \]

where \( J_{\mu \nu} = \partial \phi \partial r \cdot U \partial r \partial r U^* \).

1.2 Skyrmions

Need a parametrisation of the group:

\[ U(z) = \exp(\phi(z)), \]

where \( \alpha \) is a real scalar, \( \mathcal{V} \) a complex scalar,

\[ \mathcal{V} = \frac{1}{2\lambda}(\phi^* - \phi^* - \phi - \phi^*), \]

and \( r_{\alpha}, r_{\beta}, r_{\gamma} = (x^1, x^2, x^3) \) Pauli matrices

Charge density

\[ Q = \int d^3r \mathcal{V}, \]

yielding

\[ \mathcal{E} = \frac{\mathcal{V}^* \mathcal{V}}{2}. \]

1.3 The BPS Skyrme model

Add other terms to the Skyrme Lagrangian [2,3]

\[ \mathcal{L}_2 = -\frac{\alpha r^4}{2} J_{\mu \nu} K_{\mu \nu}, \]

Different approaches

• Properties of skyrmions in fn. of the coefficients [2]
• Further fields
• Is there a special model?
• Small interaction region
• Having all the nice properties of the Skyrmie model

BPS Skyrme model [4,5]

• Keep only 6th order term
• Drop sigma model kinetic term

Lagrangian

\[ S = \frac{1}{2} \int d^3x \epsilon^{ijk} \epsilon_{ijk} r \frac{\partial \phi}{\partial \phi_1} \frac{\partial \phi}{\partial \phi_2}, \]

Compacton cases (e.g., Skyrme \( \alpha = 1 \)): coefficient of highest derivatives vanishes outside the skyrmion! [4,5,6]

2. Dynamics of the BPS Skyrme model

2.1 BPS Skyrme model dynamics

Fitting parameters to properties of the proton: quasi-classical quantization of internal oscillations and rotations

Radial oscillations: \( \phi = \phi(r), \)

equation of motion:

\[ \mathcal{V}_0 \mathcal{V} \left( \frac{\partial^2 \phi}{\partial r^2} - \frac{\partial \phi}{\partial r} \right) + \mathcal{V}_0 \mathcal{V} \left( \frac{\partial^2 \phi}{\partial r^2} - \frac{\partial \phi}{\partial r} \right) = 0. \]