The complete $\mathcal{O}(\alpha_s^2)$ non-singlet heavy flavor corrections to DIS structure functions and sum rules

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Deep-inelastic scattering

- Lepton-nucleon scattering experiment characterized by
  \[ Q^2 = -(k_1 - k_2)^2 = -q^2, \]
  \[ x = \frac{Q^2}{2P \cdot q} \]  \hspace{1cm} \text{(1)}
  \[ Q^2 \gg P^2, \ x \text{ fixed: Bjorken limit.} \]

- Leading-order cross section factorized into leptonic and hadronic tensors

  \[ \frac{d\sigma}{dx \, dQ^2} \propto L_{\mu\nu}(k_1, q) \, W^{\mu\nu}(P, q) \]  \hspace{1cm} \text{(2)}

- \( W^{\mu\nu}(P, q) \) is parameterized by structure functions: e.g. for unpolarized e.m. \( e \, p \) scattering

  \[ W^{\mu\nu}_S = \left( g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) \frac{F_L(x, Q^2)}{2x} + \left[ P_\mu P_\nu + \frac{q_\mu P_\nu + P_\mu q_\nu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu} \right] \frac{2x}{Q^2} F_2(x, Q^2) \]  \hspace{1cm} \text{(3)}
Introduction

Factorization

- QCD radiative corrections to the structure functions are computed as convolution

\[ F_i(x, Q^2) = x \int_x^1 \frac{d\xi}{\xi} C_i \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2} \right) q \left( \xi, \mu^2 \right), \]  

(4)

- \( C_i \left( x, \frac{Q^2}{\mu^2} \right) \): Wilson coefficients describing the short distance dynamics of the scattering

\[ C_i \left( \xi, \frac{Q^2}{\mu^2} \right) = \sum_{n=0}^{\infty} \alpha_s^n c_i^{(n)} \left( \xi, \frac{Q^2}{\mu^2} \right). \]  

(5)

- \( q(\xi, \mu^2) \): parton distribution functions, encoding the non-perturbative strong interaction.

- Heavy quark with mass \( m_Q \) enters \( F_i(x, Q^2) \)

\[ F_i(x, Q^2, m_Q^2) = F_i^{\text{light}}(x, Q^2) + F_i^{\text{heavy}}(x, Q^2, m_Q^2), \]  

(6)

- The precise measure of structure functions allows to extract \( \alpha_s, m_c \) and PDFs.
Neutral current DIS

Cross section for e.m. scattering of polarized electron off polarized target: $W_{\mu\nu} = W_{S}^{\mu\nu} + i W_{A}^{\mu\nu}$,

$$W_{A}^{\mu\nu} = -\frac{M}{P \cdot q} \epsilon_{\mu\nu\rho\sigma} q^\rho \left[ S^\sigma \hat{g}_1(x, Q^2) + \left( S^\sigma - \frac{S \cdot q}{P \cdot q} P^\sigma \right) \hat{g}_2(x, Q^2) \right] \tag{7}$$

$M$, $P^\mu$ and $S^\mu$ are respectively the nucleon mass, momentum and spin.

- Flavor non-singlet contributions (NS) are important in the difference $(\frac{d\sigma}{dxdQ^2})^{ep} - (\frac{d\sigma}{dxdQ^2})^{en}$
Neutral current DIS

Cross section for e.m. scattering of polarized electron off polarized target: \( W_{\mu\nu} = W_S^{\mu\nu} + i W_A^{\mu\nu} \),

\[
W_A^{\mu\nu} = -\frac{M}{P \cdot q} \epsilon_{\mu\rho\sigma} q^\rho \left[ S^\sigma \hat{g}_1(x, Q^2) + \left( S^\sigma - \frac{S \cdot q}{P \cdot q} P^\sigma \right) \hat{g}_2(x, Q^2) \right], \tag{7}
\]

\( M, P^\mu \) and \( S^\mu \) are respectively the nucleon mass, momentum and spin.

- Flavor non-singlet contributions (NS) are important in the difference \( \left( \frac{d\sigma}{dx dQ^2} \right)^{ep} - \left( \frac{d\sigma}{dx dQ^2} \right)^{en} \)

Factorization formula for NS contributions

\[
\hat{g}^{\text{NS}}_1(x, Q^2, m_Q^2) = \frac{1}{2} \int_x^1 \frac{dz}{z} \left[ C_{g_1, q}^{\text{NS}} \left( z, \frac{Q^2}{\mu^2} \right) + L_{g_1, q}^{\text{NS}} \left( z, \frac{Q^2}{m_Q^2}, \mu^2 \right) \right] \cdot \hat{\Delta} \left( \frac{x}{z}, \mu^2 \right), \tag{8}
\]

\[
\hat{\Delta}(x, \mu^2) = \sum_i e_i^2 \left[ \Delta f_i(x, \mu^2) + \Delta f_i(x, \mu^2) \right].
\]

- \( C_{g_1, q}^{\text{NS}}, \) massless Wilson coefficient, \( \mathcal{O}(\alpha_s^3) \) terms computed \((\text{Moch, Vermaseren, Vogt '09})\)

- \( L_{g_1, q}^{\text{NS}}, \) massive Wilson coefficient, asymptotic limit \( Q^2 \gg m_Q^2 \) of \( \mathcal{O}(\alpha_s^3) \) terms computed \((\text{Behring, Blümlein, De Freitas, von Manteuffel, Schneider '15})\)

- I will present the full \( \mathcal{O}(\alpha_s^2) \) contribution to \( L_{g_1, q}^{\text{NS}}, \) including power corrections in \( \frac{m_Q^2}{Q^2} \).
Inclusive scattering process $q + \gamma^* \rightarrow q + X$, heavy quarks in the final state $X$ or in loops

\[
L^{NS}_{g_1,q}(z, Q^2, m_Q^2) = \Theta \left( \frac{\xi}{\xi + 4} - z \right) L^{NS,(R)}_{g_1,q}(z, \xi) + \delta (1 - z) L^{NS,(V)}_{g_1,q}(\xi) 
\]

\[
- \left( \frac{\alpha_s}{4\pi} \right)^2 \beta_{0,Q} \log \left( \frac{m_Q^2}{\mu^2} \right) \left[ \frac{1}{2} P^{(0)}_{qq} \log \left( \frac{Q^2}{\mu^2} \right) + c^{(1)}_{g_1,q} \right],
\]

where $z = \frac{Q^2}{2p \cdot q}$, $\xi = \frac{Q^2}{m_Q^2}$, $\beta_{0,Q} = -\frac{4}{3} T_F$. $P^{(0)}_{qq}$ and $c^{(1)}_{g_1,q}$ are respectively the quark splitting function and the one-loop massless Wilson coefficient.
**Real radiation**

- Re-calculation of the Compton process

\[ q + \gamma^* \rightarrow q + Q + \bar{Q} \]

The following variables are used

\[ sq_1 = \sqrt{1 - \frac{4}{\xi} z \left( 1 - z \right)}, \quad sq_2 = \sqrt{1 - \frac{4}{\xi} z}, \quad L_i = \log \left( \frac{1 + sq_i}{1 - sq_i} \right) \quad (i=1,2), \quad L_3 = \log \left( \frac{sq_2 + sq_1}{sq_2 - sq_1} \right), \]

\[ di_1 = \text{Li}_2 \left( (1 - z) \frac{1 + sq_1}{1 + sq_2} \right), \quad di_2 = \text{Li}_2 \left( \frac{1 - sq_2}{1 + sq_1} \right), \quad di_3 = \text{Li}_2 \left( \frac{1 - sq_1}{1 + sq_2} \right), \quad di_4 = \text{Li}_2 \left( \frac{1 + sq_1}{1 + sq_2} \right). \]
Real radiation

- Re-calculation of the Compton process

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\[ di_1 = \text{Li}_2 \left[ (1 - z) \frac{1 + sq_1}{1 + sq_2} \right], \quad di_2 = \text{Li}_2 \left( \frac{1 - sq_2}{1 + sq_1} \right), \quad di_3 = \text{Li}_2 \left( \frac{1 - sq_1}{1 + sq_2} \right), \quad di_4 = \text{Li}_2 \left( \frac{1 + sq_1}{1 + sq_2} \right). \]

- Agreement with the literature (Buza, Matiounine, Smith, Migneron, van Neerven ’96)

\[ L_{g_1, q}^{\text{NS}, (R)} (z, Q^2) = \left( \frac{\alpha_s}{4\pi} \right)^2 C_F T_F \left\{ - \frac{8 L_3}{9(z - 1)\xi} \left( 50z^3 - 11\xi + z(6\xi + 20) - 2z^2(7\xi + 12) \right) \\
+ \frac{2 sq_1}{27(z - 1)^2\xi} \left( 1200z^4 + 265\xi - 4z^3(109\xi + 490) + 2z^2(389\xi + 618) - z(607\xi + 466) \right) \\
+ \frac{4 L_1}{3(z - 1)^3\xi^2} \left( 24z^4 - \xi^2 + 3z^2(\xi^2 + 6) - 2z^3(\xi^2 + 18) \right) + \frac{12z^3 - \xi^2 - z^2\xi^2}{3(z - 1)\xi^2} \right\} \times \left[ 4L_1 L_2 + 8(-di_1 + di_2 + di_3 - di_4) - 4L_1 \log \left( \frac{z^2}{1 - z} \right) \right]. \]
The virtual corrections are given by

\[ L_{g_1,q}^{\text{NS, (V)}}(\xi) = 2 \mathcal{F}_1^{(2)} \left( -\frac{Q^2}{m_Q^2} \right), \]

where \( \mathcal{F}_1^{(2)} \) come from the form factor diagrams.

Introducing the variable \( \tilde{\lambda} = \sqrt{1 - \frac{4}{\xi}} \) the result is

\[
\begin{align*}
L_{g_1,q}^{\text{NS, (V)}}(\xi) &= 2 \left( \frac{\alpha_s}{4\pi} \right)^2 C_F T_F \left\{ \frac{3355}{81} - \frac{952}{9\xi} + \left( \frac{32}{\xi^2} - \frac{16}{3} \right) \zeta(3) \\
&\quad + \left( \frac{440}{9\xi} - \frac{530}{27} \right) \log(\xi) + \tilde{\lambda} \left[ \frac{184}{9\xi} - \frac{76}{9} \right] \left\{ \text{Li}_2 \left( \frac{\tilde{\lambda} + 1}{\tilde{\lambda} - 1} \right) - \text{Li}_2 \left( \frac{\tilde{\lambda} - 1}{\tilde{\lambda} + 1} \right) \right\} \\
&\quad + \left[ \frac{8}{3} - \frac{16}{\xi^2} \right] \left\{ \text{Li}_3 \left( \frac{\tilde{\lambda} - 1}{\tilde{\lambda} + 1} \right) + \text{Li}_3 \left( \frac{\tilde{\lambda} + 1}{\tilde{\lambda} - 1} \right) \right\} \right\}. \tag{10}
\end{align*}
\]
The massive Wilson coefficient including charm and bottom quark contributions is

\[ L_{g_1,q}^{\text{NS}}(z, Q^2, m_c^2, m_b^2) = L_{g_1,q}^{\text{NS}}(z, Q^2, m_c^2) + L_{g_1,q}^{\text{NS}}(z, Q^2, m_b^2). \]  

(11)

We computed the convolution (8) of \( L_{g_1,q}^{\text{NS}}(z, Q^2, m_c^2, m_b^2) \) with PDFs\(^1\).

\[ F_L(x, Q^2, m_Q^2), F_2(x, Q^2, m_Q^2). \]

\(^1\)These plots are done with BB09 POLPDF (polarized) and abm12.3.nnlo (unpolarized), \( m_c = 1.59 \text{ GeV} \) and \( m_b = 4.78 \text{ GeV} \).
The polarized Bjorken sum rule is the first moment of $g_{1}^{NS}$:

$$\Delta g_{1}(Q^2) = \int_{0}^{1} dx \left[ g_{1}^{eP}(x, Q^2) - g_{1}^{en}(x, Q^2) \right] = K_{g_{1}}(n_f) \ A_{g_{1}}^{g_{1}}(\alpha_s, Q^2)$$

where $A_{g_{1}}^{g_{1}}$ corresponds to the first moment of the Wilson coefficients $C_{g_{1}}^{NS}$ and $L_{g_{1}}^{NS}$.
Polarized Bjorken sum rule

The polarized Bjorken sum rule is the first moment of $g_{1}^{\text{NS}}$:

$$\Delta g_{1}(Q^{2}) = \int_{0}^{1} dx \left[ g_{1}^{e,p}(x, Q^{2}) - g_{1}^{e,n}(x, Q^{2}) \right] = \underbrace{K_{g_{1}}(n_{f})}_{\text{Parton model result}} A_{g_{1}}^{\text{NS}}(\alpha_{s}, Q^{2})$$

(12)

$A_{g_{1}}$ corresponds to the first moment of the Wilson coefficients $C_{g_{1},q}^{\text{NS}}$ and $L_{g_{1},q}^{\text{NS}}$.

$$A_{g_{1}}^{\xi} = 1 - \frac{\alpha_{s}}{\pi} + \left( \frac{\alpha_{s}}{\pi} \right)^{2} \left[ - \frac{55}{12} + \frac{1}{3} n_{f} + C_{pBj}^{Q,(2)}(\xi) \right] + O(\alpha_{s}^{3}, \alpha_{s}^{4})$$

$$C_{pBj}^{Q,(2)}(\xi) = - \frac{1}{16} \frac{C_{F} T_{F}}{315 \xi^{2}} \left\{ 2100 \log \left( \frac{\lambda + 1}{\lambda - 1} \right)^{2} - \xi \left( 6\xi^{2} + 2735\xi + 11724 \right) + \lambda \log \left( \frac{\lambda + 1}{\lambda - 1} \right) \right.$$  

$$\times \xi \left( 3\xi^{3} + 106\xi^{2} + 1054\xi + 4812 \right) - \xi^{2} \left( 3\xi^{2} + 112\xi + 1260 \right) \log(\xi) \right\}$$

$$\lambda = \sqrt{1 + \frac{4}{\xi}}$$
Polarized Bjorken sum rule

The polarized Bjorken sum rule is the first moment of $g_1^{\text{NS}}$

$$\Delta g_1(Q^2) = \int_0^1 dx \left[ g_1^{e p}(x, Q^2) - g_1^{e n}(x, Q^2) \right] = K_{g_1}(n_f) \ A^{g_1}(\alpha_s, Q^2)$$  \hspace{1cm} (12)$$

$A^{g_1}$ corresponds to the first moment of the Wilson coefficients $C_{g_1, q}^{\text{NS}}$ and $L_{g_1, q}^{\text{NS}}$.

$$A^{g_1}(\xi) = 1 - \frac{\alpha_s}{\pi} + \left( \frac{\alpha_s}{\pi} \right)^2 \left[ - \frac{55}{12} + \frac{1}{3} n_f + C_{p\text{Bj}}^{Q, (2)}(\xi) \right] + O(\alpha_s^3, \alpha_s^4)$$

$$C_{p\text{Bj}}^{Q, (2)}(\xi) = -\frac{1}{16} \frac{C_F T_F}{315 \xi^2} \left\{ 2100 \log \left( \frac{\lambda + 1}{\lambda - 1} \right)^2 - \xi \left( 6 \xi^2 + 2735 \xi + 11724 \right) + \lambda \log \left( \frac{\lambda + 1}{\lambda - 1} \right) \right. \right.$$  

$$\left. \times \xi \left( 3 \xi^3 + 106 \xi^2 + 1054 \xi + 4812 \right) - \xi^2 \left( 3 \xi^2 + 112 \xi + 1260 \right) \log(\xi) \right\}, \quad \lambda = \sqrt{1 + \frac{4}{\xi}}$$

The inclusive definition of $g_1^{\text{NS}}$ implies:

- Smooth transition $n_f \rightarrow n_f + 1$ at $Q^2 \gg m_c^2$. Logarithmic enhancements are not present.
- Negative contributions at $Q^2 \sim m_c^2$. 

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**G. Falcioni**

**Two-loop power corrections**

**Erice, 14-23 June 2016**
Charged current DIS

Cross section for (anti)neutrino scattering off unpolarized target: $W_{\mu\nu} = W_{S}^{\mu\nu} + i W_{A}^{\mu\nu}$,

$$W_{A}^{\mu\nu} = i \epsilon_{\mu\nu\rho\sigma} \frac{P^\rho q^\sigma}{2P \cdot q} F_{3}^{W^\pm}(x, Q^2).$$

(13)

Crossing-antisymmetric combinations are determined by the flavor NS Wilson coefficients:
Charged current DIS

Cross section for (anti)neutrino scattering off unpolarized target: $W^{\mu\nu} = W^S_{\mu\nu} + i W^A_{\mu\nu}$,

$$W^A_{\mu\nu} = i \epsilon_{\mu\nu\rho\sigma} \frac{P^\rho q^\sigma}{2 P \cdot q} F_3^{W^\pm}(x, Q^2).$$ (13)

Crossing-antisymmetric combinations are determined by the flavor NS Wilson coefficients:

$$F_{2,L}^{W^+-W^-} = 2\left\{ x \int_x^1 \frac{dz}{z} \left[ |V_{du}|^2 d_v \left( \frac{x}{z} \right) - \left( |V_{du}|^2 + |V_{su}|^2 \right) u_v \left( \frac{x}{z} \right) \right] \left( C_{2,L,q}^{NS}(z) + L_{2,L,q}^{NS}(z) \right) + \tilde{x} \int_{\tilde{x}}^1 \frac{dz}{z} |V_{dc}|^2 d_v \left( \frac{\tilde{x}}{z} \right) H_{2,L,q}^{NS}(z) \right\}, \quad \tilde{x} = x \frac{Q^2 + m_Q^2}{Q^2}$$

$$F_{3}^{W^++W^-} = 2\left\{ \int_x^1 \frac{dz}{z} \left[ |V_{du}|^2 d_v \left( \frac{x}{z} \right) + \left( |V_{du}|^2 + |V_{su}|^2 \right) u_v \left( \frac{x}{z} \right) \right] \left( C_{3,q}^{NS}(z) + L_{3,q}^{NS}(z) \right) + \int_{\tilde{x}}^1 \frac{dz}{z} |V_{dc}|^2 d_v \left( \frac{\tilde{x}}{z} \right) H_{3,q}^{NS}(z) \right\},$$
Charged current DIS

Cross section for (anti)neutrino scattering off unpolarized target: \( W^{\mu\nu} = W^S_{\mu\nu} + i W^A_{\mu\nu} \),

\[
W^A_{\mu\nu} = i \epsilon_{\mu\nu\rho\sigma} \frac{P^\rho q^\sigma}{2 P \cdot q} F^W_{3 \mu\nu} (x, Q^2).
\] (13)

Crossing-antisymmetric combinations are determined by the flavor NS Wilson coefficients:

\[
F^{W^+ - W^-}_{2, L} = 2 \left\{ x \int_x^1 \frac{dz}{z} \left[ |V_{du}|^2 d_{\nu} \left( \frac{x}{z} \right) - (|V_{du}|^2 + |V_{su}|^2) u_{\nu} \left( \frac{x}{z} \right) \right] \left( C_{2, L, q}^{NS}(z) + L_{2, L, q}^{NS}(z) \right) ,
\]

\[
+ \bar{x} \int_{\bar{x}}^1 \frac{dz}{z} |V_{dc}|^2 d_{\nu} \left( \frac{\bar{x}}{z} \right) H_{2, L, q}^{NS}(z) \right\} , \quad \bar{x} = x \frac{Q^2 + m_Q^2}{Q^2}
\]

\[
F^{W^+ + W^-}_{3} = 2 \left\{ \int_x^1 \frac{dz}{z} \left[ |V_{du}|^2 d_{\nu} \left( \frac{x}{z} \right) + (|V_{du}|^2 + |V_{su}|^2) u_{\nu} \left( \frac{x}{z} \right) \right] \left( C_{3, q}^{NS}(z) + L_{3, q}^{NS}(z) \right) ,
\]

\[
+ \int_{\bar{x}}^1 \frac{dz}{z} |V_{dc}|^2 d_{\nu} \left( \frac{\bar{x}}{z} \right) H_{3, q}^{NS}(z) \right\} ,
\]

\( H^{NS}_{i, q} \) generated by the flavor-excitation process.

Note the CKM suppression \( |V_{cd}|^2 \approx 0.05 \).
We implemented the numerical convolution of the necessary Wilson coefficients

- $H_i^{NS,(1)}$ (Gottschalk '81; Glück, Kretzer, Reya '96; Blümlein, Hasselhuhn, Kovacikova, Moch '11) and
- the asymptotic approximation of $H_i^{NS,(2)}$ (Blümlein, Pfoh, Hasselhuhn '14),
- $C_i^{NS,(1)}$, $C_i^{NS,(2)}$ (Zijlstra, van Neerven '92; Moch, Vermaseren '99; Moch, Rogal, Vogt '07),
- $L_i^{NS}$ for $i = 2, L$; $L_{3,q}^{NS} = L_{g_1,q}^{NS}$ are known from the neutral current calculation,

We construct the structure function $F_1(x, Q^2) = \frac{1}{2x} \left[ F_2(x, Q^2) - F_L(x, Q^2) \right]$. 

**Figure:** Exact and asymptotic $L_i^{NS}$ contribution to the structure function.

**Figure:** Relative charm quark contribution to the structure function.
The unpolarized Bjorken sum rule

The unpolarized Bjorken sum rule is derived from the first moment of $F_1$

$$\Delta F_1(Q^2) = \int_0^1 dx \left[ F_{1T}^{\tau p}(x, Q^2) - F_{1\nu}^{\nu p}(x, Q^2) \right] = K_1(n_f) A^{F_1}(\alpha_s, Q^2)$$ (14)
The unpolarized Bjorken sum rule

The unpolarized Bjorken sum rule is derived from the first moment of $F_1$

$$\Delta F_1(Q^2) = \int_0^1 dx \left[ F_1^{\nu\mu p}(x, Q^2) - F_1^{\nu\mu p}(x, Q^2) \right] = K_1(n_f) A^{F_1}(\alpha_s, Q^2)$$  \hspace{1cm} (14)

Massive contributions to $A^{F_1}$ start at tree level, due to the $H_1$ Wilson coefficient.

$$A^{F_1} = \left[ 1 - |V_{cd}|^2 \left( C_{uBj}^{Q,(0)} - 1 \right) \right] - \left( \frac{\alpha_s}{\pi} \right) \left[ \frac{2}{3} + |V_{cd}|^2 \left( C_{uBj}^{Q,(1)}(\xi) + \frac{2}{3} \right) \right]$$

$$+ \left( \frac{\alpha_s}{\pi} \right)^2 \left[ -\frac{23}{6} + \frac{8}{27} n_f + C_{uBj}^{Q,(2)}(\xi) \right] + O(\alpha_s^3),$$

$$C_{uBj}^{Q,(2)}(\xi) = C_F T_F \left\{ \frac{1129}{2520} - \frac{1}{2\xi^2} \log \left( \frac{\lambda + 1}{\lambda - 1} \right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda \log \left( \frac{\lambda + 1}{\lambda - 1} \right) \right\}$$

$$\times \left( -\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840} \right) + \left( \frac{1}{6} - \frac{\xi^2}{840} \right) \log(\xi) \right\}$$

(15)
The unpolarized Bjorken sum rule

The unpolarized Bjorken sum rule is derived from the first moment of $F_1$

$$\Delta F_1(Q^2) = \int_0^1 dx \left[ F_1^{\uparrow p}(x, Q^2) - F_1^{\downarrow p}(x, Q^2) \right] = K_1(n_f) A^{F_1}(\alpha_s, Q^2)$$

(14)

Massive contributions to $A^{F_1}$ start at tree level, due to the $H_1$ Wilson coefficient.

$$A^{F_1} = \left[ 1 - |V_{cd}|^2 \left( C^{Q,(0)}_{uBj} - 1 \right) \right] - \left( \frac{\alpha_s}{\pi} \right) \left[ \frac{2}{3} + |V_{cd}|^2 \left( C^{Q,(1)}_{uBj}(\xi) + \frac{2}{3} \right) \right]$$

$$+ \left( \frac{\alpha_s}{\pi} \right)^2 \left[ -\frac{23}{6} + \frac{8}{27} n_f + C^{Q,(2)}_{uBj}(\xi) \right] + O(\alpha_s^3),$$

(15)

The typical features seen in $A^{g_1}$ are recovered:

- absence of logarithmic terms and $n_f \rightarrow n_f + 1$ at $Q^2 \gg m_c^2$,
- negative contributions from virtual diagrams at $Q^2 \approx m_c^2$. 

\[ C^{Q,(2)}_{uBj}(\xi) = C_F T_F \left\{ \frac{1129}{2520} - \frac{1}{2\xi^2} \log \left( \frac{\lambda + 1}{\lambda - 1} \right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda \log \left( \frac{\lambda + 1}{\lambda - 1} \right) \right. \]

\[ \times \left( -\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840} \right) + \left( \frac{1}{6} - \frac{\xi^2}{840} \right) \log(\xi) \right\} \]
Summary and outlook

Neutral current DIS

- We calculate the Wilson coefficients $L_{g_1,q}^{NS}$ exactly at two-loop order and compute the structure function $g_1^{NS}$: relevant deviations from the asymptotic results at $Q^2 \lesssim 10\text{GeV}^2$.
- In a recent work, our group calculated also $L_{2,q}^{NS}, L_{L,q}^{NS}$, thus obtaining the heavy flavor corrections to all the non-singlet structure functions in neutral current DIS to $O(\alpha_s^2)$.
- We study the effect of heavy flavors on the polarized Bjorken sum rule: the massless behaviour is recovered at high scales $Q^2 \gg m_c^2$, effects are smaller (and even negative) at intermediate scales $Q^2 \gtrsim m_c^2$.

Charged current DIS

- We compute all the flavor non-singlet structure functions in charged current DIS to $O(\alpha_s^2)$.
- We studied the unpolarized Bjorken sum rule, finding again the interpolation from low scale, to high scale regime, where $n_f + 1$ quark flavors are effectively massless. In recent work, we also discuss the Gross-Llewellyn Smith sum rule and verify the Adler sum rule.

Outlook

The $O(\alpha_s^2)$ calculation of heavy flavor effects will enter in the analysis of DIS data.
Summary and outlook

Neutral current DIS

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- In a recent work, our group calculated also $L_{2,q}^{NS}$, $L_{L,q}^{NS}$, thus obtaining the heavy flavor corrections to all the non-singlet structure functions in neutral current DIS to $O(\alpha_s^2)$.
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- We studied the unpolarized Bjorken sum rule, finding again the interpolation from low scale, to high scale regime, where $n_f + 1$ quark flavors are effectively massless. In recent work, we also discuss the Gross-Llewellyn Smith sum rule and verify the Adler sum rule.

Outlook

The $O(\alpha_s^2)$ calculation of heavy flavor effects will enter in the analysis of DIS data.
Summary and outlook

Neutral current DIS

- We calculate the Wilson coefficients \( L_{81,q}^{\text{NS}} \) exactly at two-loop order and compute the structure function \( g_1^{\text{NS}} \): relevant deviations from the asymptotic results at \( Q^2 \lesssim 10 \text{GeV}^2 \).

- In a recent work, our group calculated also \( L_{2,q}^{\text{NS}}, L_{L,q}^{\text{NS}} \), thus obtaining the heavy flavor corrections to all the non-singlet structure functions in neutral current DIS to \( \mathcal{O}(\alpha_s^2) \).

- We study the effect of heavy flavors on the polarized Bjorken sum rule: the massless behaviour is recovered at high scales \( Q^2 \gg m_c^2 \), effects are smaller (and even negative) at intermediate scales \( Q^2 \gtrsim m_c^2 \).

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Thank you for your attention!
Neutral current plots: $F_L(x, Q^2)$ and $F_2(x, Q^2)$
Neutral current plots: $g_2(x, Q^2)$
Charged current plots: $F_2(x, Q^2)$ and $F_3(x, Q^2)$

- $F_2(x, Q^2)$
  - Complete, 1000 GeV$^2$
  - Asymptotic, 1000 GeV$^2$
  - Complete, 100 GeV$^2$
  - Asymptotic, 100 GeV$^2$
  - Complete, 10 GeV$^2$
  - Asymptotic, 10 GeV$^2$

- $F_3(x, Q^2)$
  - Complete, 1000 GeV$^2$
  - Asymptotic, 1000 GeV$^2$
  - Complete, 100 GeV$^2$
  - Asymptotic, 100 GeV$^2$
  - Complete, 10 GeV$^2$
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